# Accounting for Nonexclusivity in Sequential Indicator Simulation of Categorical Variables

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*Reservoir flow simulation is usually performed on large size grids. The required data for flow simulation, in order to get reasonable reservoir models, are from very different scales and sources. Facies modeling is performed to better capture the reservoir heterogeneity. Facies have significant impact on reservoir flow performance. Facies are treated as categorical variables and they are often considered as mutual exclusive and exhaustive at the scale of the geological model and flow simulation model. These two assumptions are needed for sequential indicator simulation (SIS) and most other facies modeling techniques. The assumption that an entire grid block would be just one facies becomes unreasonable as the scale increases; there is almost certainly some mixing as the scale increases. This paper addresses how to relax the assumption of exclusivity in large scale and how to account for mixing of facies at large scales. A methodology is proposed with synthetic and real examples.*

## 1. Introduction

In geostatistical applications, facies simulation is done to capture important large-scale changes in rock properties. Consider $K$ facies categories $k = 1, \dots, K$. The facies are called mutually exclusive and exhaustive because it is assumed that any location $\boldsymbol{u}_\alpha, \alpha = 1, \dots, N$, in the reservoir belongs to one and only one of these $K$ categories, therefore:

$$i(\boldsymbol{u}_\alpha; k) = \begin{cases} 1, & if\ location\ \boldsymbol{u}_\alpha\ in\ category\ k \\ 0, & otherwise \end{cases} \quad ;\ k = 1, \dots, K\ \&\ \alpha = 1, \dots, N$$

The two assumptions of mutual exclusivity and exhaustivity are summarized by:

$$i(\boldsymbol{u}_\alpha; k) \cdot i(\boldsymbol{u}_\alpha; k') = 0 \quad ;\ \forall k \neq k'\ \&\ k, k' = 1, \dots, K\ \&\ \alpha = 1, \dots, N$$

$$\sum_{k=1}^{K} i(\boldsymbol{u}_\alpha; k) = 1 \quad ;\ \alpha = 1, \dots, N$$

Arithmetic upscaling of facies converts the categories from discrete indicators to proportions, $p_k(\boldsymbol{u}_\alpha), k = 1, \dots, K\ \&\ \alpha = 1, \dots, N$:

$$p_k(\boldsymbol{u}_\alpha) = \frac{1}{V} \int_V i(\boldsymbol{u}_\alpha; k).dv \quad ; k = 1, \dots, K\ \&\ \alpha = 1, \dots, N$$

The facies proportions must sum to one at any scale:

$$\sum_{k=1}^{K} p_k(\boldsymbol{u}_\alpha) = 1 \quad ;\ \alpha = 1, \dots, N$$

As an aside, let's consider the size of a grid cell. Suppose that the grid size of flow simulation is 50 m x 50 m x 1 m which has a volume of 2500 $m^3$ with a total porosity of 15%. Sixteen giant mining haul trucks (with payload capacity of 363 tons) would be needed just to move the mass of one grid cell.

$$\left.\begin{array}{c} (No.\ of\ haul\ trucks) = \dfrac{(Bulk\ Volume) \times (Grain\ Density) \times (1 - \phi_{total})}{(Payload\ capacity\ of\ a\ haul\ truck)} \\ (Bulk\ Volume) = 2500\ m^3 \\ (Grain\ Density) = 2.67\ \dfrac{tons}{m^3} \\ \phi_{total} = 15\ \% \\ (Payload\ capacity\ of\ a\ high\ capacity\ haul\ truck) = 363\ tons \end{array}\right\} \Longrightarrow (No.\ of\ haul\ trucks) \cong 16$$

In many cases, there would be some mixing of facies in such a large volume. To account for nonexclusivity at large supports, a *quality variable*, $q(\boldsymbol{u})$, is defined and modeled to improve the petrophysical models and honor the nonexclusivity of facies at large scales. Another variable that is used to account for the effect of exclusivity assumption in petrophysical models is *anisotropic distance to nearest other facies*, $\mathcal{D}(\boldsymbol{u})$. Once the facies model at

large support is created with best available practice (SIS, BlockSIS, etc) the gridded model of anisotropic distance to nearest other facies can be constructed. This variable is used to better model the quality variable.

*Quality variable*

The quality variable is a function of scale of modeling and facies data at small scale. The quality variable contains the information of mixing of facies at large scales. The distribution of the quality variable in the model gives an idea about mixing of facies. In petroleum cases the vertical well logging scale is 0.5 ft (15.24 cm), a reasonable vertical scaling size for modeling is about 0.5 m-1.5 m (it depends on the application, geometry of reservoir and production mechanisms/schemes) and the maximum number of facies is about 4-5. Facies data at fine scale should be upscaled to the modeling scale by major rule, and then these upscaled data are being used to create the facies model. The upscaled facies variable and the quality variable are calculated simultaneously. The calculated facies proportions should be converted to a single categorical value. The upscaled categorical value is the facies that has the maximum facies proportion which is equal to $l$ where:

$$p_l(u_\alpha) = max\{p_k(u_\alpha); k = 1, \dots, K\} \quad ; \alpha = 1, \dots, N$$

The quality variable used for this methodology is derived from *K*-tuple facies proportion, $p_k(\boldsymbol{u}_\alpha), k = 1, \dots, K \;\&\; \alpha = 1, \dots, N$ and global effective properties for each facies:

$$\left.\begin{matrix}\{p_k(\boldsymbol{u}_\alpha), k = 1, \dots, K \;\&\; \alpha = 1, \dots, N\} \\ \{\mathcal{K}_{eff,k}, k = 1, \dots, K\}\end{matrix}\right\} \Rightarrow \{q(\boldsymbol{u}_\alpha), \alpha = 1, \dots, N\}$$
$$q(\boldsymbol{u}_\alpha) = f\left(p_1(\boldsymbol{u}_\alpha), \dots, p_K(\boldsymbol{u}_\alpha); \mathcal{K}_{eff,1}, \dots, \mathcal{K}_{eff,K}\right)$$

The global effective properties should be chosen in such a way that they show the distinctions between the flow properties of each facies. The function $f(\cdot)$ can be any reasonable (in terms of geology and flow properties) function that uses all of the information. In this methodology, $f(\cdot)$ is defined as geometric mean of the global effective properties for each facies weighted by facies proportions, that is:

$$q(\boldsymbol{u}_\alpha) = \prod_{k=1}^{K} \mathcal{K}_{eff,k}^{p_k(\boldsymbol{u}_\alpha)}$$

Therefore there are two values at each upscaled location data; (1) upscaled facies indicator and (2) quality variable. The quality variable data will be used to create the quality model for further modeling of petrophysical properties. The algorithm on how to use quality data will be presented later. The schematic diagram for calculating quality variable is shown in Figure 1.

The maximum number of possible values that $q(\boldsymbol{u}_\alpha)$ can take is a function of the number of facies, $K$, and the number of fine scale points used for upscaling of facies (scaling size), $V$. $p_k(\boldsymbol{u}_\alpha)$ takes a value from set of $\left\{0, \frac{1}{V}, \frac{2}{V}, \dots, \frac{V-1}{V}, 1\right\}$. In the case of upscaling $V$ fine grids in the presence of $K$ facies, the problem is analog to multiset problem, therefore the number of possible combinations for this case is:

$$n_{comb} = \binom{V + K - 1}{K - 1} = \frac{(V + K - 1)!}{V! \cdot (K - 1)!}$$

$n_{comb}$ has the same value as the number of possible lattice points on $(K-1)$-Simplex ($\Delta^{K-1}$) with the domain of $V$ equally spaced values from 0 to 1 for each facies. The first few values of $n_{comb}$ is tabulated in Table 1. In other words $n_{comb}$ means the total number of combinations that leads to distinct facies proportion of $(p_1, \dots, p_k)$; for example in the case of $K = 3$ and $V = 4$ (that leads to $n_{comb} = 15$), the total number of combinations are shown in Figure 3. The total number of arrangements for each case (combination) can be calculated as:

$$\binom{V}{V_1, \dots, V_K} = \frac{V!}{(V_1)! \cdots (V_K)!}$$
$$\sum_{k=1}^{K} V_k = V \quad \& \quad 0 \le V_1, \dots, V_K \le V$$
$$p_k = \frac{V_k}{V} \quad \& \quad \sum_{k=1}^{K} p_k = 1$$

| | $p_1$ | $p_2$ | $p_3$ | $q$ |
|---|---|---|---|---|
| **1** | 1 | 0 | 0 | $\mathcal{K}_{eff,1}$ |
| **2** | 0 | 1 | 0 | $\mathcal{K}_{eff,2}$ |
| **3** | 0 | 0 | 1 | $\mathcal{K}_{eff,3}$ |
| **4** | $\frac{3}{4}$ | $\frac{1}{4}$ | 0 | $\mathcal{K}_{eff,1}^{\frac{3}{4}} \cdot \mathcal{K}_{eff,2}^{\frac{1}{4}}$ |
| **5** | $\frac{3}{4}$ | 0 | $\frac{1}{4}$ | $\mathcal{K}_{eff,1}^{\frac{3}{4}} \cdot \mathcal{K}_{eff,3}^{\frac{1}{4}}$ |
| **6** | $\frac{1}{4}$ | $\frac{3}{4}$ | 0 | $\mathcal{K}_{eff,1}^{\frac{1}{4}} \cdot \mathcal{K}_{eff,2}^{\frac{3}{4}}$ |
| **7** | 0 | $\frac{3}{4}$ | $\frac{1}{4}$ | $\mathcal{K}_{eff,2}^{\frac{3}{4}} \cdot \mathcal{K}_{eff,3}^{\frac{1}{4}}$ |
| **8** | $\frac{1}{4}$ | 0 | $\frac{3}{4}$ | $\mathcal{K}_{eff,1}^{\frac{1}{4}} \cdot \mathcal{K}_{eff,3}^{\frac{3}{4}}$ |
| **9** | 0 | $\frac{1}{4}$ | $\frac{3}{4}$ | $\mathcal{K}_{eff,2}^{\frac{1}{4}} \cdot \mathcal{K}_{eff,3}^{\frac{3}{4}}$ |
| **10** | $\frac{2}{4}$ | $\frac{2}{4}$ | 0 | $\mathcal{K}_{eff,1}^{\frac{2}{4}} \cdot \mathcal{K}_{eff,2}^{\frac{2}{4}}$ |
| **11** | $\frac{2}{4}$ | 0 | $\frac{2}{4}$ | $\mathcal{K}_{eff,1}^{\frac{2}{4}} \cdot \mathcal{K}_{eff,3}^{\frac{2}{4}}$ |
| **12** | 0 | $\frac{2}{4}$ | $\frac{2}{4}$ | $\mathcal{K}_{eff,2}^{\frac{2}{4}} \cdot \mathcal{K}_{eff,3}^{\frac{2}{4}}$ |
| **13** | $\frac{2}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\mathcal{K}_{eff,1}^{\frac{2}{4}} \cdot \mathcal{K}_{eff,2}^{\frac{1}{4}} \cdot \mathcal{K}_{eff,3}^{\frac{1}{4}}$ |
| **14** | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{1}{4}$ | $\mathcal{K}_{eff,1}^{\frac{1}{4}} \cdot \mathcal{K}_{eff,2}^{\frac{2}{4}} \cdot \mathcal{K}_{eff,3}^{\frac{1}{4}}$ |
| **15** | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\mathcal{K}_{eff,1}^{\frac{1}{4}} \cdot \mathcal{K}_{eff,2}^{\frac{1}{4}} \cdot \mathcal{K}_{eff,3}^{\frac{2}{4}}$ |

*Sequential indicator simulation*

Sequential indicator simulation is used to build the facies model in this paper. The existing SIS has been developed by Alabert (1987) and Journel (1989). SIS consists of visiting each grid node in a random order. At each grid the nearby data and previously simulated grid nodes are considered for simulation, and then the conditional distribution is constructed by using kriging. Once the conditional distribution is constructed a simulated facies from the set of probabilities can be drawn. The input data and parameters that are needed for indicator simulation are as follows: (1) number of categories and the global facies proportions for each facies, (2) upscaled facies data at the grid block of simulation (by major rule), (3) indicator variogram models for each facies.

*Anisotropic distance to nearest other facies*, $\mathcal{D}(\boldsymbol{u})$

As mentioned calculating the anisotropic distance to nearest other facies is a post-processing step for facies indicator simulation and is needed to build the quality model. It will be used to classify the distribution of quality data and calculate the stepwise conditional distribution of quality variable for modeling. Figure 2 shows a typical facies model (the number of facies is 3**,** $K = 3$) with corresponding anisotropic distance to nearest other facies model. The distance calculated in Figure 2 is based on the anisotropic distance and the minimum distance for a grid block is obtained based on anisotropic direction. $\mathcal{D}(\boldsymbol{u})$ should have the same spatial configuration as facies model. The only differences are that in the area with low proportion of uniform facies, $\mathcal{D}(\boldsymbol{u})$ has low values but the area of influence is high; in contrast in the area with high proportion of uniform facies, $\mathcal{D}(\boldsymbol{u})$ has high values but the area of influence is low. The low values of $\mathcal{D}(\boldsymbol{u})$ are around the contacts between facies while the high values are in the area where there are no changes in facies values. For the purpose of stepwise conditional simulation of quality variable, we need the shortest distance to nearest other facies at data locations. The distance model, $\mathcal{D}(\boldsymbol{u})$, magnifies the mixing regions (contact regions or borders between two distinct facies). $\mathcal{D}(\boldsymbol{u})$ calculations are performed on gridded model of facies indicators. Once the nearest grid with different facies is found, below calculation is done to get $\mathcal{D}(\boldsymbol{u})$ at location $\boldsymbol{u}$:

$$\mathcal{D}^2(\boldsymbol{u}) = [\tau(\boldsymbol{u})]^T \cdot \mathcal{R}^T \cdot \mathcal{R} \cdot [\tau(\boldsymbol{u})]$$

Where $\tau(\boldsymbol{u})$ is the nearest distance vector between the two grids with different facies and $\mathcal{R}$ is the rotation matrix:

$$\tau(\boldsymbol{u}) = \begin{bmatrix} \tau_x(\boldsymbol{u}) \\ \tau_y(\boldsymbol{u}) \\ \tau_z(\boldsymbol{u}) \end{bmatrix}$$

$$\mathcal{R} = \begin{bmatrix} Cos\alpha \cdot Cos\varphi - Sin\alpha \cdot Sin\beta \cdot Sin\varphi & -Sin\alpha \cdot Cos\varphi - Cos\alpha \cdot Sin\beta \cdot Sin\varphi & Cos\beta \cdot Sin\varphi \\ \frac{1}{r_1} \cdot Sin\alpha \cdot Cos\beta & \frac{1}{r_1} \cdot Cos\alpha \cdot Cos\beta & \frac{1}{r_1} \cdot Sin\beta \\ \frac{1}{r_2} \cdot (-Cos\alpha \cdot Sin\varphi - Sin\alpha \cdot Sin\beta \cdot Cos\varphi) & \frac{1}{r_2} \cdot (Sin\alpha \cdot Sin\varphi - Cos\alpha \cdot Sin\beta \cdot Cos\varphi) & \frac{1}{r_2} \cdot Cos\beta \cdot Cos\varphi \end{bmatrix}$$

The rotation matrix honors the anisotropic ellipsoid. The anisotropy ellipsoid is characterized by five parameters of $(\alpha, \beta, \varphi, r_1, r_2)$ where $\alpha$ is strike, $\beta$ is dip, $\varphi$ is plunge, $r_1$ is the ratio between the minor and major ranges of correlation and $r_2$ is the ratio between the vertical and major ranges of correlation.

*Stepwise conditional transformation*

The stepwise conditional transformation is the same as normal score transformation in univariate case. In the bivariate case the normal transformation of the second variable is conditional to the probability class of the first variable. The first variable is anisotropic distance to nearest other facies and the second variable is the quality variable. The Gaussian transformation is used for the grouped data in each probability class separately. Stepwise conditional transformation removes any correlation between the variables. This type of transformation can be generalized for any number of variables. Figure 4 shows the steps for stepwise conditional transformation in the bivariate case. The resulting correlation coefficient between the transformed variables is zero because each class of the secondary variable is independently transformed to a normal distribution. The property of zero correlation between the stepwise conditional transformed variables helps to have no cosimulation between the variables (the transformed variables are simulated independently). Ordering matters in stepwise conditional transformation. The transformation of primary variable is the same as univariate normal score transformation. The simulation result of primary variable does not contain any information of secondary variable but the results for secondary variable honor both primary and secondary variables. In this paper the anisotropic distance to nearest other facies is considered as primary variable while the quality variable is considered as secondary variable because building the quality model is the goal in this research and distance model helps to have more realistic model for quality. Since the distance model shows clearly the mixing regions; therefore, the simulated quality model better captures mixing of facies. In presence of sparse data, Kernel smoothing technique is applied. In bivariate case the kernel density is characterized by the bivariate Gaussian probability density function. Kernel smoothing technique helps to have more data in each probability class. After simulation of stepwise transformed variables, the output models are back transformed to original units.

As an example for primary and secondary variables for this application, a synthetic case is run and the scatter plot of the corresponding quality variable and anisotropic distance is plotted in Figure 5. For this synthetic example, three facies are considered with very different flow properties. The solid lines in the scatter plot show the pure facies points in the model. There exist points in low quality facies that have high anisotropic distance to nearest other facies. Based on the distributions of quality and distance, it is concluded that a large portion of the reservoir belongs to a region with low quality facies and small anisotropic distance to nearest other facies. Stepwise conditional simulation helps to preserve the non-linear feature that might exist in scatter plot of quality and distance.

**2. Methodology**

The steps of the methodology are summarized below:

1. Prepare the needed input data and parameters:

- Petrophysical and facies data are needed at fine scale (e.g. well logging scale, 15 cm or 0.5 ft), the effective properties (e.g. permeability) for each facies, the upscaled grid block size in X, Y and Z coordinates
- Group facies together if necessary

2. Upscale facies by major-rule approach and petrophysical properties by averaging (arithmetic, geometric, power law, etc.) and calculating quality variable at each upscaled location using the facies proportions of upscaled grid block and the global effective properties for each facies
3. Model facies with best available practice (e.g. SIS)
4. Build the gridded model for the distance to nearest other facies from facies model in step 3.
5. Assign distance values from step 4 to the corresponding data locations and the upscaled data set.
6. Stepwise conditional simulation of quality variable for each facies
    - Consider distance variable as primary and quality variable as secondary (apply Kernel smoothing technique as needed)
    - Conditional transformation of the variables
    - Variogram modeling for stepwise conditional (SC) transformed of quality variable
    - Sequential Gaussian simulation (SGS) of the SC transformed quality variable

After obtaining the by-facies quality models, petrophysical variables can be simulated using SGS. The by-facies simulated quality variable in step 6 should be used as secondary model for modeling of by-facies petrophysical properties.

**3. Application example**

The proposed methodology is applied on a real example (Hekla data set) in this section. The Hekla reservoir is a section of a fluvial deposit in the Statfjord formation offshore Norway; it is divided into two zones of H1 and H2. The conceptual geological model for Hekla area is depicted in Figure 6. The minimum (across channel) and maximum (along channel) directions of continuity are N60°W and N30°E respectively (Journel, 1998). Figure 7 shows the locations of wells for Hekla data set. There are 20 wells in the study zone. There are 5 facies in the reservoir. The cumulative distribution functions (CDFs) of all by-facies porosity and permeability give information about the quality of reservoir rock. Figure 8 shows the CDFs of core porosity and horizontal permeability data. From the CDFs it is obvious that facies 2 has high porosity and permeability (pay) while facies 5 has low porosity and permeability (non-pay). These five categories are defined based on geological considerations and conceptual models. Table 2 summarizes statistics for facies and petrophysical properties of Hekla data set. Figure 9 shows the probability density function of facies before and after grouping facies. The facies are grouped into three facies associates (FA) in order to have more realistic facies model and avoid any artifacts in the model. The grouping of facies is based on geological interpretations and considerations. The host rock (facies 5) is considered as FA 1, the border sands (facies 3 and 4) is considered as FA 2 and channel sand (facies 1 and 2) is considered as FA 3. The indicator variograms are calculated and fitted for these three new facies for the purpose of modeling. Figure 10 shows the indicator variograms for the three facies associates in vertical direction. The global effective property is considered as average horizontal core permeability for this data set. The global by-facies effective properties should be chosen in such a way that they capture the distinct flow properties between the facies. The global values are as follows (see Figure 11):

$$\mathcal{K}_{eff,1} = 0.0460\ mD$$
$$\mathcal{K}_{eff,2} = 424.8521\ mD$$
$$\mathcal{K}_{eff,5} = 6405.5000\ mD$$

Facies model for this data set is built using SIS. Stratigraphic transformation is performed to have corrected Z coordinate. The modeling is performed in H1 layer. The number of grids in X, Y and Z are as follows: 100, 130 and 40 and the grid sizes are 50 m, 50 m and 0.79 m respectively. The resulting 3D model and the associated PDF are shown in Figure 12. The stepwise conditional simulation is performed, distance variable is considered as the first variable and quality variable is considered as second variable in stepwise conditional transformation. The by-facies quality model is constructed using sequential Gaussian simulation (SGS) with stepwise conditional transformed

quality. The simulated values need to be back transformed to original units. Since there are three facies therefore three by-facies quality models are constructed. One can use these created by-facies quality model as secondary variables for simulation of porosity (or any other petrophysical property that reflects flow properties of the reservoir rock) in SGS. The by-facies models along with the facies, distance and merged models are shown in Figure 13 for a fixed slide in XY plane. Figure 14 shows three different slides of facies model and the corresponding distance to nearest other facies and quality model (note that the quality model in this figure is merged based on three by-facies model and the created facies model).

**4. Conclusion**

The reality of nonexclusivity in large scale facies models is honored by constructing by-facies quality models. The quality models can be used as secondary variable in SGS for petrophysical variables. As a future work, this methodology should be validated with flow simulator. It can be compared to conventional approaches for petrophysical simulation. The quality map is a reasonable indicator of mixing and accounts for the closeness to facies transitions. The distance variable considers the contacts between facies (where the probability of mixing is high) and regions where there is only one facies. Stepwise conditional transformation is used to preserve the non-linear features between distance and quality. The results of conventional approaches and this methodology for petrophysical properties are the same in the regions with pure facies. The problem of nonexclusivity appears mostly in the region where facies are mixed.

Although the application is straightforward, testing of the methodology through flow simulation and by comparing to very high resolution models is still required.

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| $V$ \ $K$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | **1** | **2** | **3** | **4** | **5** |
| 2 | **1** | **3** | **6** | **10** | **15** |
| 3 | **1** | **4** | **10** | **20** | **35** |
| 4 | **1** | **5** | **15** | **35** | **70** |
| 5 | **1** | **6** | **21** | **56** | **126** |

**Table 1** first few values for $n_{comb} = \binom{V+K-1}{K-1}$

| | | Facies 1 | Facies 2 | Facies 3 | Facies 4 | Facies 5 |
|---|---|---|---|---|---|---|
| Global Proportion | | 0.0166 | 0.2383 | 0.0512 | 0.0863 | 0.6076 |
| Core Porosity | mean | 0.2294 | 0.2829 | 0.2398 | 0.2285 | 0.1792 |
| | std. dev. | 0.0875 | 0.0663 | 0.0767 | 0.0734 | 0.0755 |
| | coef. of var. | 0.3815 | 0.2343 | 0.3200 | 0.3211 | 0.4213 |
| Core K_h | mean | 146.6634 | 524.4570 | 650.3132 | 132.0820 | 110.8002 |
| | std. dev. | 163.9453 | 795.7706 | 1480.7098 | 462.4265 | 337.6700 |
| | coef. of var. | 1.1178 | 1.5173 | 2.2769 | 3.5011 | 3.0476 |
| Core K_v | mean | 93.6042 | 371.6645 | 67.3343 | 241.6001 | 159.5658 |
| | std. dev. | 115.2620 | 680.8011 | 157.9108 | 515.7867 | 363.1393 |
| | coef. of var. | 1.2314 | 1.8318 | 2.3452 | 2.1349 | 2.2758 |
| Log Porosity | mean | 0.1433 | 0.2525 | 0.1455 | 0.1090 | 0.0413 |
| | std. dev. | 0.0831 | 0.0671 | 0.0778 | 0.0871 | 0.0631 |
| | coef. of var. | 0.5799 | 0.2657 | 0.5346 | 0.7992 | 1.5273 |
| Log Permeability | mean | 66.5841 | 6050.1439 | 126.8849 | 15.2380 | 0.4229 |
| | std. dev. | 106.6462 | 656.1804 | 338.9524 | 45.5389 | 6.8061 |
| | coef. of var. | 1.6017 | 1.0667 | 2.6713 | 2.9885 | 16.0926 |

**Table 2** summarized statistics (global proportions, mean, standard deviation and coefficient of variation) for facies and petrophysical properties (porosity and permeability) of Hekla data set

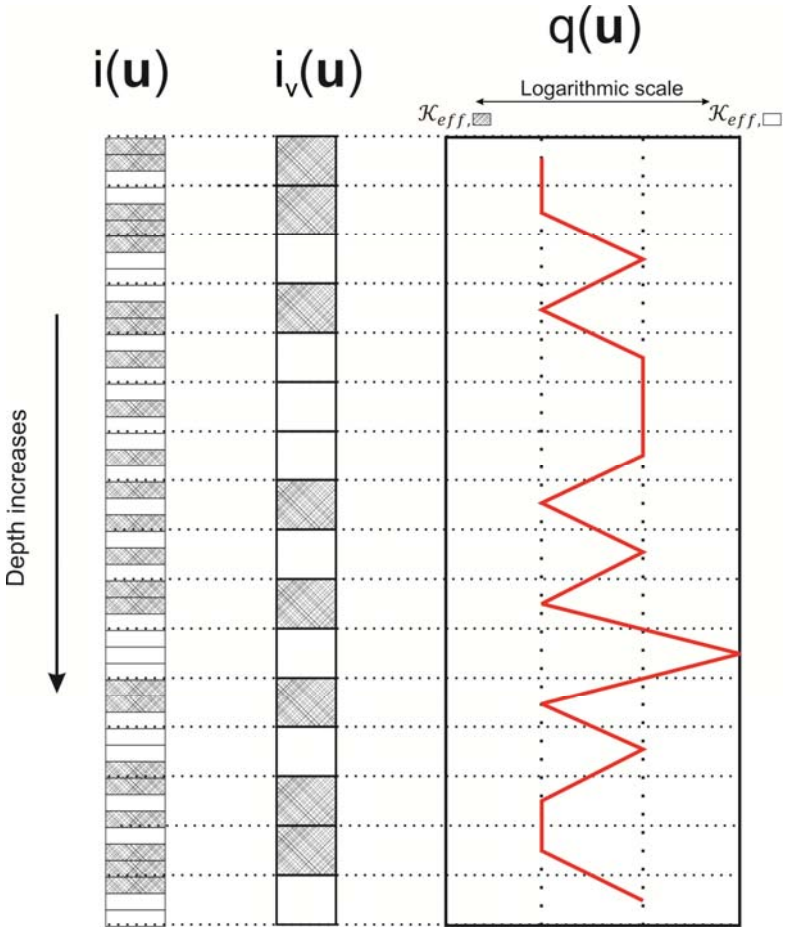



**Figure 1** schematic diagram for arithmetic upscaling of facies by major rule and calculating the corresponding quality variable with geometric averaging (the number of facies is 2 (black and white), $K = 2$, and the scaling size is 3, $V = 3$)

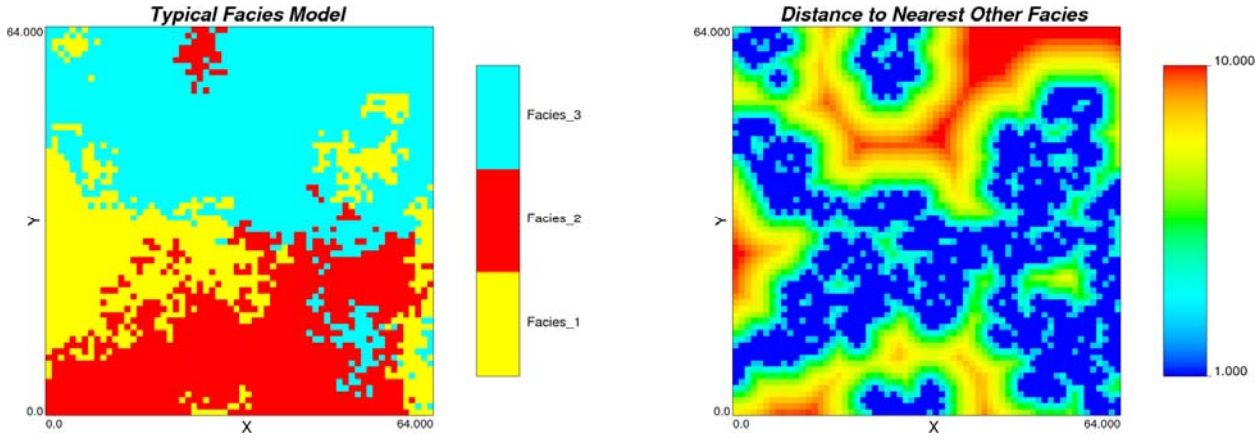



**Figure 2** typical facies model with three facies (left) along with corresponding model of distance to nearest other facies in logarithmic scale (right)

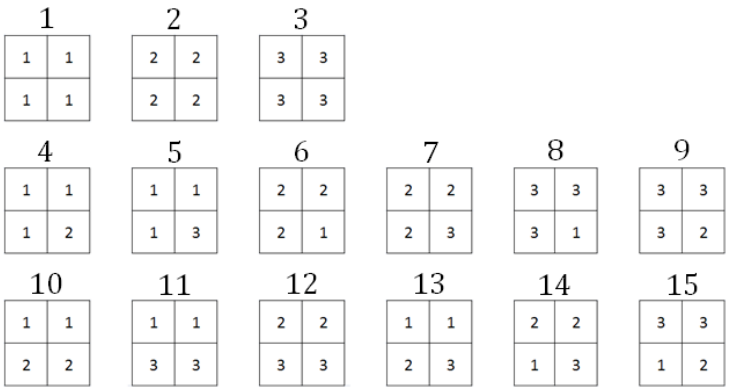



**Figure 3** total possible combinations for calculating $q$ in the case of $K = 3$ and $V = 4$

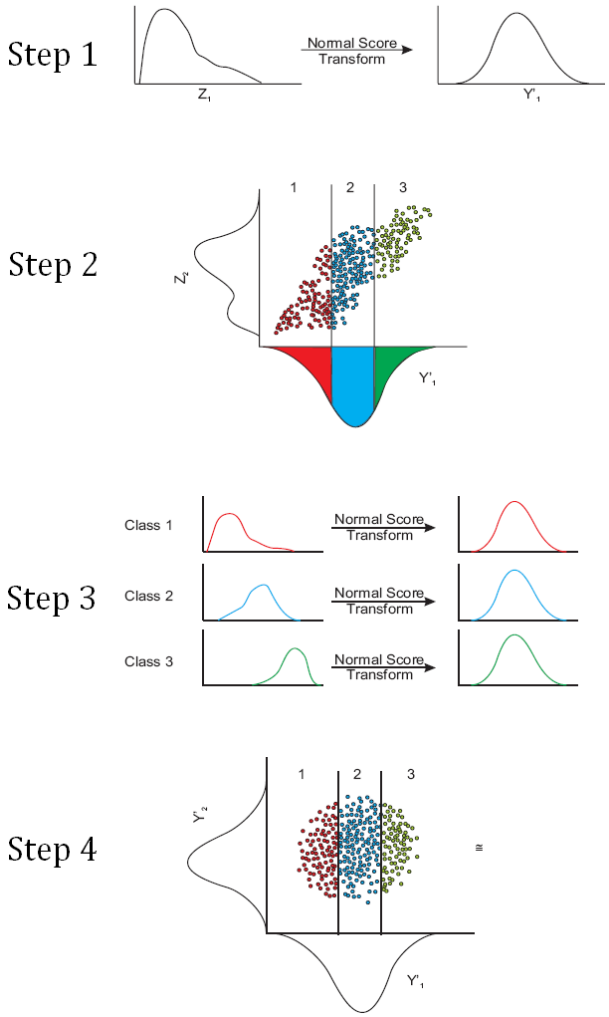



**Figure 4** schematic steps for stepwise conditional transformation for bivariate case ($Z_1$ and $Z_2$ ); Step 1 is the normal score transformation of $Z_1$ to get $Y'_1$, Step 2 is to classify $Z_2$ conditional to $Y'_1$, Step 3 is the normal score transformation $Z_2$ classes separately and Step 4 is to lump all of the normal score transformed classes of $Z_2$ together to get $Y'_2$ and crossplot of stepwise conditional transformed variables (further geostatistical simulation are done with these two new variables but finally the models should be back transformed to original values) (Leuangthong, 2003)

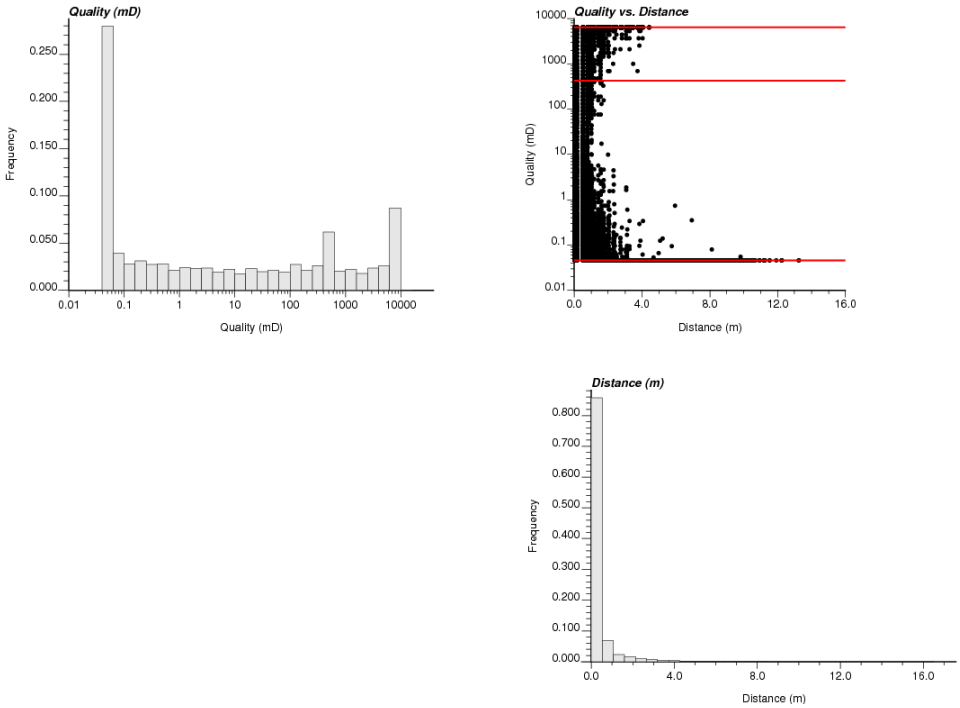



**Figure 5** synthetic scatter plot and distributions of quality variable and anisotropic distance to nearest other facies for a reservoir with three facies of low, intermediate and high quality. Here the quality is considered as permeability

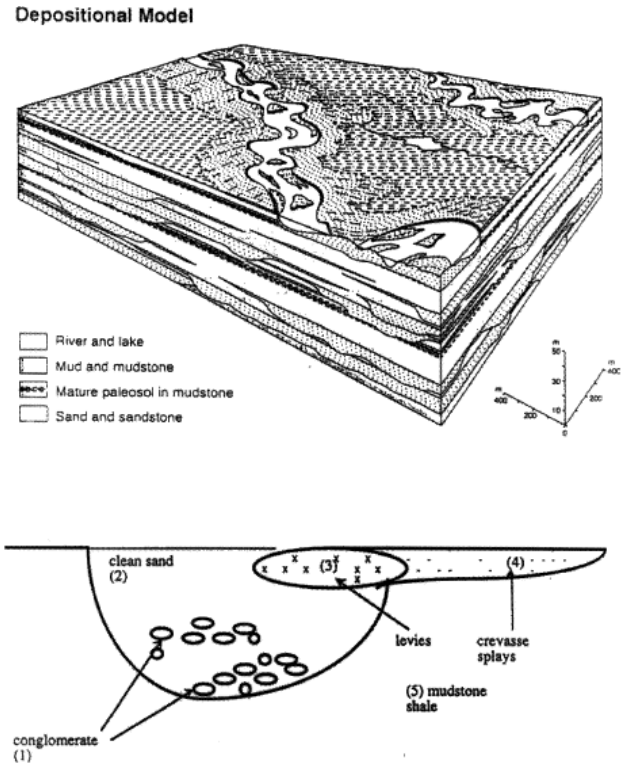



**Figure 6** Conceptual geological model for Hekla area (Journel, 1998)

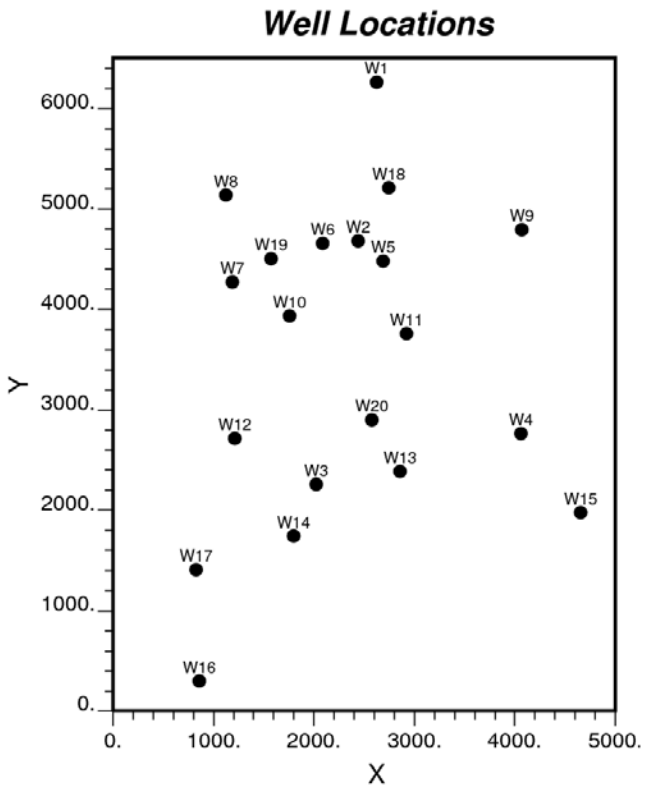



**Figure 7** well locations for Hekla data set

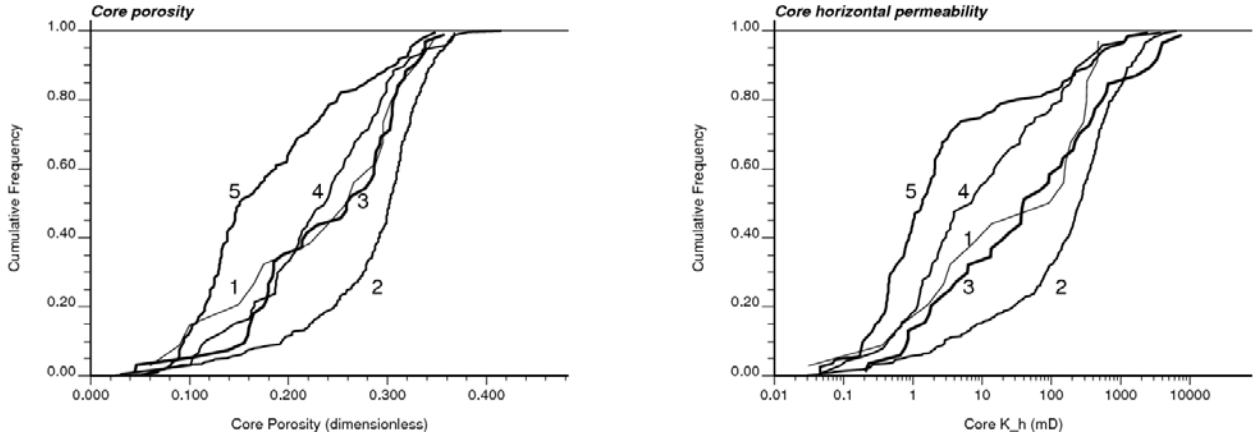
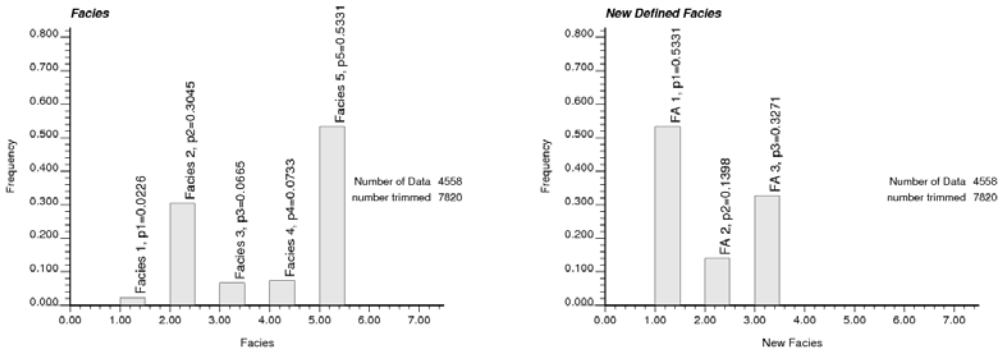



**Figure 8** cumulative distribution functions (CDF) for by facies core porosity and horizontal permeability; CDFS show that facies 2 is the best portion of reservoir while facies 5 is the poorest.




FA 1 = Facies 5

FA 2 = Facies 3 + Facies 4

FA 3 = Facies 1 + Facies 2

**Figure 9** probability density function (PDF) for facies; the global proportions for each class is written on top of each column; the original PDF is shown on the left hand side with five facies while the right hand side plot shows the new defined facies (the number of facies is reduced to three); the grouping of facies is based on geological and flow properties of the reservoir

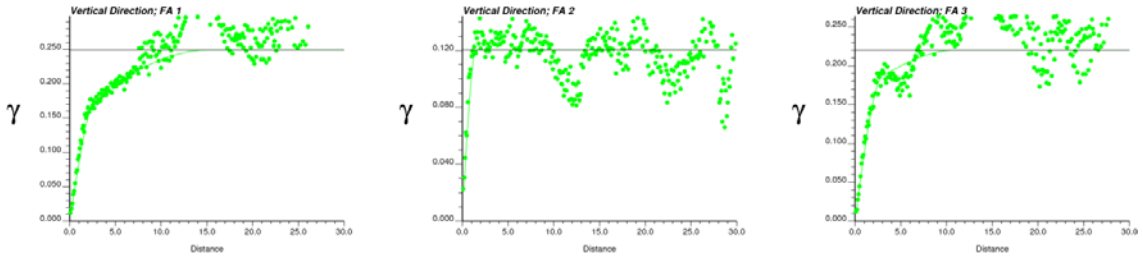



**Figure 10** the indicator experimental variogram points in vertical direction along with the corresponding fitted models for three new defined facies

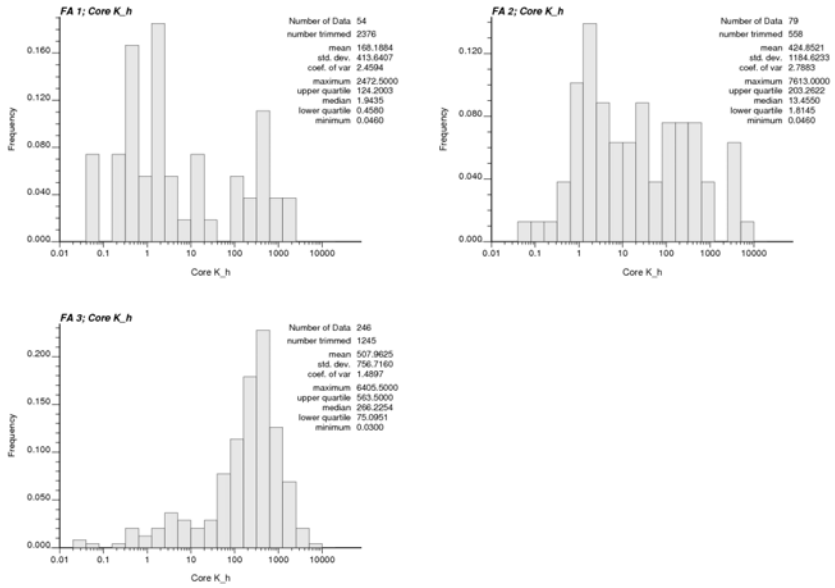



**Figure 11** probability density functions (PDFs) for by-facies horizontal permeability (core data)

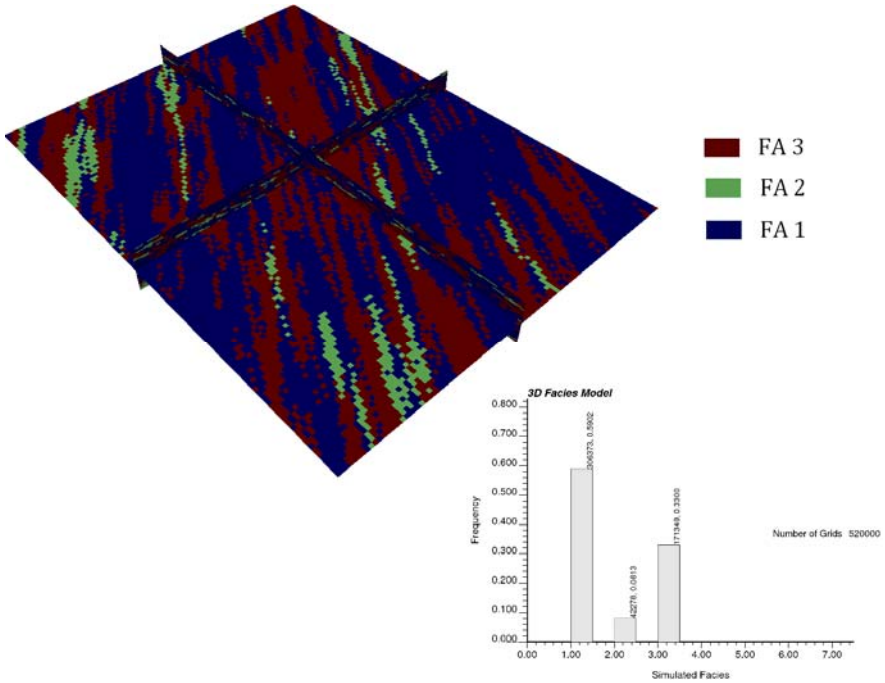



**Figure 12** 3D facies model for Hekla; the number of facies is three

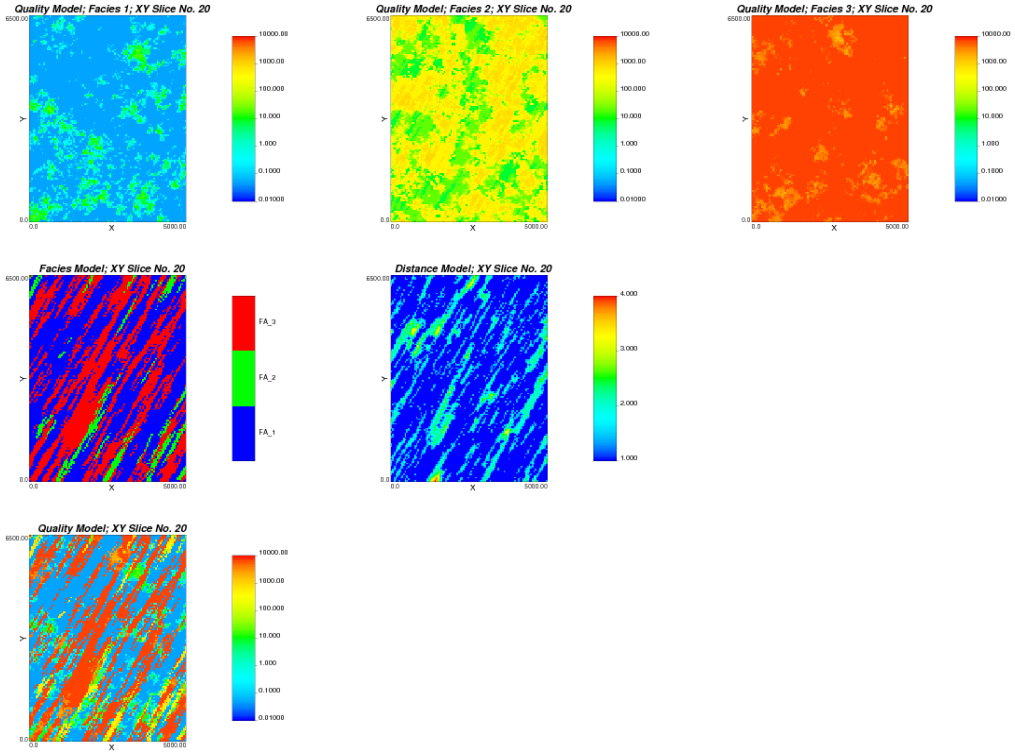



**Figure 13** the by-facies quality models (top row); the facies model and the corresponding anisotropic distance model (middle row) and the merged quality model (bottom row)

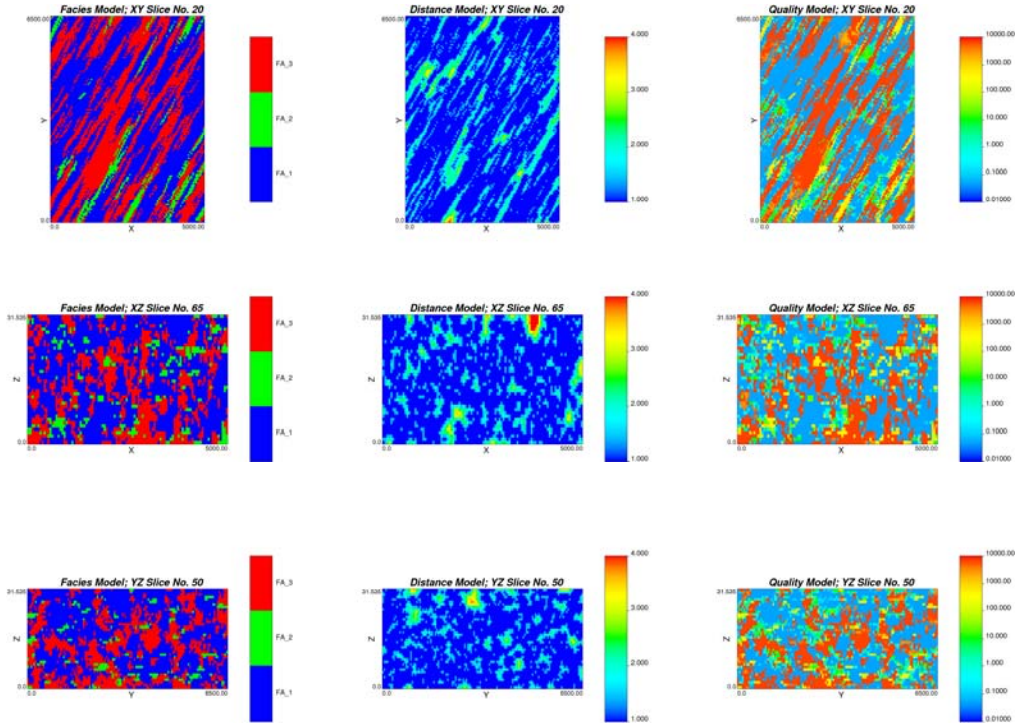



**Figure 14** the left hand side plots are different slides on XY, XZ and YZ planes of facies model, the middle plots are the associated distance to nearest other facies model and the right hand side plots are the corresponding stepwise conditional simulated quality models

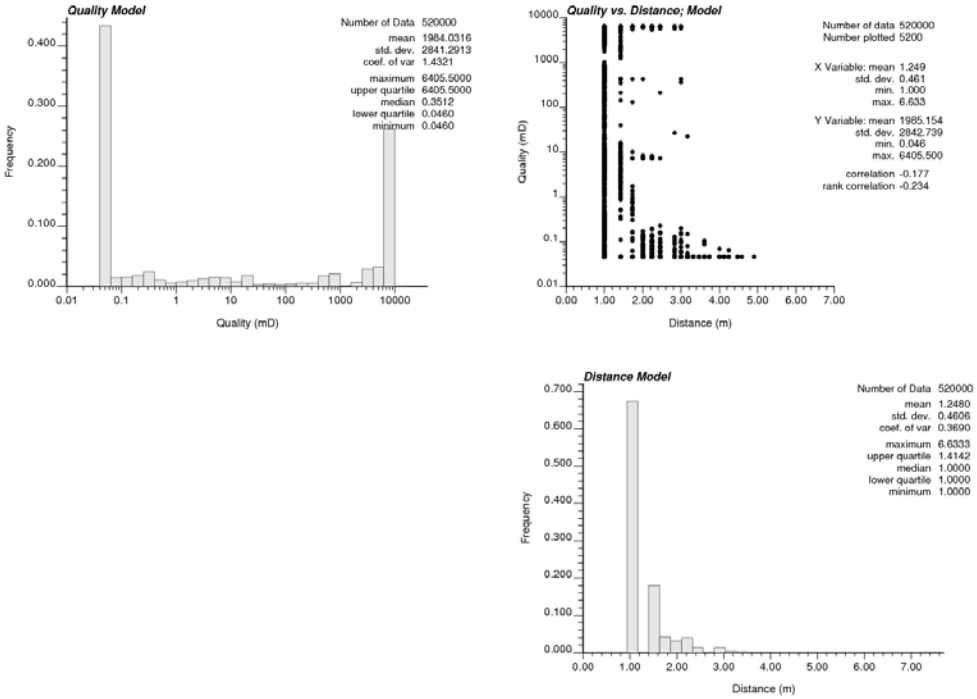



**Figure 15** the scatter plot and distributions of quality variable and anisotropic distance to nearest other facies for Hekla data set