Paper 104, CCG Annual Report 12, 2010 (© 2010)

Kriging and SGS with Multi-gridding and Locally Varying Anisotropy (LVA)

J.B. Boisvert

Consideration of nonlinear or nonstationary features in geostatistical models is important. Often techniques to incorporate these features have excessive computer memory and time requirements. The proposed methodology is to generate a coarse scale model with order(s) of magnitude fewer blocks that considers the nonlinear features. This coarse scale model is then used as point data input in a second step where the final model is built. The final model is built with traditional second order stationary kriging or sequential Gaussian simulation which has lower computer resources demand. The LVA features are reproduced due to the high level of conditioning data supplied by the initial coarse scale modeling with LVA. This methodology is suggested only when there are insufficient computer resources to directly apply the modeling methodology (i.e. LVA kriging or multiple point statistics) on the desired fine scale grid.

1. Introduction

The incorporation of locally varying anisotropy (Figure 1) into geostatistical modeling has been a theme of research for a number of years. A practical methodology to krige or simulate with LVA is presented in Boisvert and Deutsch (2009) or Boisvert (2010); the associated run times and CPU requirements are normally acceptable but extremely large grids do have high RAM demands. The main benefit to modeling is the incorporation of nonlinear geologically realistic features into numerical models as demonstrated by the synthetic examples in Figure 2. The methodology is summarized by Figure 3 and contains the following 5 steps:

Step 1: Generate the LVA field (paper 103, CCG report 11).
Step 2: Calculate the initial distance matrix between points with the Dijkstra algorithm (paper 110, CCG report 10).
Step 3: Perform MDS to embed all cells in a high dimensional Euclidean space (paper 111, CCG report 10).
Step 4: Model an isotropic variogram. This variogram is used to obtain the covariance between locations given the shortest path distance.
Step 5: For every grid cell:
  a. Determine the nearest *n* neighbors
  b. Calculate the required *n* by *n* distance matrix
  c. From the *n* by *n* distance matrix, calculate the covariance matrix using the modeled variogram
  d. Solve the resulting system of equations to determine weights for each datum
  e. Calculate the kriging mean and error variance

Simulation is performed by drawing random residuals as each location is visited in step 5.

Incorporating the types of features shown in Figure 2 is difficult with traditional two point geostatistical methods based on the variogram, this is a clear benefit of the methodology. The main drawback of the LVA methodology is the memory requirements for very large models. There are two instances in the LVA methodology where memory requirements peak, the first is during the determination of the shortest path distance (Figure 4 left) the second is during simulation when the coordinates of each grid location in *q* dimensions must be stored in addition to the realizations (Figure 4 right). This article proposes a multi-gridding approach to reduce the CPU requirements of both memory peaks (Figure 4). First an initial kriging/simulation model is generated on a coarse grid that considers LVA. This coarse model is used as input data to a traditional kriging/simulation with a globally stationary covariance function.

2. Methodology

The methodology considers kriging/simulation on a coarse grid (i.e. larger block size than is required in the final model). This coarse model uses the underlying LVA field and captures the necessary features. As discussed in the next paper in this report (paper 105, CCG report 12) the scale of the anisotropic features is important. The coarse block size must be selected such that the LVA features of interest are captured. Once the coarse model is completed, the model is used as input to a traditional kriging/simulation methodology that does not consider LVA (Figure 5). Because of the dense data coming from the coarse scale model, the final fine scale model contains the LVA features of interest. Overall, the methodology is:

1) Model the LVA field for the features of interest at a coarse resolution.
2) Krige/simulate a coarse scale model of the property of interest (say at a 5 unit resolution)
3) Using the coarse scale model from step 2 as input data, krige/simulate at the fine scale (say at a 1 unit resolution) using traditional kriging/SGS programs that have very low RAM requirements.

The purpose of this methodology is to allow for the generation of models with a large number of cells on computers that are RAM limited. This methodology is not recommended if there is sufficient RAM capacity to simply use the methodology presented in Boisvert (2010) directly. There is error introduced by not kriging/simulating at the final fine scale resolution. This error is assessed for the 2D case study presented in Boisvert (2010).

3. Example: Net Continuous Bitumen (NCB)

From Boisvert (2010):

> *The data used in this case study can be obtained from www.ercb.ca. The data was used by the Energy Resources Conservation Board (ERCB) to determine which gas pools are in pressure communication with bitumen for the assessment of risk associated with extracting hydrocarbons with steam assisted gravity drainage (Warren 2003). The data contains the thickness of a number of oil bearing stratigraphic layers. The thickest and most significant layer is the McMurray Channel which contains a significant proportion of the hydrocarbon resource in the region.*

See Boisvert (2010) for details on the generation of the LVA field for the NCB data.

The desired fine scale final grid has a 50m cell size and ~8M grid blocks (2300x3600). In the implementation of kriging with LVA the Dijkstra algorithm is implemented with an offset=2. Increasing the offset allows for more accurate shortest paths in the methodology but requires more memory (Boisvert, 2010) than using a single offset, see Figure 4. As such, the LVA kriging for the fine scale grid requires ~26GB of RAM. Some computers may not contain 26GB of RAM, thus the proposed methodology is implemented with a coarse grid of size 200m with ~500,000 blocks and requires 1.7 GB of RAM. This coarse grid is used as input data to a traditional kriging which assumes second order stationarity. The LVA features in the final model are reproduced (Figure 7) due to the conditioning data supplied by the coarse scale model.

There will be some error when considering a coarsening of the grid (Figure 7). Consider the error histograms with an increasing level of coarsening. In this example, the grid can be coarsened up to 200m with minimal error; also note that the majority of the error is located in areas near the perimeter of the modeling domain where there is sparse data and the models are not likely to be used. The apparent additional bias with the 1000m regridding (average error of -0.6924m) is due to extrapolation into the boundaries around the main modeling domain and is not a concern unless the models are to be used to extrapolate in these areas. The magnitude of error depends on the level of coarsening, the LVA field and the conditioning data configuration. It should be noted that this methodology should only be used if the available computer resources are restrictive as the error induced is avoidable.

4. Conclusions

The discussion above was limited to kriging with a multi-grid implementation to reduce CPU requirements of incorporating LVA. The extension to simulation is straightforward; a coarse grid is simulated that considers the LVA present and each realization is used as input to the final simulation using a standard SGS program. Simulation with a multigridding approach is considered in the next paper in this report (paper 105, CCG report 12). Kriging was the focus of this paper as the error when implementing the multi-gridding is assessable, with SGS issues such as the random number selected or the order of nodes visited becomes distracting; kriging allows for a meaningful comparison between the truth (kriging the fine grid with LVA) and the approximation (kriging a coarse grid with LVA and using the result as input to a fine grid kriging).

The proposed methodology is not limited to the LVA kriging/SGS framework as proposed by Boisvert (2010). The memory requirements of any CPU intensive methodology could be reduced by considering this approach; most notably, any multiple point simulation (MPS) algorithm that requires excessive memory could be substituted for the LVA algorithm used in this article.

No new programs/tools were written. The intention is that the user would utilize the programs provided with Boisvert and Deutsch (2009) to perform the LVA kriging/SGS on the coarse grid and the standard GSLIB programs (`kt3d` and `sgsim`) for the fine scale modeling.

References

Boisvert J and Deutsch CV. 2009, Advances in Locally Varying Anisotropy with MDS. CCG report 11. 14 p.

Boisvert J, 2010. Geostatistics with Locally Varying Anisotropy. PhD Thesis. University of Alberta. 175 p.

Figures

Figure 1: Cross sections displaying LVA. Left: Folding and faulting caused by the San Andreas Fault (www. strike-slip.geol.ucsb.edu). Right: Folding in the northern Rocky Mountains (www. mkutis.iweb.bsu.edu).

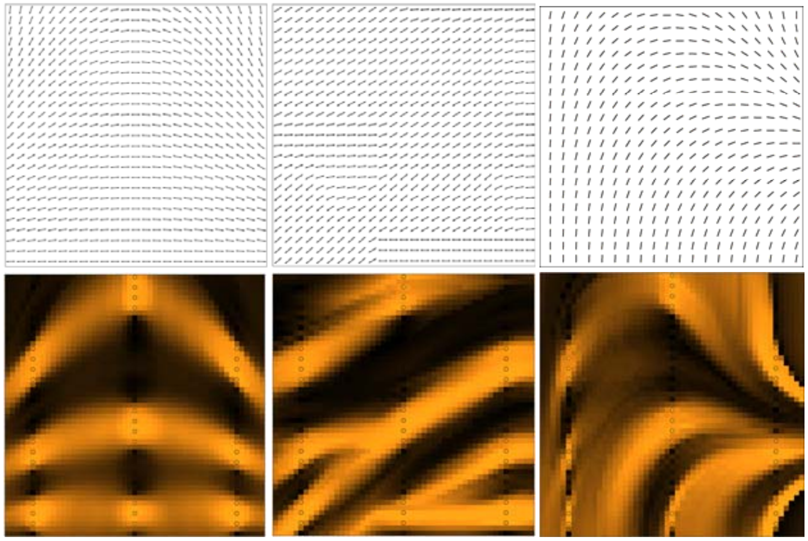

Figure 2: Examples of kriging with LVA. Above: LVA field direction, anisotropy ratio is constant 10:1. Below: Estimates with kriging with an exponential variogram with a range of 200 units. Dimensions of all plots are 51x51 units. Boisvert (2010).

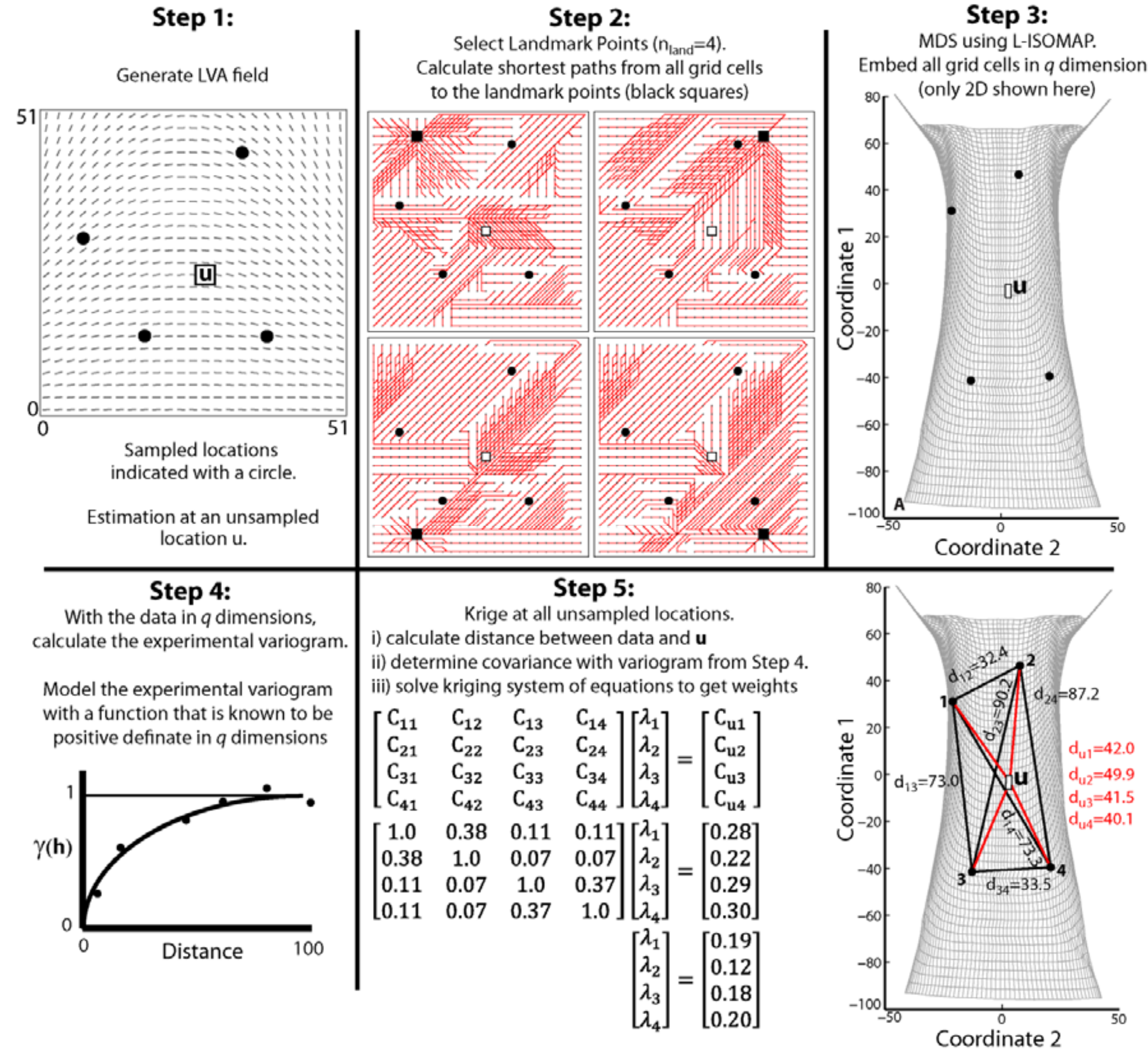

Figure 3: Kriging with LVA explained. Boisvert (2010)

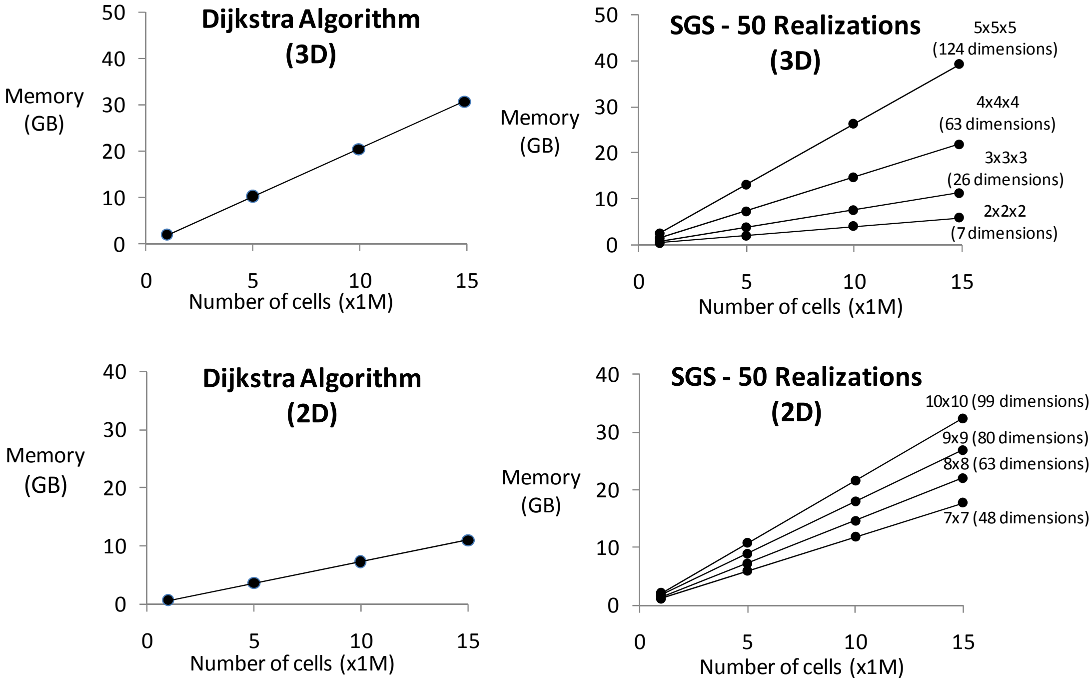

Figure 4: Above: RAM requirements in 3D. Below: RAM requirements in 2D. Left: RAM requirements for the Dijkstra algorithm. Right: RAM requirements for simulation.

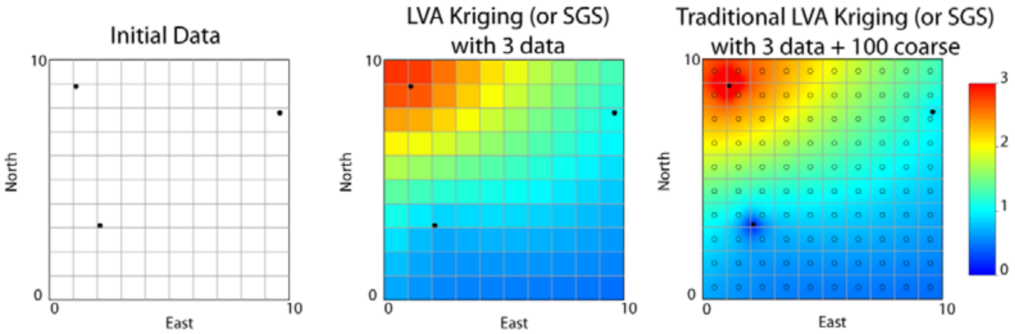

Figure 5: Left: Initial three samples. Middle: Model is built at a coarse resolution. Right: coarse data (gray 10x10 grid) is used as input to the final model (100x100 grid) using traditional kriging or SGS.

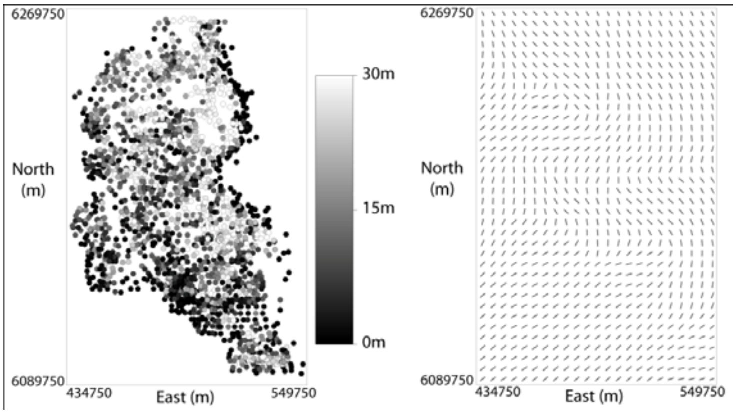

Figure 6: Left: 2342 wells with NCB measurements. Right: LVA field with a constant 0.45 anisotropy (2.22:1), after Boisvert (2010).

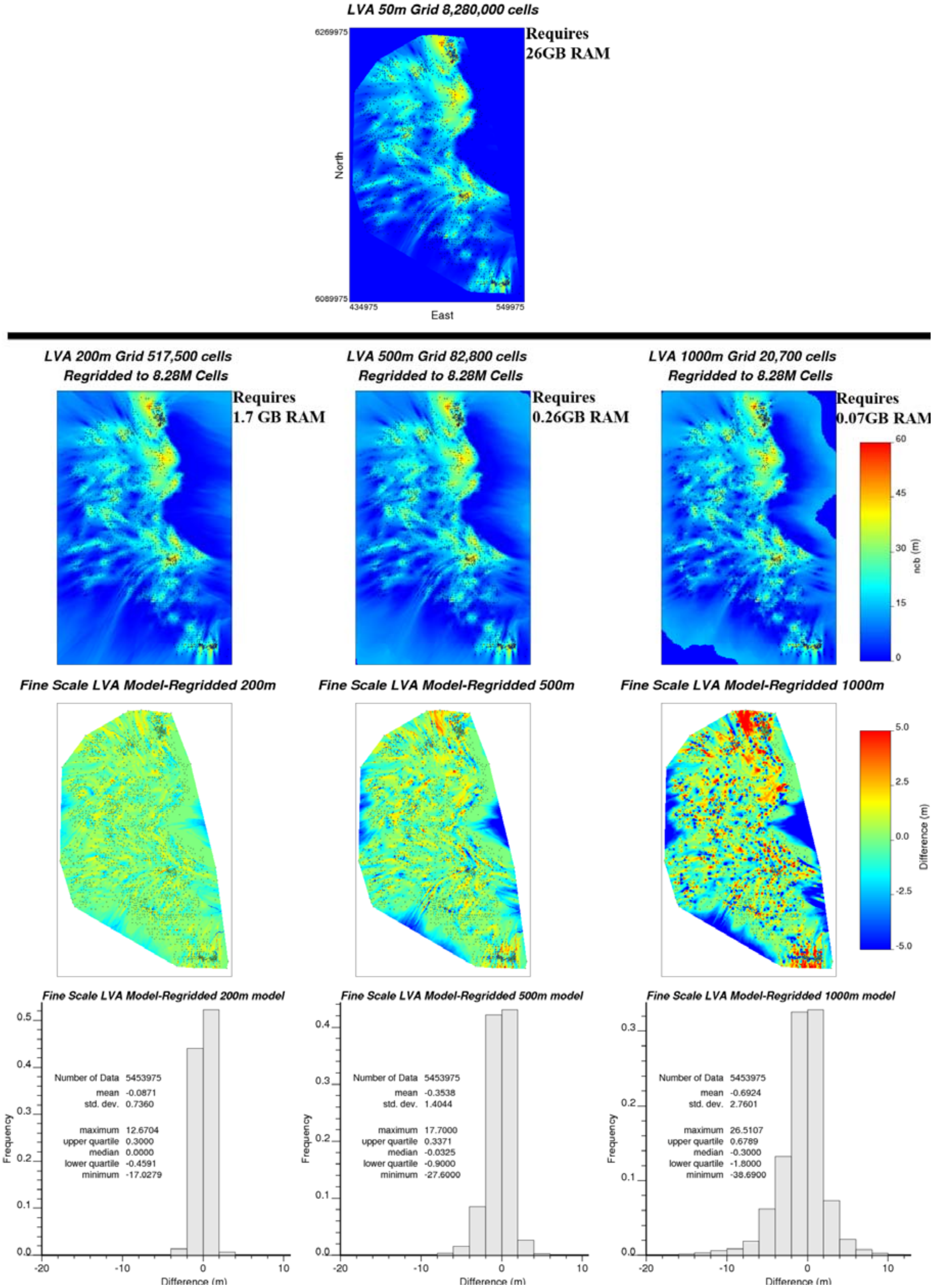

Figure 7: Above: Kriging using LVA on the 50m grid. Upper Row: Kriging result using a 200m, 500m and 1000m coarse LVA kriging regridded to 50m. Middle Row: Difference between kriging directly on a 50m grid and the regridding. Lower row: Histogram of errors.