Thoughts on Practical Inverse Modeling in Petroleum Reservoir Characterization

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Petroleum reservoir modeling frequently faces inverse problems, where large scale secondary variable information is used to assess the distribution of a primary variable, whose measurements are scarce. A good example of this is permeability modeling with production data such as flow rate and pressure measurements. Production data may be more accessible and are directly related to what is being predicted. This short note consists of thoughts on practical inverse modeling in petroleum reservoir characterization. Different available techniques for inverse problems are discussed. More attention is focused on the Ensemble Kalman Filter (EnKF) and Sequential Self Calibration (SSC) inverse modeling methods.

1. Problem Formulation

Numerical models for petroleum reservoir characterization help to increase oil production rate and to improve oil recovery. The numerical model discretizes the volume of interest into blocks of a predefined size. Each block is characterized by a set of variables such as facies, porosity and permeability that together form the numerical model. Thus, a model may consist of many thousands of variables. The data to assign these variables come from various sources at different scales and different coverage. The task of modeling is to define the value of each variable for every block. The goal would be to have the values as close as possible to the reality, but there is great uncertainty and combining all available observations is one of the only approaches to reduce this uncertainty.

Some variables are rock properties that we must have for resource and reserve calculation, for example, facies (to identify the appropriate relative permeability and other rock properties), porosity (storativity – very important for resources) and permeability (transmissibility – important for flow rates and reserves); these are called primary variables. Other variables inform on the primary variables and are called secondary variables. Primary and secondary variables can be a model response or a model state and a model parameter, which are defined by dynamic and static quantities, respectively. In most cases the relationship between variables is established at least approximately by experimental observation. If insufficient primary variable measurements are present, then estimation of primary variable values is complicated and must rely on a conceptual or physical model. Extensive measurements of secondary variables can be used for the estimation of primary variables using a known relationship between them. The volumetric support of the primary and secondary variables is often different and this can be accounted for in the geostatistical calibration.

Some secondary data is dynamic data, that is, the result of flow in the reservoir. The forward problem of predicting the flow response of a reservoir from a numerical model of primary variables is quite well understood. The inverse problem, that is, constraining the primary variables to observations of the flow process is much more difficult in practice. There are different inverse modeling techniques. In this short note Ensemble Kalman Filter (EnKF) and Sequential Self Calibration (SSC) inverse modeling techniques are mainly discussed. The objective function is employed as a criterion of estimation goodness, which should be minimized. In a petroleum reservoir characterization context inverse modeling techniques are efficiently applicable to history matching and can be used for real time updating and reservoir management.

Some previous research on EnKF application in petroleum reservoir characterization can be found in (Nævdal et al., 2002), where EnKF is used for permeability field and production data in form of bottom hole pressure, in (Gu and Oliver, 2005) – for porosity, permeability and well bottom hole pressure, well gas-oil ratio, well water cut, and oil production rate, and in (Gu and Oliver, 2006) for permeability, porosity and water saturation. SSC inverse modeling technique was applied for transmissivity and piezometric head fields (Gómez-Hernánez, 2007), for permeability and reservoir temperature (Hassanpour and Deutsch, 2010). Recent overview papers on EnKF are (Aanonsen et al, 2009) and (Oliver and Chen, 2010).

The short note is organized in following manner. First, inverse modeling techniques are briefly reviewed. Implementation details of inverse modeling are presented next and are not tied to any specific technique. Then, computational challenges of EnKF and SSC are summarized and possible solutions are recommended. Finally, promising avenues of EnKF-based inverse modeling are discussed.

2. Inverse Modeling Techniques

Some inverse modeling techniques that may be applied for petroleum reservoir characterization are techniques based on Kalman Filter (Extended Kalman Filter (EKF), Unscented Kalman Filter (UKF), EnKF, Ensemble Kalman Smoother (EnKS), Particle Filter (PF)), Sequential Self Calibration (SSC), Pilot Point (PP), Maximum Likelihood (ML), Linearized Cokriging (LC), Linearizaed Semianalytical (LS), Fast Fourier Transform (FF) and Fractal Simulation (FS).

The Kalman Filter is a data assimilation method that improves the quality of a model by updating a model to honor extra measurements of primary and secondary variables using covariance as the measures of how the all of the data relate to each other. This method did not gain popularity in reservoir characterization because of its inability to handle large-scale systems and nonlinear problems. Later EKF and UKF were introduced to cope with nonlinear systems. EKF employs deterministic approach and treats one 'best' possible model, while UKF applies sigma point concept (restricted stochastic approach), where small set of samples from variable distribution are taken around its mean and only chosen point values are used in estimation and, thereby, several possible plausible realizations of a model are obtained. However, EKF and UKF are limited in large-scale system applications as well. So, recursive EnKF was devised and utilizes a fully stochastic approach. The method requires several realizations of a model and the covariance matrix is replaced with the sample covariance matrix and only part of it is calculated. Thus, EnKF is good for large models, but all estimates tend to be Gaussian which limits its application to non-Gaussian problems.

The EnKF consists of two steps: nonlinear prediction or a forward step and linear update or analysis step (Equations (1) and (2) respectively). The update step utilizes Bayesian update technique, where posterior probability distribution function (distribution of variable conditional to data) is minimized. Note that, in EnKF the solution is conditional only to data obtained at previous time steps, while in EnKS solution is conditional to data obtained at future time steps as well. PF is a generalized form of EnKF where the update is nonlinear; thereby nonlinear systems can be more easily modeled. But in most cases the PF technique is limited to small models and estimates are much more dependent on initial realizations or prior information of variable distribution. The two steps in the EnKF technique:

$$X_t^f = M\left(X_{t-1}^a\right) + E_{t-1}^{\text{model}} \tag{1}$$

$$X_t^a = X_t^f + \hat{C}_t^f \cdot H^T \cdot \left(H \cdot \hat{C}_t^f \cdot H^T + R\right)^{-1} \cdot \left(D - H \cdot X_t^f\right) \tag{2}$$

where X^f_t is the model matrix consisting of model parameters and states at forecast step n_t; X^a_t is the updated model matrix at analysis step n_t; M is the model operator that establish relationship between all variables, not necessarily linear; E^{model} is the model error, usually is assumed to be zero; D is the data matrix; H is the observation matrix consisting of 0s and 1s; $\hat{C}_t^f$ is the sample covariance matrix at time n_t calculated from matrix X^f_t. Theoretical aspects of the EnKF can be found in (Evensen, 2007).

The SSC technique is an iterative inverse modeling technique, whose goal is to estimate primary variables using additional measurements of a second variable and known relationship between them. A set of realizations is used and each realization represents plausible and possible actual distribution of estimated variable. Each realization is created one at a time – an ensemble of results is created by repeated application of the technique. The quality of prediction is based on objective function that should be minimized:

$$O^{(n)} = \sum_{i=1}^{N_{obs}} w_i \cdot \left(X^{obs} - X^{cal,(n)}\right)^2 \tag{3}$$

where, $O^{(n)}$ is the objective or penalty function at iteration step n; X^{obs} is the measured value of variable X; $X^{cal,(n)}$ is the estimated value of variable X at step n; N_{obs} is the number of variable measurements; w_i is the weight of i^{th} observation proportional to inverse of covariance matrix [R] of variable observation error, which consists of measurement and estimation errors, $[W] \propto [R]^{-1}$.

A comparison of other inverse modeling techniques on different examples can be found in (Zimmerman et al., 1998). These two techniques are quite different and have promise for practical application.

3. Implementation Details

An advantage of EnKF over other inverse modeling techniques is that less iteration/recursion steps need to be conducted in order to obtain reasonable estimation results and ability cope with large-scale systems. However, EnKF estimates heavily depend on initial realizations, whose number can be large and the method becomes computationally expensive for large-scale systems; many evaluations of the forward problem are required to converge the ensemble of realizations. So, first all prior information should be used for EnKF initialization and computational shortcuts should be devised to lower computational requirements.

An appropriate site-specific variogram model with reasonable ranges and directions of anisotropy help to decrease number of realizations used in EnKF ensemble and preserve estimation accuracy. Also, realizations can be ranked based on primary, secondary variables (static and dynamic) or both and the most likely realizations retained in a model, while others should be dismissed. Perhaps the 'best' realizations from different assimilation steps should be kept in a model unmodified at further steps; thereby, realizations from various assimilation steps are mixed. So, following question arises whether ensemble should be dynamic or partially static at some assimilation steps.

For large-scale models, the dimension of covariance matrix can be reduced to make EnKF more efficient. For this reason, gradual deformation (Hu, 2000), Karhunen–Loève (K-L) expansion (Zhang and Chen, 2005), covariance localization techniques may be applied. The covariance matrix could be stabilized and improved during the application of the EnKF.

The model error in EnKF becomes an important parameter that can accelerate convergence. It should be treated separately from the uncertainty in model structure (Evensen, 2007).

4. Computational Issues

A number of issues arise when solving inverse problems using numerical approaches. The challenges can be classified in three broad groups: inverse modeling technique limitation, data source, and model quality. Each group should be treated separately.

Constraint of specific inverse modeling method can be eliminated with different mathematical techniques. For EnKF possible improvements were discussed in previous paragraphs.

Measurements usually come at different scales with different measurement errors and of different amount. First data should be analyzed and grouped according to main geological features or information source, where measurement error is minimized. Later observations can be upscaled or downscaled to bring all data and discretized model to one support, which depends on several things, e.g. task requirements, model specification. If data are upscaled, it can be clustered by averaging out using assigned weights. The relationship between variables is used to support estimation of one variable with low measurements by other variables with sufficient observations.

Model quality is characterized by knowledge level of modeller on specific system mechanism. In petroleum reservoir context trade off between assumptions and closeness to reality is present, e.g. whether flow process is single or multiflow. Also it is not always a good idea to use model with full mathematics and physics that establish relationship between variables, since computational requirements can rocket. Instead proxy or approximative solution can be used that produces almost same quality of results.

5. Promising Avenues

Facies is one of the most important factors in petroleum reservoir characterization. The entire model largely depends on distribution of facies in space. Facies are discrete variables and should be treated differently from continuous ones. There is huge promising EnKF application on categorical variables. Also ambiguities about EnKF implementation for non-Gaussian and highly nonlinear problems should be resolved. Possible solutions are data transformation to Gaussian space and usage of Particle Filter instead, respectively.

6. Conclusion

Inverse modeling in petroleum reservoir characterization is promising numerical modeling field, since lots of secondary information is available. Inverse modeling techniques have been applied on different examples and satisfactory results are obtained. The EnKF is a suitable technique for petroleum reservoir modeling, since it is simple and requires relatively low computational resources. However, several implementation and computational issues should be solved, such as handling non-Gaussian, nonlinear, complex, and extensively large-scale systems.

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