

Application of Block LU Simulation with an Approximate Model of Coregionalization

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*The simulation of multiple variables using the Lower –Upper (LU) decomposition method is practical for a local area. The Block LU simulation with an approximate model of coregionalization (AMC) algorithm is proposed under the multi-Gaussian assumption. This methodology overcomes the cumbersome task of modeling the linear model of coregionalization (LMC). Only direct variograms and correlation between variables is required when doing approximate modeling. An application with the GSLIB like program *blusim_AMC* demonstrates the utility of the method.*

Introduction

Geostatistical simulation methods are used to quantify the spatial uncertainty in the attributes of natural phenomena of interest be they characteristics of mineral deposits or petroleum reservoirs. Although methods for simulating a single attribute are generally efficient, the usefulness of the existing methods for modeling correlated multivariables spatial phenomena is limited in practice (Alexander, 2008). There are many methods for joint simulation, for example the Sequential cosimulation, which is limited only one secondary variable. The LU matrix approach was first proposed by Davis (1987). Myers (1988) extends the conditional univariate LU decomposition for joint simulation of two variables. Verly (1933) proposed a method for joint simulation of two variables. Boisvert (2007) provides a method based on the local joint simulation using LU with several variables.

For joint simulation several variables, the current LU algorithm requires a positive definite linear model of coregionalization, in this paper, the new method Approximate Model of coregionalization (AMC) was proposed, it overcome the need to fit the cross variogram. It only needs to fit the direct variogram models from data. A program, *Blusim_AMC*, will be shown that only the direct variables variograms and the correlation matrix of the variables are needed do define the cross variogram model. Software implementation is illustrated by a case study of a nickel deposit with three variables of interest.

Review of Methodology

The LU unconditional method for one variable is as follows:

- Transform the data to a standard Gaussian distribution
- Calculate and model the variogram
- Compute covariance matrix C for the location in the search neighborhood
- Decompose the covariance via the Cholesky decomposition into a lower and upper triangular matrix, where the upper triangular matrix is the transpose of the lower triangular matrix $C = LU$ where $L^T = U$
- Now consider a random vector $y = LW$ where w is a vector of independent N (0, 1) distributed random numbers
- Back-transform the simulated values to original units.

If the local transformation option is considered, then the local data are transformed to a standard Gaussian distribution and at last the back-transform is done by that local transformation table. The bigger search radius will search all the data following in the radius for transformation. The max kriging option will only using the most relevant data to simulate. So then the high value area will have higher simulated values than global transformation options.

Multivariate block LU simulation is obtained by using the LMC. The method is developed for the joint simulation of multiple data locations. The covariance matrix of data location the grid to be generated and the partition matrix as following:

$$C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

Where C_{11} is the covariance and cross-covariance between data points, C_{12} is covariance and cross-covariance between search data points and grid discretization points. The C_{22} is the covariance and cross-covariance matrix between discretization location. Each sub matrix in C_{11}, C_{12}, C_{22} , contains the covariance among the v, u, and w data and cross covariance between v, u and w.

Linear model of coregionalization is the most extensively used model for joint spatial simulation. The LMC for the two random variables is given by the following (Deutsch, 2002).

$$\gamma_U(h) = u_0\gamma_0(h) + u_1\gamma_1(h) + \dots + u_m\gamma_m(h)$$

$$\gamma_V(h) = v_0\gamma_0(h) + v_1\gamma_1(h) + \dots + v_m\gamma_m(h)$$

$$\gamma_{UV}(h) = w_0\gamma_0(h) + w_1\gamma_1(h) + \dots + w_m\gamma_m(h)$$

Each of them has m nested structures. For all of the variogram, they all have the same variogram nested structures, but the constraints they have to obey are:

$$u_j > 0 \text{ and } v_j > 0$$

$$u_j v_j > w_j w_j$$

For all $j=0, \dots, m$

The new algorithm gets the cross variogram model between the variables and correlation matrix. That is obtained by multiply correlation coefficient and the geometric average of the two direct variogram.

$$\gamma_{k,k'} = \rho_{k,k'} \sqrt{\gamma_k * \gamma_{k'}}$$

The other option is obtained by multiply correlation coefficient and the arithmetic average of the two direct variogram.

$$\gamma_{k,k'} = \rho_{k,k'} \cdot \frac{\gamma_k + \gamma_{k'}}{2}$$

Here, we recommend the first one and the following case study are also used by this method. The geometric average is always less than or equal to the arithmetic mean and this is more stable than the arithmetic average. By using the AMC method, the disadvantage is that it will not assure that the big correlation C matrix will be positive definite. If found C is not positive definite, there will be correcting scheme that a small number will be divided for the off-diagonal. 0.05 is chosen for the constant as the diagonal of C matrix is 1. In the fowling case there are no corrections found in the simulation process.

Implementation:

BLUSIM_AMC is a GSLIB like FORTRAN program that requires an input parameter file. See the following for detail.

- Information about the data (line 6-10). Name of the data file, columns for the coordinates X, Y, Z ant the variables ID, variables and the weights. The variables ID must be indicated in the data file. The number of variables for line 8 must equal to the highest ID of the variables in the data file.
- Correlation information and trimming options (line 10-11). Line 10 is the upper diagonal correlation matrix of among the variables. Line 11 is the trimming options
- Transformation options (line 12-19). Line 12 and 13 is the option weather to transform the data and the output file. If consider the transformation of local data then the minimum number of data is needed. Line 14 to 16 is weather to consider a transformation table. This is in one file with each column represents different variables.
- Debugging options and output file (line 20-22). This option allows the user to check the calculation, but if set higher debugging level; the debugging file could be large. Line 22 is the name of output file.
- Parameters for simulation grid and discretization (line 23-28). Line 23 is number of realizations, Line 24-26 is the gird definition. Line 27 is number of discretization points in the panel; Line 28 is the

number of Selective Mining Unit in the panel. Note that number of discretization points divided by number of SMU should be a whole number.

- Parameters for cutoff report (line 29- 34). Line 29 and 30 is number to cutoffs and value of cutoffs. Line 31 is the option to consider equivalent grade, which requires a factor for each data simulated. Line 32 is the option weather to write out all SMU realizations, which could be large if the realization is large.
- Seed for random number (line 35)
- Search information (line 36-39). Line 36 is the min and max data used in simulation. Line 37 is whether to consider octant search. Line 38 is the search radii.
- Variogram model parameters (line 40-end). These are only the direct variogram models for the data.

Example:

A case study will show the methodology. We consider using Ni, Fe and MgO as our test data. Before doing the simulation some preprocess of the data should be done. For each location there should be 3 variables, if there are some locations that have only one or two variables information, the third one should also be set to 0 and not be trimmed by the simulation process. The location map (Figure 1) shows the pattern of the drill well, and in the left bottom edge and the center of deposit the Ni and Fe is high, while in the right corn the MgO is high. The univariate statistics for the variables is presented in Figure 2. The scatter plots are shown in Figure 3 and summarized in Table 1. From the table, we can see that the relationship between Ni and Fe, and Ni and MgO is not very high, while Fe and MgO have a relative high negative correlation.

In Figure 4, the experimental Omni-directional variogram and cross variogram for variables Ni Fe and MgO are computed and presented. We use the program varfit_lmc to fit these variograms and cross variograms. And a positive definite Linear model of coregionalization is the following:

Variograms:

$$\gamma_{Ni}(h) = 0.01 + 0.751Exp_{a=37.838} + 0.248Exp_{a=204.47}$$

$$\gamma_{Fe}(h) = 0.01 + 0.737Exp_{a=37.838} + 0.262Exp_{a=204.47}$$

$$\gamma_{MgO}(h) = 0.01 + 0.223Exp_{a=37.838} + 0.776Exp_{a=204.47}$$

Cross variograms:

$$\gamma_{Ni-Fe}(h) = 0.01 + 0.066Exp_{a=37.838} + 0.234Exp_{a=204.47}$$

$$\gamma_{Ni-MgO}(h) = 0.01 + 0.084Exp_{a=37.838} + 0.042Exp_{a=204.47}$$

$$\gamma_{Fe-MgO}(h) = 0.01 - 0.388Exp_{a=37.838} - 0.137Exp_{a=204.47}$$

For the Approximate model only the variograms are needed to modeled, no cross variogram models are need. So, we model the three variograms separately, which are shown in Figure 5. The variogram models are:

$$\gamma_{Ni}(h) = 0.01 + 0.726Exp_{a=35} + 0.273Sph_{a=200}$$

$$\gamma_{Fe}(h) = 0.01 + 0.710Sph_{a=30} + 0.289Sph_{a=200}$$

$$\gamma_{MgO}(h) = 0.01 + 0.510Sph_{a=35} + 0.489Sph_{a=200}$$

Let's only take the cross variogram between Fe and MgO for example, Figure 6 shows that how AMC model is close to the LMC model. After doing the modeling work, no corrections in the big C matrix are found in the block simulations for this case. The comparative results are based on blusim_mv using the LMC model (Jeff 2008). Figure 6 shows that the block averaged simulation scatter plots comparison using AMC and LMC for Ni, Fe and MgO. The correlations are all very high. The AMC estimate mean is changing a little smaller, but the shape of the estimate using the AMC is approximately unchanged. Maps of the estimates confirm the similarity in the two methods

The local transformation application was also considered with this program. When set the local transformation option, a larger search radius should be given because a small search radius will results in smaller number of local transformation table. A proper minimum locally transformation data also should be reasonable, high minimum data will leave lots of area unsimulated, low minimum data will results the transformation table not stable. In this test case, we choose 30 as the minimum data to use.

Conclusion

In this paper a new local LU simulation using approximate model of coregionalization methodology is proposed, which uses only the direct variables and the correlation matrix between the variables. By using the geometric average approximate model, the simulation result are similar with the LMC model. A FORTRAN program called Blusim_AMC was prepared for performing block multivariable LU simulation. Although no change in the C matrix has been found, it does not assure that other case will not. Future work will incorporate other features into this methodology.

References

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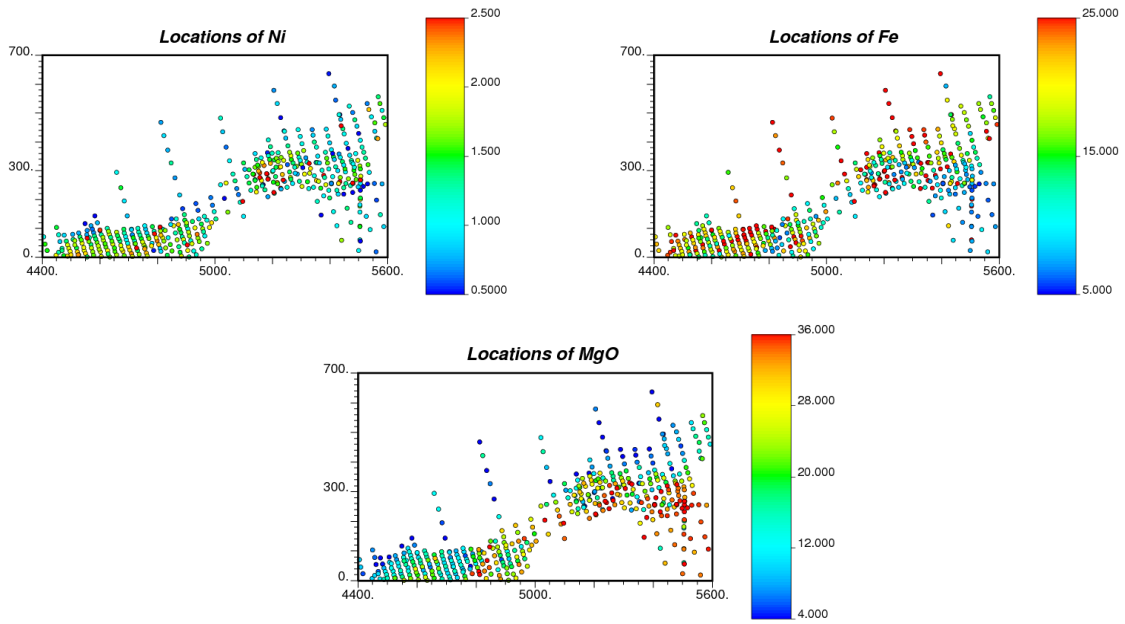


Figure 1. Location maps for the three variables.

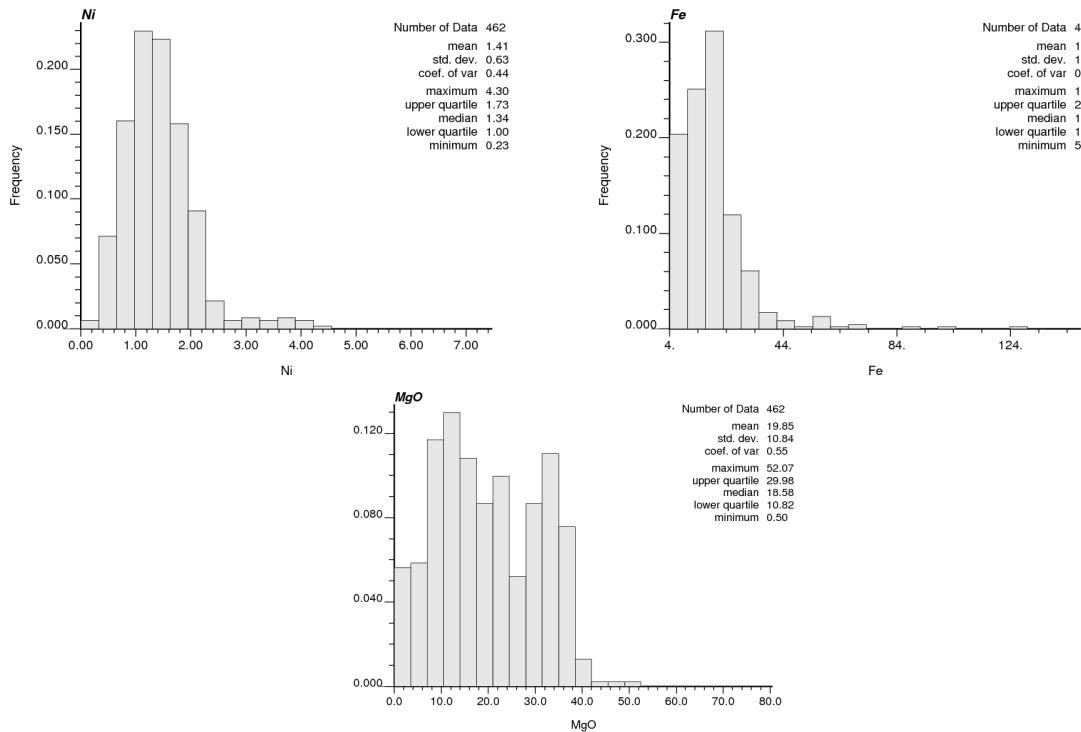


Figure 2. Histograms for the three variables.

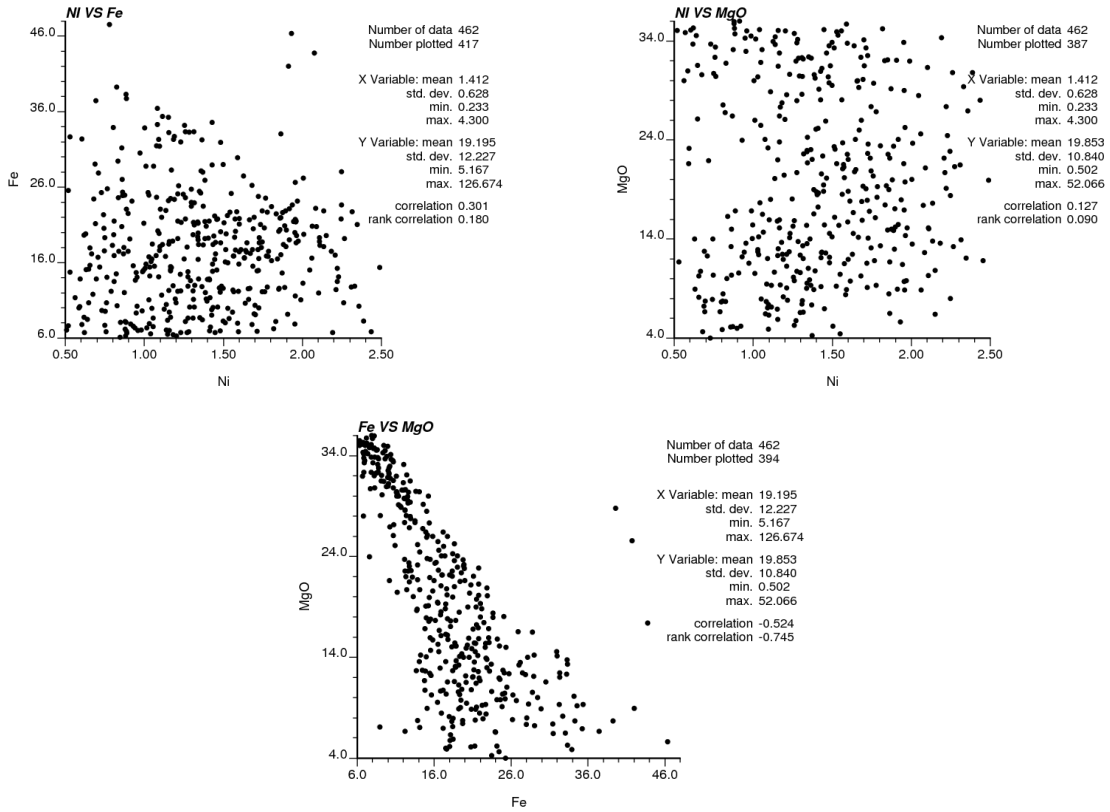
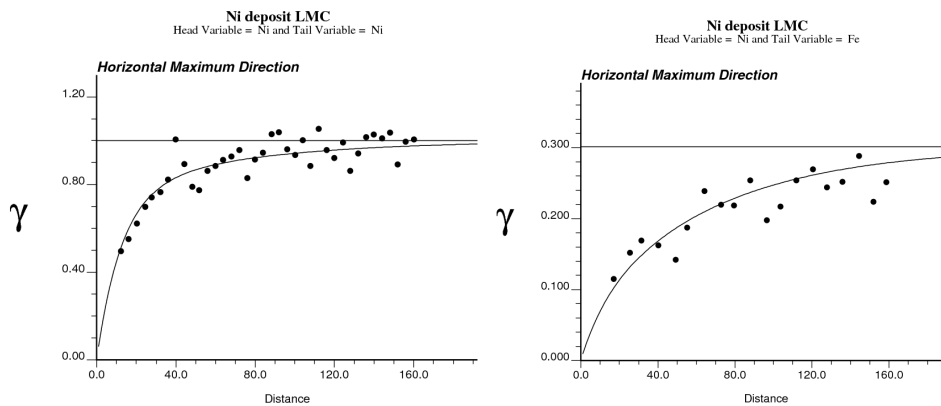


Figure 3. scatter plots of the three variables.

1	0.301	0.127
0.301	1	-0.524
0.127	-0.524	1

Table 1. Correlation between variables.



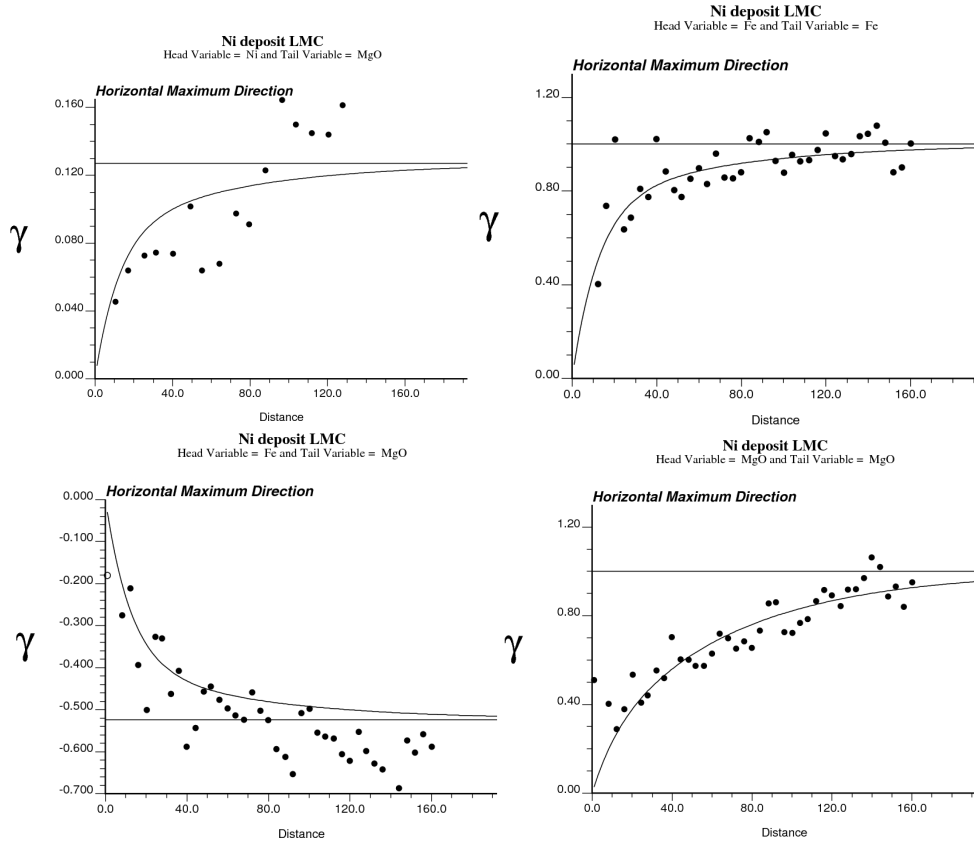


Figure 4. Ni, Fe and MgO variogram and cross variogram fitted by LMC.

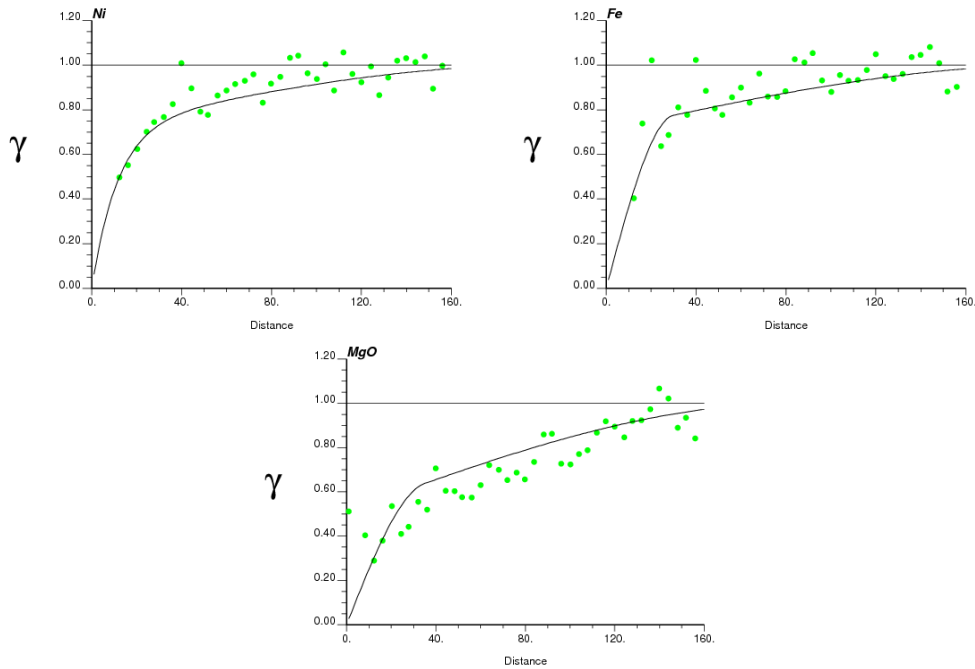


Figure 5. Ni, Fe and MgO variogram model fitted separately.

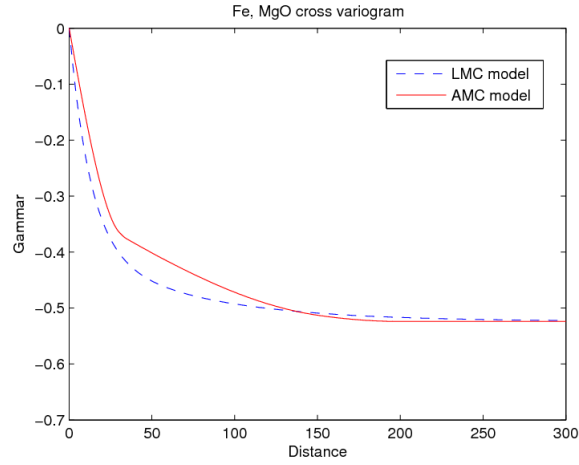


Figure 6. Comparative Fe, MgO cross variogram model between LMC and AMC

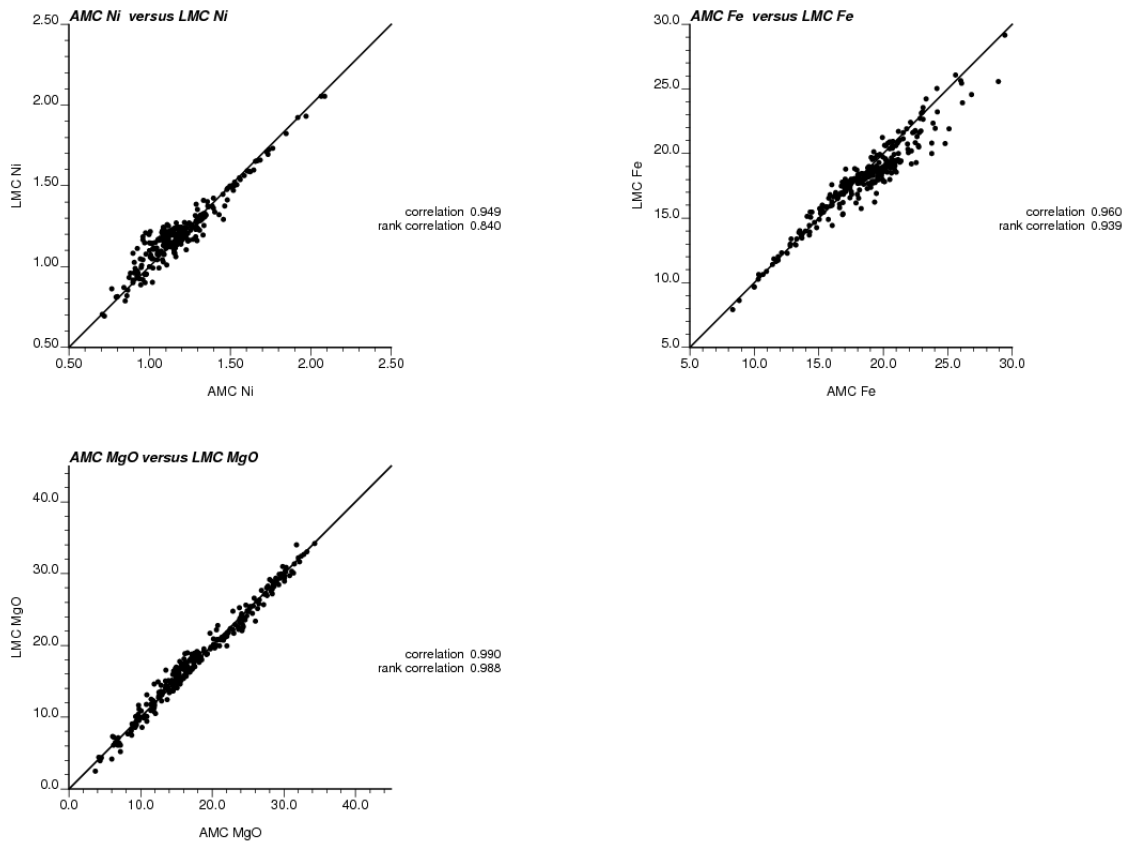


Figure 7. Block averaged simulation results comparison using AMC and LMC for Ni, Fe and MgO.