A Color Scale for Ternary Mixtures

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The purpose of this paper is to introduce different ways of plotting the ternary proportions of a facies model. The ternary facies model is plotted using an equilateral triangle. The mixing of three facies is plotted using the rule of mixing colors (red, green and blue). The visualization of the mixing of three facies proportions is shown using the color theory. As a comparison, the inverse distance coloring algorithm is also implemented and performed.

Introduction

The definition of facies in reservoir characterization and geostatistical modeling applications is common for better capturing the reservoir heterogeneity. The number of defined facies is different based on the application. For geostatistical modeling application the number of facies is limited to four or five. Geostatistical facies modeling with more than four or five facies is not usually performed. For reservoir engineering applications usually three facies is enough. Three facies might be defined as low permeability rocks (shale, barriers), medium permeability rocks (shaley sand) and high permeability rocks (sand, conduits). Binary and ternary mixtures can be plotted on two dimensional surfaces. Facies models with more than three facies cannot be plotted on two dimensional surfaces.

Ternary Plot

Since the summation of facies proportion at any location in the reservoir is unity therefore the three facies proportion models can be plotted on ternary plot. A ternary plot is drawn as an equilateral triangle. The vertices represent the three facies. The three sides represent the three binary combinations of three facies. Each vertex of the triangle represents a proportion of 100 % with the base of the equilateral triangle opposite that vertex representing the proportion of 0 %. The constant proportion lines for each facies are parallel to the 0 % proportion base (side of equilateral the triangle). Cartesian coordinates are useful for plotting points in the triangle. Suppose that we have three facies of S_1, S_2 and S_3 with proportions of p_1, p_2 and p_3 respectively. If $p_1 = 100\ \%$ is located at $(x, y) = (0,0)$ and $p_3 = 100\ \%$ is placed at $(x, y) = (1,0)$ then based on the trigonometry $p_2 = 100\ \%$ must be placed at $(x, y) = (0.5, 0.5\sqrt{3})$. Based on the Cartesian coordinates of three vertices the triple (p_1, p_2, p_3) is located at $(p_3 + 0.5p_2, 0.5\sqrt{3}p_2)$. Figure 1 shows the representation of some points on a ternary plot.

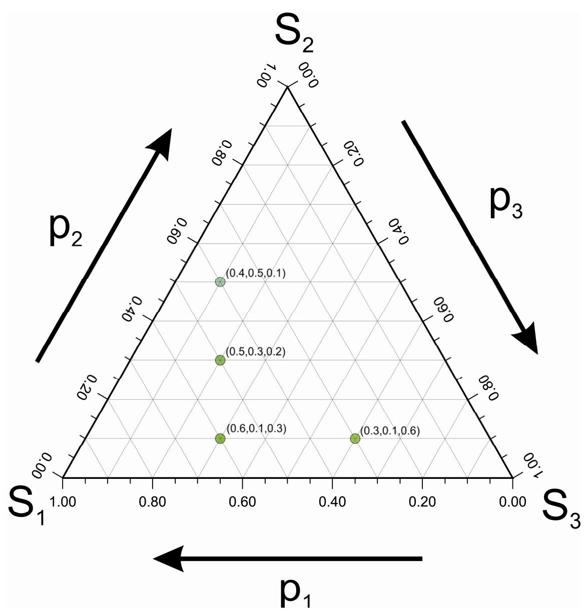

Figure 1 plotting facies proportion points on ternary plots

Indexing in Ternary Plot

Each side of the equilateral triangle in Figure 2 is divided into N intervals. The total number of small unit equilateral triangles that can be is N^2. The indexing of the small triangles are performed in two different ways (1) one dimensional indexing, $k = 1, \dots, N^2$; in this type of indexing the numbering is started from the left lower (vertex S_1) small triangle. The numbering is performed from left to the right in each row and when the entire row is numbered, the numbering is shifted to the upper row and continued from left to right. (2) Two dimensional indexing, (i, j), j is the row index, $j = 1, \dots, N$, and i is the index for the small triangles that are located in each row from left to right, $i = 1, \dots, (2N - 2j + 1)$. The relations between i, j and k are written below:

$$k = i + (j-1)\cdot(2N - j + 1) \;\; \Leftrightarrow \begin{cases} i = k - (j-1)\cdot(2N - j + 1) \\ j = N - int\left(\sqrt{N^2 - k}\right) \end{cases}$$

$$i = 1, \dots, (2N - 2j + 1) \;\; \& \;\; j = 1, \dots, N \;\; \& \;\; k = 1, \dots, N^2$$

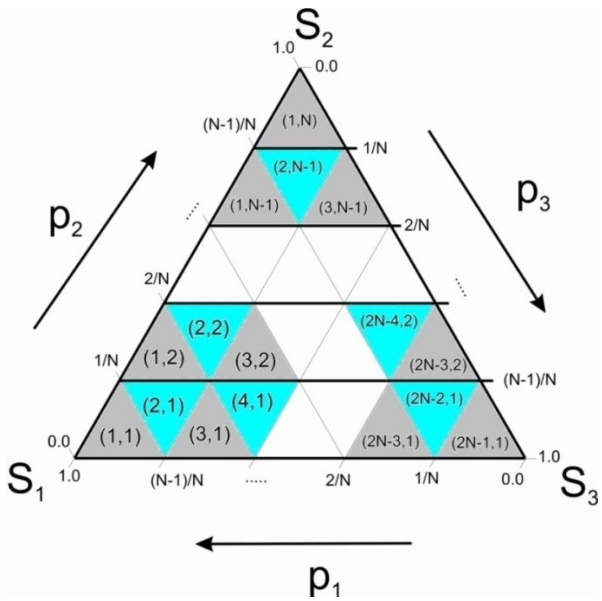

Figure 2 indexing the ternary plot

The centroid of each small unit equilateral triangle with the index of (i, j) has the ternary coordinate of $(p_1, p_2, p_3)_{(i,j)}$. The transformation from Cartesian coordinates to ternary coordinates is:

$$\left.\begin{array}{r} S_1\colon (1,0,0) \equiv (0,0) \\ S_2\colon (0,1,0) \equiv \left(\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right) \\ S_3\colon (0,0,1) \equiv (1,0) \end{array}\right\} \;\; \Longrightarrow \;\; (p_1, p_2, p_3)_{(i,j)} \equiv \left(p_3 + \frac{p_2}{2}, \frac{p_2\sqrt{3}}{2}\right)_{(i,j)}$$

The ternary coordinate $(p_1, p_2, p_3)_{(i,j)}$ can be expressed as a function of i, j and N:

$$i = 1, \dots, (2N - 2j + 1) \;\; \& \;\; j = 1, \dots, N$$

$$(p_1, p_2, p_3)_{(i,j)} = \left(\frac{6N - 6j - 3i + 4 + i(mod)2}{6N}, \frac{6j - 2 - 2 \times i(mod)2}{6N}, \frac{3i - 2 + i(mod)2}{6N}\right)$$

The *mod* function is used to identify the regular triangles (i is odd or $i(mod)2 = 1$) and the upside-down triangles (i is even or $i(mod)2 = 0$) .

Using the ternary coordinates of the centroids of the triangles and whether it is regular (i is odd) or upside-down (i is even) the coordinates of three vertices of small triangle with index of (i,j) are determined:

$$Centroid: \ (p_1, p_2, p_3)_{(i,j)}$$

$$Vertix\ 1: \left(p_1 - \frac{(-1)^{i(mod)2} \times 2}{3N}, p_2 + \frac{(-1)^{i(mod)2}}{3N}, p_3 + \frac{(-1)^{i(mod)2}}{3N}\right)_{(i,j)}$$
$$Vertix\ 2: \left(p_1 + \frac{(-1)^{i(mod)2}}{3N}, p_2 - \frac{(-1)^{i(mod)2} \times 2}{3N}, p_3 + \frac{(-1)^{i(mod)2}}{3N}\right)_{(i,j)}$$
$$Vertix\ 3: \left(p_1 + \frac{(-1)^{i(mod)2}}{3N}, p_2 + \frac{(-1)^{i(mod)2}}{3N}, p_3 - \frac{(-1)^{i(mod)2} \times 2}{3N}\right)_{(i,j)}$$

Figure 3 shows the way of calculating the ternary coordinates of three vertices using the coordinates of centroid for the regular case.

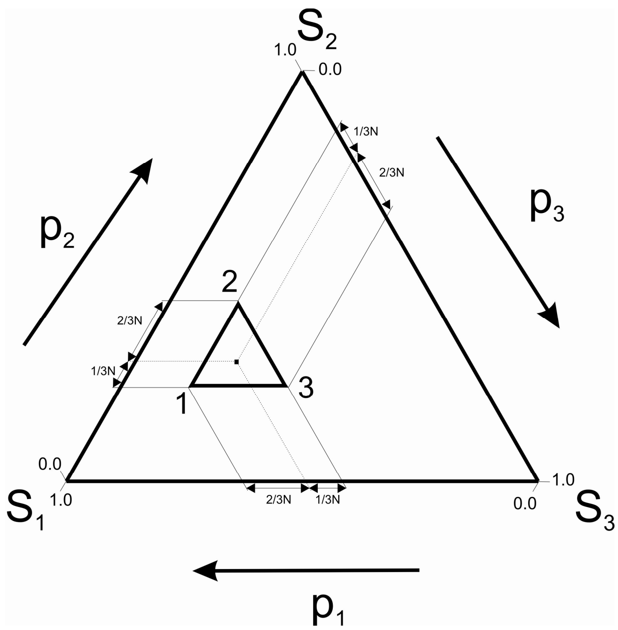

Figure 3 the small regular triangle with index of (i,j) along with its centroid and vertices

Plotting and Coloring the Mixtures

Two different approaches are presented to display the mixture of three facies using three fundamental colors of red, green and blue. The first approach is based on the inverse distance approach. The color of each point inside the ternary plot with the index of (i,j) is obtained using the four points of red (facies S_1), green (facies S_2), blue (facies S_3) and white (the global proportion point). The color value for each point is obtained using the inverse distance estimation using the four known points (Figure 4). In RGB color space each color is a combination of three colors of red, green and blue. For estimating the color each of three components are estimated independently. The color of the point (p_1, p_2, p_3) is denoted by (r, g, b). The simplified formulas for the inverse distance coloring approach are:

$$r = \frac{\frac{1}{d_R^2} + \frac{1}{d_W^2}}{\frac{1}{d_R^2} + \frac{1}{d_G^2} + \frac{1}{d_B^2} + \frac{1}{d_W^2}}$$

$$g = \frac{\frac{1}{d_G^2} + \frac{1}{d_W^2}}{\frac{1}{d_R^2} + \frac{1}{d_G^2} + \frac{1}{d_B^2} + \frac{1}{d_W^2}}$$

$$b = \frac{\frac{1}{d_B^2} + \frac{1}{d_W^2}}{\frac{1}{d_R^2} + \frac{1}{d_G^2} + \frac{1}{d_B^2} + \frac{1}{d_W^2}}$$

Where d_B, d_G, d_R and d_W are the distance from four known points of red (facies S_1), green (facies S_2), blue (facies S_3) and white (the global proportion point) respectively to the point of interest and are calculated using below formulas. The global proportion point (white) is denoted by $\left(p_{1g}, p_{2g}, p_{3g}\right)$.

$$d_R^2 = \left(p_3 + \frac{p_2}{2}\right)^2 + \frac{3}{4}p_2^2$$

$$d_G^2 = \left(p_3 + \frac{p_2 - 1}{2}\right)^2 + \frac{3}{4}(p_2 - 1)^2$$

$$d_B^2 = \left(p_3 + \frac{p_2}{2} - 1\right)^2 + \frac{3}{4}p_2^2$$

$$d_W^2 = \left(p_3 - p_{3g} + \frac{p_2 - p_{2g}}{2}\right)^2 + \frac{3}{4}\left(p_2 - p_{2g}\right)^2$$

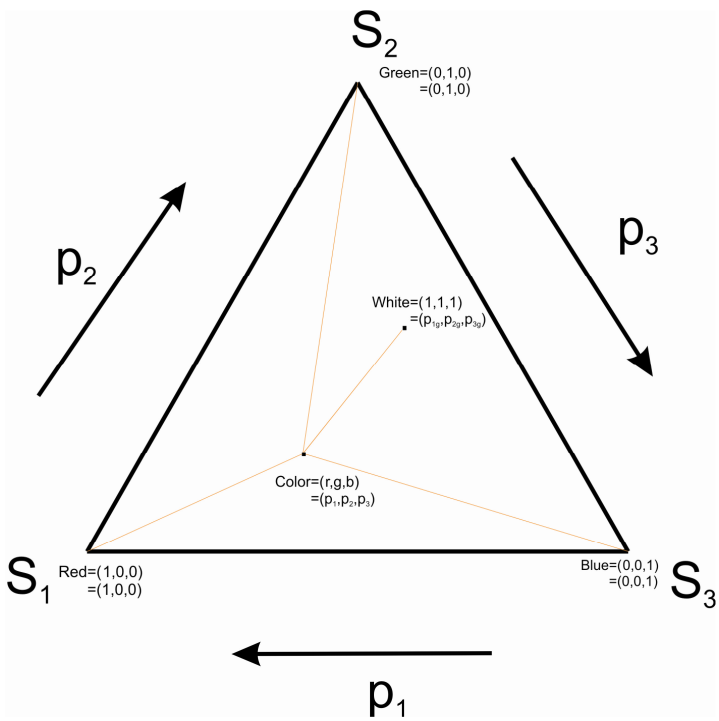

Figure 4 inverse distance coloring approach

The second approach of coloring the ternary mixtures is based on the chromaticity diagram (Fairman 1997) which is well established in optics and physics. The CIE (International Commission on Illumination) color space is used to define the color of ternary mixture. The color of mixtures in this case is close to reality for the human eyes (Fairman 1997). Figure 5 shows the IE color space in two dimensional space.

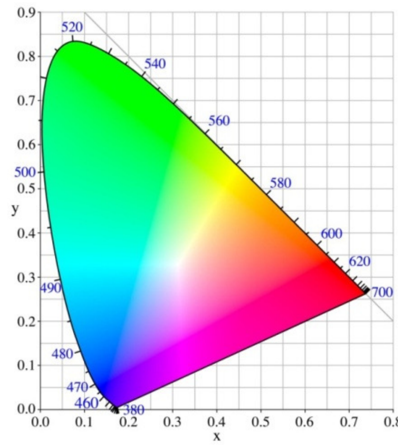

Figure 5 CIE chromaticity diagram of RGB color space (http://en.wikipedia.org/wiki/Gamut)

There are lots of literature and articles for defining such color spaces in optical physics science. Here is just a summary of transforming from proportion to RGB colors with some modifications. The modification is to change the white point (global proportion point) and to color the triangle based on the white point and CIE approach. CIE approach states that the RGB values of a color is a result of a multiplication of a data derived 3x3 matrix and a 3x1 matrix of three facies proportion (Fairman 1997):

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} = \begin{bmatrix} 2.3649 & -0.8969 & -0.4679 \\ -0.5153 & 1.4265 & 0.0887 \\ 0.0052 & -0.0142 & 1.0091 \end{bmatrix} \times \begin{bmatrix} \frac{p_1}{p_{1g}} \\ \frac{p_2}{p_{2g}} \\ \frac{p_3}{p_{3g}} \end{bmatrix}$$

The data derived 3x3 matrix is based on Wright-Guild lab data (Fairman 1997).

Example

A three dimensional facies model is considered with three global proportions of 0.34, 0.42 and 0.24. The field size is 256x256x128. The variogram model for the three categorical variables is as below:

$$\gamma_{facies\ 1}(\boldsymbol{h}) = \gamma_{facies\ 2}(\boldsymbol{h}) = \gamma_{facies\ 3}(\boldsymbol{h}) = Sph_{\substack{a_{h-max=128} \\ a_{h-min=128} \\ a_{vert=64}}}(\boldsymbol{h})$$

The global proportions are calculated and plotted on ternary diagrams using the two algorithms. Figure 6 shows a view of the facies model. The histograms for three facies are shown in Figure 7.

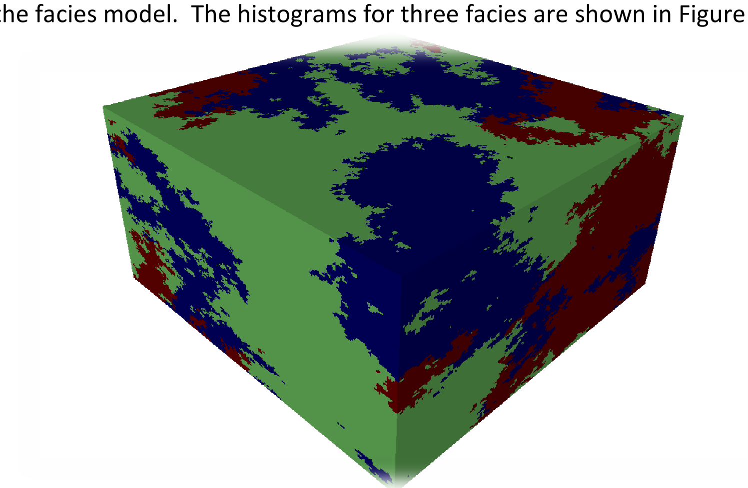

Figure 6 a geostatistical facies model with three facies

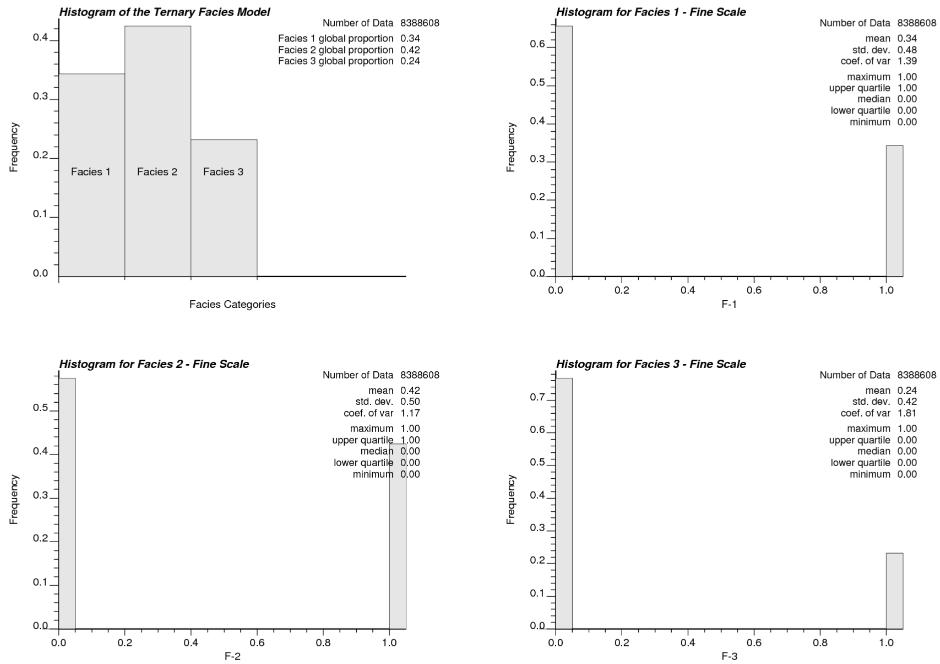

Figure 7 the histograms for three different facies

The ternary mixture plots for this facies model with specified global proportions are shown in Figure 8 and Figure 9 with inverse distance and CIE coloring approaches respectively. The ternary plot is equivalent to the histograms of three facies. Both the inverse distance and CIE coloring approaches are used to show the ternary coloring.

Conclusion

An algorithm is developed to plot the ternary mixtures with color scale. The inverse distance coloring approach uses four known points of three facies and global proportions point to color the ternary plot. The CIE approach uses a 3x3 matrix which is derived from experiments in optical physics to color the ternary plot.

References

1. Deutsch, C.V. and Journel, A.G., 1998: *GSLIB - Geostatistical software library and users guide*. Oxford University press, 2nd Edition.
2. Fairman, H.S., Brill, M.H. and Hemmendinger, H., *How the CIE 1931 Color-Matching Functions Were Derived from Wright-Guild Data, COLOR Research and Applications, Volume 22, Number 1,* February 1997
3. http://en.wikipedia.org/wiki/Gamut

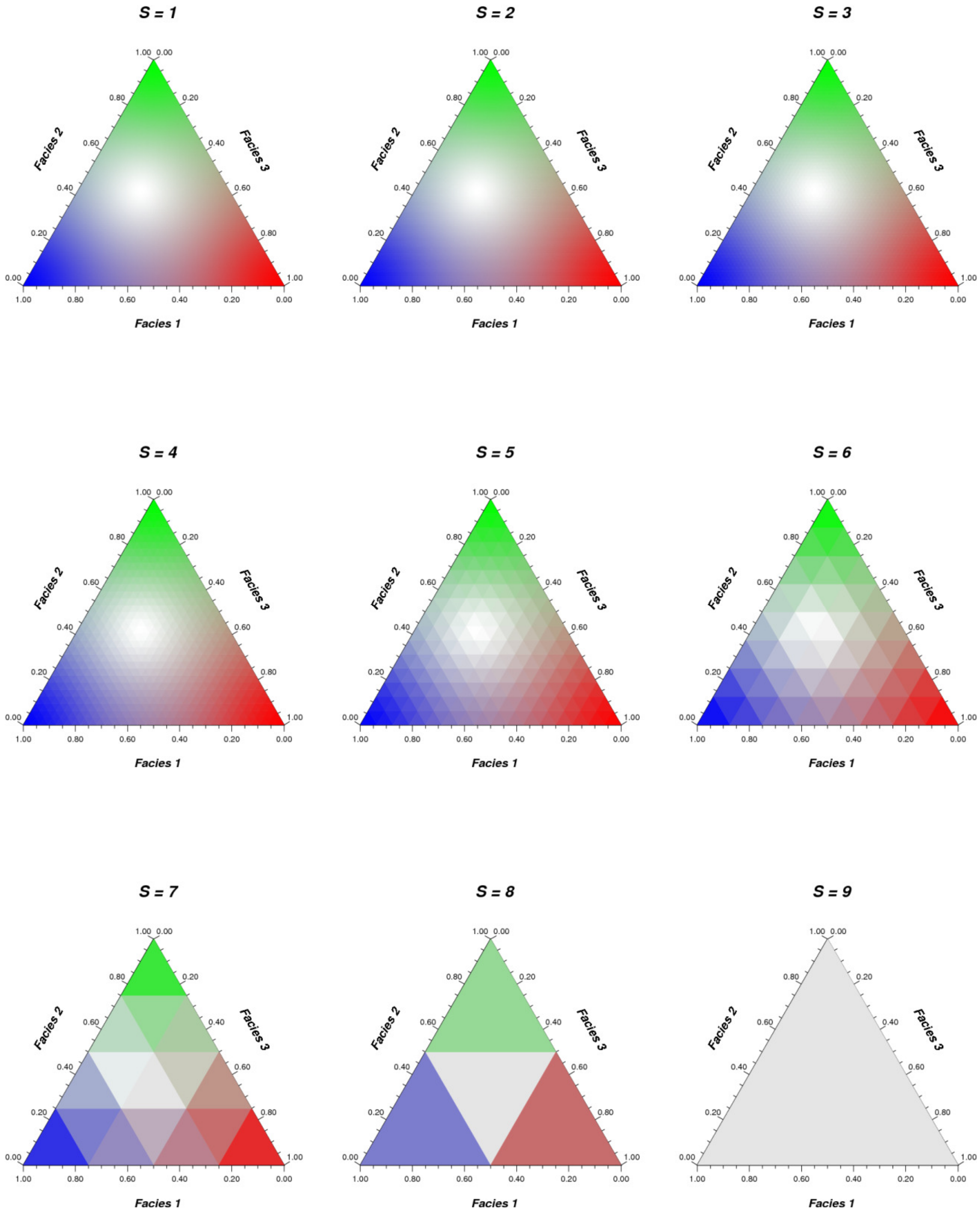

Figure 8 ternary color mixture plots using inverse distance coloring method the three facies model with global proportions of 0.34, 0.42 and 0.24 for different scales; the top left plot is for very fine scale model and the bottom right plot is for very large scale model

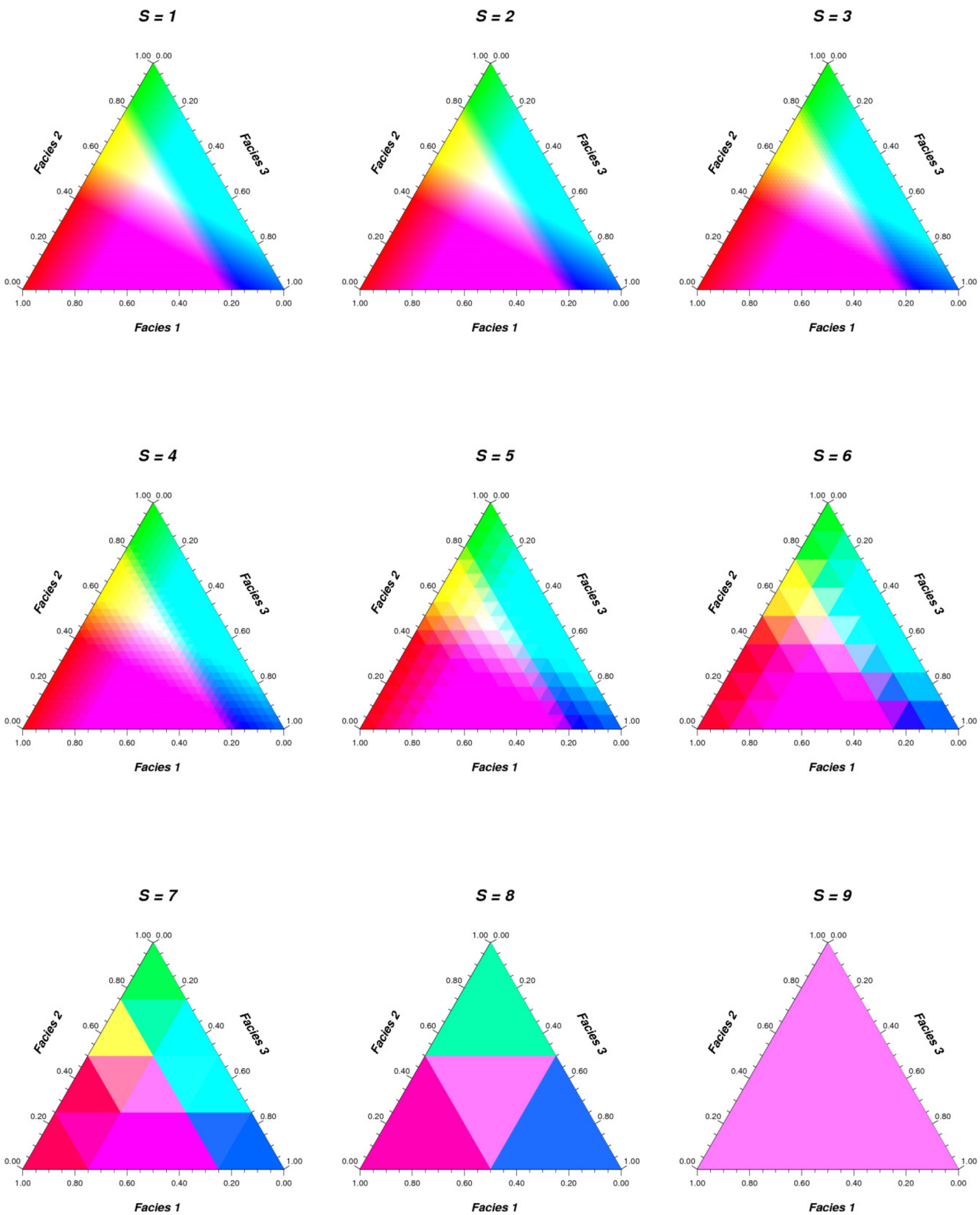

Figure 9 ternary color mixture plots using CIE coloring method the three facies model with global proportions of 0.34, 0.42 and 0.24 for different scales; the top left plot is for very fine scale model and the bottom right plot is for very large scale model