Mathematical Programming Formulation for Long-Term Mine Planning in Presence of Grade Uncertainty

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Uncertainty is inevitable with sparse geological data. The optimality of the solution for an open pit production scheduling problem is affected dramatically by grade uncertainty. Recent research initiatives have attempted to consider the effect of grade uncertainty on production schedules. These methods either are aimed to minimize the risk or to maximize the net present value (NPV) without taking into account grade uncertainty explicitly. Another major problem in open pit production scheduling is the size of the optimization problem. The mathematical programming formulation of real size long-term open pit production schedules are beyond the capacity of current hardware and optimization software. In this paper, a mathematical programming formulism is presented to find a sequence in which ore and waste blocks should be removed from a predefined open pit outline and their respective destinations, over the life of mine, so that the net present value of the operation is maximized and the deviations from the annual target ore production is minimized in the presence of geological and grade uncertainty. An oil sand deposit at north of Alberta is used to generate an optimum schedule.

1. Introduction

Mine planning is a process to find a feasible block extraction schedule that maximizes net present value (NPV) and is one of the critical processes in the mining engineering. Also, there are some technical, financial and environmental constraints that should be considered. Whittle (Whittle, 1989) defined open pit mine planning as 'Specifying the sequence of blocks extraction from the mine to give the highest NPV, subject to variety of production, grade blending and pit slope constraints.

The uncertainty of ore grade may cause some shortfalls at the designed production and discrepancies between planning expectations and actual production (Koushavand and Askari-Nasab, 2009; Osanloo et al., 2008; Vallee, 2000). Different authors present methodologies to transfer grade uncertainty and show the impact of uncertainty at mine planning.

Geological characteristics of each point (grade) are assigned using available estimation techniques. Kriging (Deutsch and Journel, 1998; Goovaerts, 1997) is the most common estimation method used in industry; however, kriging results do not capture uncertainty and may be smooth leading to systematically biased reserve estimates. Mine plans that are generated based on one input block model fail to quantify the geological uncertainty and its impact on the future cash flows and production targets. Geostatistical simulation algorithms are widely used to quantify and assess this uncertainty. The generated realizations are equally probable and represent plausible geological outcomes (Deutsch & Journel, 1998; Goovaerts, 1997; Journel & Huijbregts, 1981). Choosing one or some of these realizations will not be objective to fair uncertainty assessment. Also, generated final pit limit and production schedule based on one block model would not necessarily be the optimum one. Therefore to get robust and optimum long-term production planning (LTPP), a sufficient number of realizations should be used simultaneously.

Godoy and Dimitrakopoulos (Godoy and Dimitrakopoulos, 2003) and Leite and Dimitrakopoulos (Leite and Dimitrakopoulos, 2007) present a new risk inclusive LTPP approach based on simulated annealing. A multistage heuristic framework is presented to generate a final schedule, which considers geological uncertainty so as to minimize the risk of deviations from production targets. A basic input to this framework is a set of equally probable scenarios of the orebody, generated by the technique of conditional simulation. They report significant improvement on NPV in presence of uncertainty.

Dimitrakopoulos and Ramazan (Dimitrakopoulos and Ramazan, 2004) proposed a probabilistic method for long-term mine planning based on linear programming. This method uses probabilities of being above or below a cut-off to deal with uncertainty. The LP model is used to minimize the deviation from target production. This method does not directly and explicitly account for grade uncertainty and also does not maximize the NPV.

Leite and Dimitrakopoulos (Leite and Dimitrakopoulos, 2007) present a technique that generates an optimal schedule for each realization. Afterwards, simulated annealing is used to generate a single schedule based on all schedules, such that the deviation from target production is minimized. The main drawback of simulated annealing is that it does not necessarily find the optimum solution. Figure 1 shows the stages of the stochastic production schedule framework presented by Leite and Dimitrakopoulos (Leite and Dimitrakopoulos, 2007) . For each of conditional simulation realizations, an optimum schedule is generated and using simulated annealing technique, a single schedule is generated based on all schedules such that deviation from target production be minimized.

Dimitrakopoulos and Ramazan (Dimitrakopoulos and Ramazan, 2008) present a stochastic integer programming (SIP) model to generate the optimal production schedule using equally probable simulated orebody models as input, without averaging the related grades. This model has a penalty function that is the cost of deviation from the target production and is calculated from geological risk discount rate (GDR) that is discounted unit cost of deviation from a target production. They use linear programming to maximize a function equal to NPV minus penalty costs. They concluded that the generated production schedule is the optimum solution that can produce the maximum achievable discounted total value from the project, given the available orebody uncertainty described through a set of stochastically simulated orebody models. The proposed scheduling approach considers multiple simulated orebody models without increasing the required number of binary variables and thus computational complexity. With this method, it is not clear how to define the GDR parameter. For different GDR values there are different optimal solution. A high GRD value causes blocks with less uncertainty to be extracted. The authors do not suggest any method to choose the optimum GDR parameter. At the presented model, mixed integer programming has been used. A variable is defined for each block. Adding constraints increases the complexity and CPU time to solve the optimization. The short-term production schedule is not taken into account. It is not dynamic and flexible to new information that is acquired during the mine life.

Appendix

$v_{l,n}^{t}$ The discounted revenue generated by selling the final product within block n in period t at realization number l minus the extra discounted cost of mining all the material in block n as ore and processing

$q_{l,n}^{t}$ The discounted cost of mining all the material as waste in block n in period t at realization number l.

E Number of element of interests in each block.

T Total number of periods

L Total number of simulation realizations

$o_{l,n}$ Ore tonnage in block n at realization l.

$g_{l,n}^{e}$ Average grade of element e in one portion of block n at realization l.

$r^{e,t}$ Processing recovery; the proportion of element e recovered in time period t

$p^{e,t}$ Price in present value terms obtainable per unit of product, element e

$cs^{e,t}$ Selling cost in present value terms obtainable per unit of product, element e

$cp^{e,t}$ Extra cost in present value terms per unit of production, element e

$w_{l,n}$ Waste tonnage in block n at realization l.

cm^{t} Cost in present value terms of mining a tone of waste in period t.

$a_{n}^{t} \in \{0,1\}$ Binary integer variable controlling the precedence of extraction of blocks. It is equal to one if extraction of block n has started by or in period t, otherwise it is zero

$z_{n}^{t} \in [0,1]$ Continues variable, representing the portion of bock n to be extracted as ore and processed in period t

$y_{n}^{t} \in [0,1]$ Continues variable, representing the portion of bock n to be mined in period in period t, fraction of y characterizes both ore and waste.

op_{l}^{t} is the over produced amount of ore tonnage above a desired tonnage, or upper limit, in period t and realization number l.

up_{l}^{t} is the under produced amount of ore tonnage bellow a desired tonnage, or upper limit, in period t and realization number l.

c_{op}^{t} is the discounted unit cost of op^{t} at period t.

c_{up}^{t} is the discounted unit cost of up_{l}^{t} at period t

V_{n}^{t} is the expected discounted revenue over all simulation realizations.

$Q_{l,n}^{t}$ is the expected discounted cost over all simulation realizations.

Tables

Table 1 shows the summary of the uncertainty based methods to solve LTPP problem. The major shortcomings of the current mine planning methods reviewed in this literature are:

- Most of the methods show the effect of uncertainty on the mine plan, but do not suggest a method to minimize the risk of uncertainty.
- There are some methods to deal with grade uncertainty, these methods either are trying to minimize the risk or maximizing NPV without using uncertainty explicitly. It is critical to maximize NPV and to minimize risk in presence of uncertainty explicitly.
- The methods are not suitable for real-size mining problems.
- They are not dynamic and flexible to the new information that is produced overtime.

2. Linear Programming (LP) formulation for mine planning

Mine planning is a process that defines a sequence of extraction of blocks with the objective of net present value (NPV) maximization. The mine production scheduling can be formulated as an optimization problem. NPV is the discounted revenue that discounted cost has been deducted from it:

$$\text{discountedprofit} = \text{discounted revenue} - \text{discounted costs} \tag{2.1}$$

Askari-Nasab and Awuah-Offei (2009) have presented the objective functions of the LP formulations that maximizes the net present value of the mining operation. Is it needed to define a clear concept of economic block value based on ore parcels which could be mined selectively. The profit from mining a block depends on the value of the block and the costs incurred in mining and processing. The cost of mining a block is a function of its spatial location, which characterizes how deep the block is located relative to the surface and how far it is relative to its final dump. The spatial factor can be applied as a mining cost adjustment factor for each block according to its location to the surface. The discounted profit from block is equal to the discounted revenue generated by selling the final product contained in block n minus all the discounted costs involved in extracting block, this is presented at eq.(2.1) The discounted cost can rewrite as eq (2.2) and eq (2.3):

$$v_n^t = \sum_{e=1}^{E} o_n \times g_n^e \times r^{e,t} \times \left(p^{e,t} - cs^{e,t} \right) - \sum_{e=1}^{E} o_n \times cp^{e,t} \tag{2.2}$$

$$q_n^t = \left(o_n + w_n \right) \times cm^t \tag{2.3}$$

Therefore the mathematical form of optimal mining schedule is presented at eq (2.4):

$$Max \quad \sum_{t=1}^{T} \sum_{n=1}^{N} \left(v_n^t \times z_n^t - q_n^t \times y_n^t \right) \tag{2.4}$$

Subject to:

$$g_l^{t,e} \le \frac{\sum_{n=1}^{N} g_n^e \times o_n \times z_n^t}{\sum_{n=1}^{N} o_n \times z_n^t} \le g_u^{t,e} \qquad \forall\, t = 1, 2, \ldots, T \;\; , \;\; e = 1, 2, \ldots, E \tag{2.5}$$

$$p_l^t \le \sum_{n=1}^{N} o_n \times z_n^t \le p_u^t \qquad \forall\, t = 1, 2, \ldots, T \tag{2.6}$$

$$m_l^t \le \sum_{n=1}^{N} \left(o_n + w_n \right) \times z_n^t \le m_u^t \qquad \forall\, t = 1, 2, \ldots, T \tag{2.7}$$

$$z_n^t \le y_n^t \qquad \forall t = 1,2,\ldots,T \;,\; n = 1,2,\ldots,N \tag{2.8}$$

$$a_n^t - \sum_{i=1}^{t} y_l^i \le 0 \qquad \forall t = 1,2,\ldots,T \;,\; n = 1,2,\ldots,N \;,\; l = 1,2,\ldots,C(L) \tag{2.9}$$

$$\sum_{i=1}^{t} y_n^i - a_n^t \le 0 \qquad \forall t = 1,2,\ldots,T \;,\; n = 1,2,\ldots,N \tag{2.10}$$

$$a_n^t - a_n^{t+1} \le 0 \qquad \forall t = 1,2,\ldots,T-1 \;,\; n = 1,2,\ldots,N \tag{2.11}$$

Where Eq. (2.5) is grade blending constraints; these inequalities ensure that the head grade of the elements of interest and contaminants are within the desired range in each period. There are two equations (upper bound and lower bound) per element per scheduling period in Eq.(2.5). Eq. (2.6) is processing capacity constraints; these inequalities ensure that the total ore processed in each period is within the acceptable range of processing plant capacity. There are two equations (upper bound and lower) per period per ore type. Eq. (2.7) is mining constraints; these inequalities ensure that the total tonnage of material mined (ore, waste, overburden, and undefined waste) in each period is within the acceptable range of mining equipment capacity in that period. There are two equations (upper bound and lower bound) per period. Eq. (2.8) represents inequalities that ensure the amount of ore of any block which is processed in any given period is less than or equal to the amount of rock extracted in the considered time period.

Eqs. (2.9) to (2.11) control the relationship of block extraction precedence by binary integer variables at block level. This model only requires the set of immediate predecessors' blocks on top of each block to model the order of block extraction relationship. This is presented by set C(L) in Eq. (2.9).

3. Case Study

An oil sands deposit in Fort McMurray, Alberta, Canada is used. Location of boreholes and histogram of data is presented at Figure 2 and Figure 3 respectively.

Directional experimental variograms are calculated and fit. The Azimuths of major and minor directions are 50 and 140 degrees. Figure 4 shows the experimental and the fitted variogram models in major (Figure 4a), minor (Figure 4b) and vertical (Figure 4c) directions.

Ordinary kriging is used to estimate the bitumen grade (with no normal score transform) at each block location. Multiple realizations of the bitumen grade are generated using Sequential Gaussian Simulation (SGS) (Isaaks and Srivastava, 1989) at a very high resolution three-dimensional grid at the point scale, this method is the means of constructing uncertainty models of bitumen grades. Figure 5a and 5b illustrate the map of the bitumen grade for the kriged, the E-type and realization 26 models. It is well-known that kriging is conditionally biased (Isaaks, 2005) and on the other hand "there is no conditional bias of simulation when the simulation results are used correctly" (McLennan and Deutsch, 2004). Conditional biasness of kriging can be reduced by tuning estimation parameter but it can not be eliminated (Isaaks, 2005). Grade-Tonnage curve is the good tool to check the impact of kriging biasness. Figure 6 shows the grade tonnage curve of simulation realization (dashed lines), krig (bold solid line) and Etype (bold dashed line). The systematic biasness of kriging was tried to be minimized but still the there are differences between kriging and simulation results. Also Etype is slightly different than kriging; Theoretically Etype model is identical with simple kriging result at Gaussian space (Journel and Huijbregts, 1981).

Histogram and variogram reproduction are checked. Figure 7 shows the histogram reproduction and Figure 8a to Figure 8c show the variogram reproduction at major and minor horizontal, and vertical directions. Generally Sequential Gaussian simulation would reproduce histogram and variogram of original data if it implements carefully. In this case reproduction of histogram and variograms are acceptable.

The sizes of the blocks used in mine planning are a function of the selective mining unit (SMU). The high resolution grid is up-scaled to get the right block scale values. Arithmetic averaging of point scale grades provides the up-scaled SMU grades

The final pit limit design is carried out based on the industry standard Lerchs and Grossmann algorithm (Lerchs and Grossmann, 1965) using the Whittle strategic mine planning software (Gemcom Software International, 1998-2008). The kriged, E-type, and fifty SGS realizations models are imported into the Whittle software.

The ultimate pit limit design is carried out based on the Syncrude's costs in CAN$/bbl of sweet blend for the third quarter of 2008 (Jaremko 2009). Price of oil was considered US$45 with an exchange rate of 1.25:1 equal to CAN $56.25/bbl SSB for the same time period. We assume that every two tonnes of oil sands with an average grade of 10% mass will produce one barrel of sweet blend, which is approximately 200 kg. We also assume a density of 2.16 tonne/m³ for oil sands, and a density of 2.1 tonne/m³ for waste material, including clay and sand.

Table 2 summarizes the costs used in the pit limit design. The mining cost of $12.18 is per tonne of oil sands ore, we assumed a stripping ratio of 1.8:1, and this would lead to a cost of $4.6/tonne of extracted material (ore and waste).

Table 3 shows the pit design and production scheduling input parameters. Thirty three pitshells are generated using 49 fixed revenue factors ranging between 0.1 to 2.5, based on the kriged block model. The number of pitshells is reduced to 14 after applying the minimum mining width of 150 meters for the final pit and the intermediate pits. Table 4 summarizes the information related to the final pit limit based on kriged block model at 6% bitumen cut-off grade.

The kriged model is the basis for production scheduling. The aim is to maintain a uniform processing feed throughout the mine life. Five years of pre-stripping is considered to provide enough operating space and ore availability. No stockpile is defined and the target production is set to 20 million tonnes of ore per year with a mining capacity of 35 million tonnes per year. There are 14607 blocks inside the final pit; it is almost imposible to solve the LP algorithm with this much of blocks. Therefore clustering algorithms has been applied and 500 mining cuts have been generated. Figure 9 shows the plane view of mining cuts at 280m. LP algorithm is used to maximize NPV. At this study we don't consider grade uncertainty and only used kriging to maximize NPV. Figure 10 illustrates the kriged block model schedule over 21 years of mine-life. Figure 11shows the plan view of block schedule and their year of extraction. Figure 12 shows the discounted cash flow. The cash back was happened during period 7 for all realizations. Black line is krig model which we optimized NPV using it. This schedule is followed for EType (blue dashed line) and simulation realizations (dashed red lines). NPV of kriging is 1.17 billion dollars. Table 5 shows the statistical information of krig, Etype and realization schedules.

Figure 13 shows the tonnage of feed to the plant. One should take into account that the shortfall in production has happened although we have used five years of pre-stripping. The effect of the grade uncertainty on the production targets would be more severe if the pre-stripping strategy was not adopted.

Figure 14 shows the input average head grade for each period. Kriged model (bold solid line), Etype model (bold dashed line) and the simulation realizations (dashed lines). LP optimization have tried to extract high grade zoons at first years to maximize Net present value.

4. Grade uncertainty based MILP formulation

A Stochastic integer Programming (SIP) model for optimizing long term production scheduling in open pit mines is developed with an objective function that maximizes the total NPV of the project under a managed grade risk profile. Grade uncertainty causes shortfalls from target productions. Therefore to get optimum solution, NPV must be maximized and deviation from target production must be minimized simultaneously among all simulation realizations:

$$\begin{cases} \text{Max. NPV} \\ \text{Min. Deviationfrom target production} \end{cases}$$

Dimitrakopoulos and Ramazan (Dimitrakopoulos and Ramazan, 2004) defined a geological risk discount rate (GDR) to allow the management of risk to be distributed between periods. They defined some costs for deviation from the target tonnage production and a discounted cost is calculated.

A new profit function is defined with two more extra variables per period. Therefore the new profit function for any block in the model is calculated by Eq.(4.1).

$$F_l = \sum_{t=1}^{T} \left\{ \sum_{n=1}^{N} \left(v_{l,n}^t \times z_n^t - q_{l,n}^t \times y_n^t \right) - \left(c_{cp}^t \times op_l^t + c_{up}^t \times up_l^t \right) \right\} \tag{4.1}$$

The mathematical programming model maximizes the expected profit function represented by Eq. (4.2) through all simulation realizations to get optimum solution in presence of grade uncertainty:

$$\begin{aligned}
& Max \frac{1}{L} \sum_{l=1}^{L} F_l \\
& = Max \frac{1}{L} \sum_{l=1}^{L} \left[\sum_{t=1}^{T} \left\{ \sum_{n=1}^{N} \left(v_{l,n}^t \times z_n^t - q_{l,n}^t \times y_n^t \right) - \left(c_{cp}^t \times op_l^t + c^t \times up_l^t \right) \right\} \right] \\
& = Max \ \frac{1}{L} \sum_{t=1}^{T} \left\{ \sum_{n=1}^{N} \left(\sum_{l=1}^{L} \left(v_{l,n}^t \right) \times z_n^t - \sum_{l=1}^{L} \left(q_{l,n}^t \right) \times y_n^t \right) - \sum_{l=1}^{L} \left(c_{cp}^t \times op_l^t + c^t \times up_l^t \right) \right\} \\
& = Max \ \frac{1}{L} \sum_{t=1}^{T} \left\{ \sum_{n=1}^{N} \left(L \times V_n^t \times z_n^t - L \times Q_{l,n}^t \times y_n^t \right) - \sum_{l=1}^{L} \left(c_{cp}^t \times op_l^t + c^t \times up_l^t \right) \right\} \\
& = Max \ \sum_{t=1}^{T} \left\{ \sum_{n=1}^{N} \left(V_n^t \times z_n^t - Q_{l,n}^t \times y_n^t \right) - \frac{1}{L} \sum_{l=1}^{L} \left(c_{cp}^t \times op_l^t + c^t \times up_l^t \right) \right\}
\end{aligned} \tag{4.2}$$

Both of V_n^t and $Q_{l,n}^t$ come from the Etype mean that is the average of all realization and are constant values. The constraints of this model are the same as model in Appendix 1.

There will be two constraints per period for each realization defined by Eq.(4.3) and (4.4):

$$\sum_{n=1}^{N} \left(o_{n,l} \times z_n^t - op_l^t \right) \le P_u^t \qquad \forall t = 1,2,\ldots,T \ , \ l = 1,2,\ldots,L \tag{4.3}$$

$$\sum_{n=1}^{N} \left(o_{n,l} \times z_n^t - up_l^t \right) \ge P_l^t \qquad \forall t = 1,2,\ldots,T \ , \ l = 1,2,\ldots,L \tag{4.4}$$

The model defined by Eq. (4.2) is superior to the model defined by Dimitrakopoulos and Ramazan (Dimitrakopoulos and Ramazan, 2008), in that, the amount of ore processed and amount of material mined are controlled by two separate continues variable rather than binary integer variables. This method enables us to have a high resolution solution for selection of ore and processing. This approach reduces the number of binary integer variable in the model drastically. Therefore the CPU time will be decreased and the model will be more efficient.

In this model, the number of variables equals the number of blocks multiplied by the number of periods. Therefore, it would be time consuming process to solve this linear programming. Boland et.al. (Boland et al., 2009) and Askar-Nasab and Awuah-Offeri (Askari-Nasab and Awuah-Offeri, 2009) tried to solve this problem with clustering the blocks to reduce the number of variables. Using some grade aggregation methodology and based on lithological information, similar blocks are summarized to a group and are dealt as one variable which will be extracted at the same period. Each group of blocks is called a mining cut. Grouping the blocks into mining-cuts is done without sacrificing the accuracy of the estimated (or simulated) values and to model a more realistic equipment movement strategy. The mining-cut clustering algorithm developed uses fuzzy logic clustering (Kaufman and Rousseeuw, 1990). Coordinates of each mining-cut has been represented by the centre of the cut and its spatial location.

The proposed linear programming was formulated in MATLAB environment (MathWorks Inc., 2007). TOMLAB/CPLEX (Holmström, 1989-2009) was used as the Linear programming Solver. TOMLAB/CPLEX efficiently integrates the solver package CPLEX (ILOG Inc, 2007) with MATLAB.

5. Value of stochastic optimization

The input grade is uncertain. Therefore LTPP is a stochastic programming. A number of realizations are generated by geostatistical conditional simulations algorithm. These realizations are equal probability. Assume that there are 10 simulation realizations that fully describe the uncertainty of the orebody. Therefore each of realizations has exactly 10% chance to be the same as actual deposit. Each of realizations is used to generate an optimum solution to maximize NPV. Assume that realization number 3 is the true realization. Therefore the NPV that is calculated with this realization is the highest NPV that can be achieved. This NPV obtained from scheduling of perfect information. For example if realization number 1 is used to generate a schedule, because the blocks value are not what is expected, the schedule based on this realization will not be an optimum solution. Since it is not known that which realization is the true one, the average of NPVs that has been calculated from all 10 realizations is the expected net present value of the project and is called 'expected solution of perfect information (ESPI)'.

Etype block model is calculated by averaging all realization values and maybe used as an input block model. To assess the uncertainty, 10 realizations can be used to get different NPVs by using the schedule that has been calculated by Etype values. The average of these NPVs is called the 'expected value solution (EVS)'

The more efficient method is to use scholastic optimization algorithms by using all 10 realizations simultaneously. At this method, all realizations and constrains are used to generate a single schedule that.

Again, 10 different NPVs can be calculated by using this schedule and 10 realizations. The average value of the 10 NPVs is called the 'expected stochastic solution (ESS)'.

Dimitrakopoulos and Ramazan (Dimitrakopoulos and Ramazan, 2008) defined the value of stochastic programming (VSP), that is the differences between the stochastic solutions and the expected value solution:

$$\mathrm{VSP} = \mathrm{ESS} - \mathrm{EVS} \tag{5.1}$$

The VSP is a positive quantity if the optimality of the problem depends on the uncertainty of the variables. VSP represents the cost of ignoring uncertainty in making a decision.

By using Stochastic Integer Programming (SIP), all possible outcomes (simulated orebody models) are taken into account. If only a single orebody model is used in optimization, the NPV may appear to be the highest for that specific orebody model. However during the mine operation, the grade and amount of ore for each block will be deferent than the estimated value. This causes some shortfalls and decreases the actual NPV. The SP considers these possible variations through the available simulated models, and as a result, quantifies and manages the risk of not meeting production targets properly.

Note that the VSP can only be 0 for some extreme cases where the optimal solution is not sensitive to the uncertain variables. This means that regardless of the scenario used as input to the mathematical programming model, the result would be exactly the same. It is a highly unlikely case to occur in mine production scheduling that the schedule does not depend on the grades and quality parameters involved. Although there is no general rule for the magnitude of the VSP, VSP is expected to increase with increasing variance in the variables related to the optimization.

5. Conclusions

The LP optimization has been applied to the kriged block model and the effect of grade uncertainty is illustrated by following the same schedule for all 50 realizations. Even an optimum schedule base on one block model is not optimum for other realizations and grade uncertainty cause over and under production at plant feed. To get optimum results, all conditional simulation realizations should take into account. A stochastic linear mathematical programming has been presented to minimize risk of uncertainty. A new profit function is defined. Any over and under production amount of ore at conditional realizations are penalized. Therefore by maximizing the new profit function, NOV is maximized and risk of under and over production was reduced. Future work is to write MATLAB code to implement the new profit function.

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Appendix

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$p^{e,t}$ Price in present value terms obtainable per unit of product, element e

$cs^{e,t}$ Selling cost in present value terms obtainable per unit of product, element e

$cp^{e,t}$ Extra cost in present value terms per unit of production, element e

$w_{l,n}$ Waste tonnage in block n at realization l.

cm^{t} Cost in present value terms of mining a tone of waste in period t.

$a_{n}^{t} \in \{0,1\}$ Binary integer variable controlling the precedence of extraction of blocks. It is equal to one if extraction of block n has started by or in period t, otherwise it is zero

$z_{n}^{t} \in [0,1]$ Continues variable, representing the portion of bock n to be extracted as ore and processed in period t

$y_{n}^{t} \in [0,1]$ Continues variable, representing the portion of bock n to be mined in period in period t, fraction of y characterizes both ore and waste.

op_{l}^{t} is the over produced amount of ore tonnage above a desired tonnage, or upper limit, in period t and realization number l.

up_{l}^{t} is the under produced amount of ore tonnage bellow a desired tonnage, or upper limit, in period t and realization number l.

c_{op}^{t} is the discounted unit cost of op^{t} at period t.

c_{up}^{t} is the discounted unit cost of up_{l}^{t} at period t

V_{n}^{t} is the expected discounted revenue over all simulation realizations.

$Q_{l,n}^{t}$ is the expected discounted cost over all simulation realizations.

Tables

Table 1: summarizes the uncertainty-based algorithms to solve LTPP problems.

Type of model	Author	Year	Solution	Advantages	Disadvantages
Risk analysis using deterministic algorithm	Rovenscroft	1992	Conditional simulation technique and deterministic LTPP algorithm	Shows the impact of grade uncertainty on LTPP	It can not quantify the risk of a project It does not optimal solution in presence of grade uncertainty
Risk analysis using dynamic programming	Dowd	1994	Conditional simulation and DP	Quantify risk associated in a project	It does not give any criteria to accept or reject the risk Does not produce optimal solution in the presence of grade uncertainty
Linear programming	Dimitrakopoulos and Ramazan	2003	Linear goal programming	Generates the schedule that reduces the risk at early production sages Consider equipment mobility and block access in production planning	Will schedule some blocks partially It does not generate maximum NPV in presence of grade uncertainty
Metaheuristic	Gody and Dimitrakopoulos	2003	Conditional simulation and Simulated annealing	Integration of ore body uncertainty, waste management and economic and mining consideration to generate optimal mining rates Produces a single optimum production planning in presence of uncertainty	Implementation is complicated The optimality of this method cannot be guaranteed It does not consider equipment access Grade uncertainty is not incorporated explicitly in the production planning process
Mixed integer Programming	Ramazan and Dimitrakopoulos	2004	Mixed integer Programming	Maximizes NPV explicitly with the consideration of equipment mobility and block access	It cannot implement on large deposit Grade uncertainty has not been used directly
Metaheuristic	Laite and Dimitrakopoulos	2007	Conditional simulation and Simulated annealing	Integration of ore body uncertainty, waste management and economic and mining consideration to generate optimal mining rates Produces a single optimum production planning in presence of uncertainty The method minimize deviation from target production that courses by grade uncertainty	It cannot implement on large deposit The optimality of this method cannot be guaranteed Simulated annealing technique is very complex and need lots of parameters. It is very slow and time consuming
Stochastic Mix integer programming	Dimitrakopoulos and Ramazan	2008	Linear stochastic programming	Maximizes NPV Grade uncertainty has been used directly Produces a single optimum production planning for a geological risk discount rate (GDR) value.	It cannot implement on large deposit. GDR parameter is not very clear. Short-term production schedule issue and blast data set are not taken into account
Risk analysis	Koushavand, Askari and Deutsch	2009	Conditional simulation technique	Shows the impact of grade uncertainty on LTPP	It can not quantify the risk of a project It does not optimal solution in presence of grade uncertainty

Table 2: Summary of costs used in pit limit design.

Description	Value	Description	Value
Mining Costs (CAN $/ bbl SSB)	24.35	Mining Costs (CAN $/tonne)	12.18
Upgrading Costs (CAN $/ bbl SSB)	10.05	Upgrading Costs (CAN $/tonne)	5.025
Others (CAN $/bbl SSB)	1.5	Others (CAN $/tonne)	0.75
Total Costs (CAN $/ bbl SSB)	35.9	Total Costs (CAN $/tonne)	17.28

Table 3: Final pit limit and mine planning input parameters.

Description	Value	Description	Value
Cutoff grade (%mass bitumen)	6	Processing limit (M tonne/year)	20
Mining recovery fraction	0.88	Mining limit (M tonne/year)	35
Processing recovery factor	0.95	Overall slope (degrees)	20
Minimum mining width (m)	150	Pre-stripping (years)	5

Table 4: Material in the final pit using the kriged block model.

Description	Value
Total tonnage of material (M tonne)	653.61
Tonnage of ore (M tonne)	280.5
Tonnage of material below cutoff (M tonne)	37.4
Tonnage of waste (M tonne)	335.71
Bitumen recovered (M tonne)	27.52
Stripping ratio (waste:ore)	1.33

Table 5: Statistical comparison of schaduels created with kriging, Etype and realization

	Tonnage of ore M tonne	Strip Ratio	Input Bitumen M tonne	Average %m	Produced Bitumen M tonne	NPV B Dollors
Krig	282.4	1.31	29.11	10.3	27.7	1.169
Max	282.3	1.43	29.03	10.5	27.6	1.168
Q1	277.2	1.39	28.5	10.3	27.1	1.126
Q2	274.7	1.37	28.1	10.3	26.7	1.095
Q3	272.3	1.350472423	28.0	10.2	26.61092591	1.074
Max	268.0	1.296943	27.4	10.0	26.052034	1.028
Etype	281.4	28.435457	28.4	10.1	27.013685	1.103

Figures

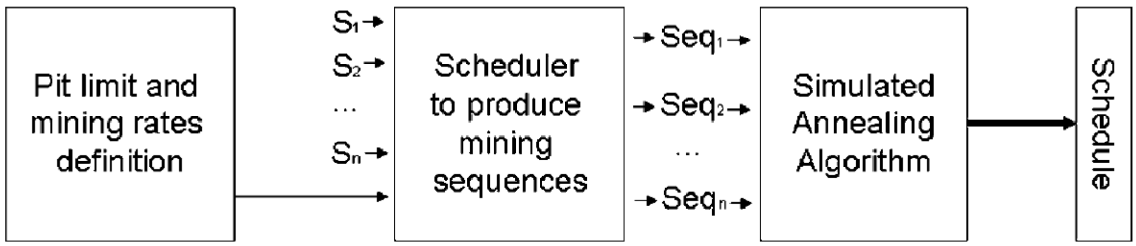

Figure 1. Three stages of mine production scheduling process (Leite and Dimitrakopoulos, 2007).

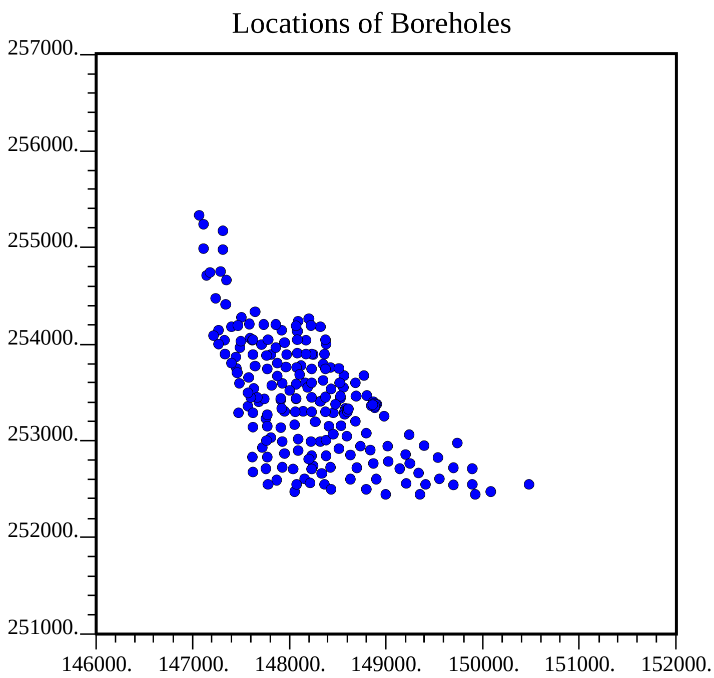

Figure 2: location map of boreholes.

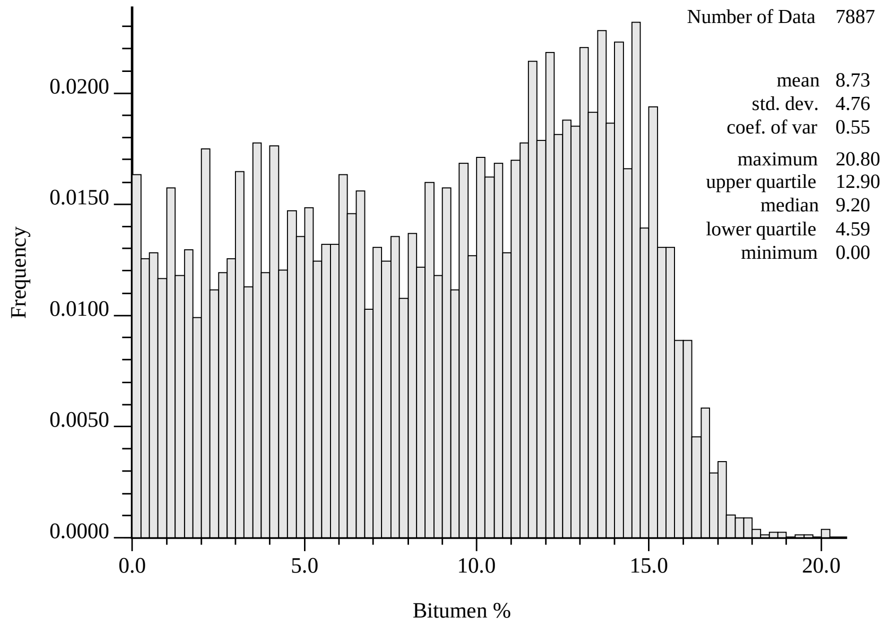

Figure 3: histogram of Bitumen

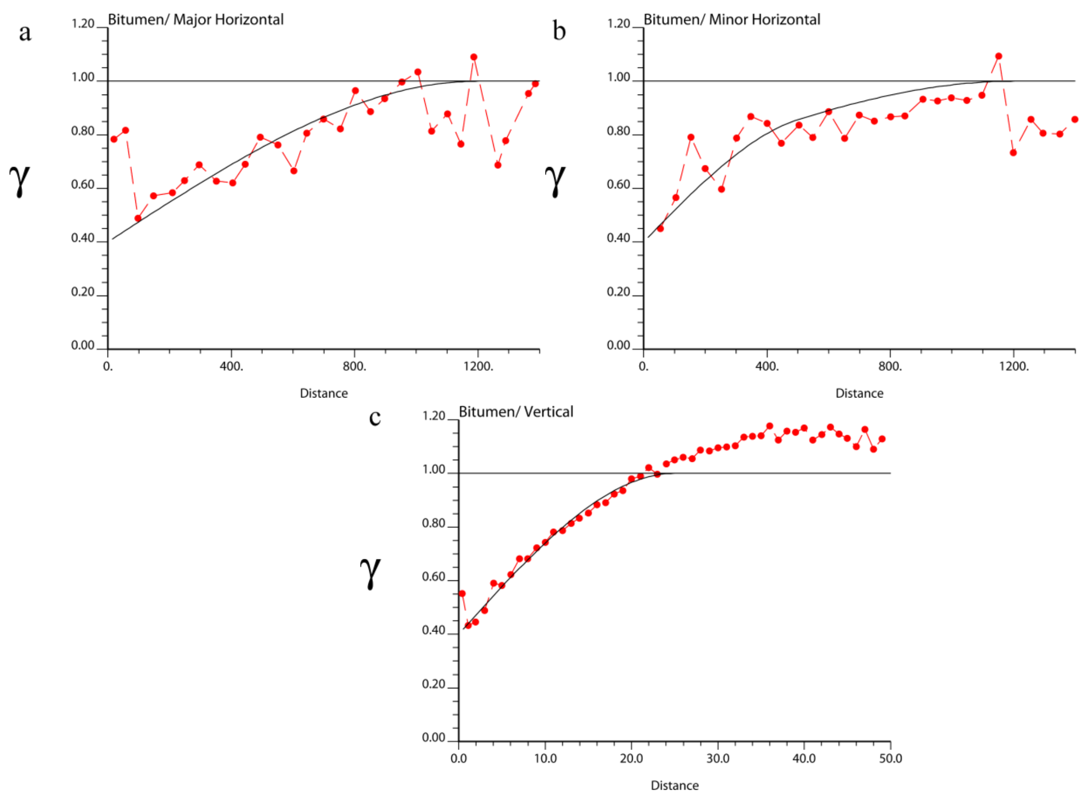

Figure 4: Experimental directional variograms (dots) and the fitted variogram models (solid lines), distance units in

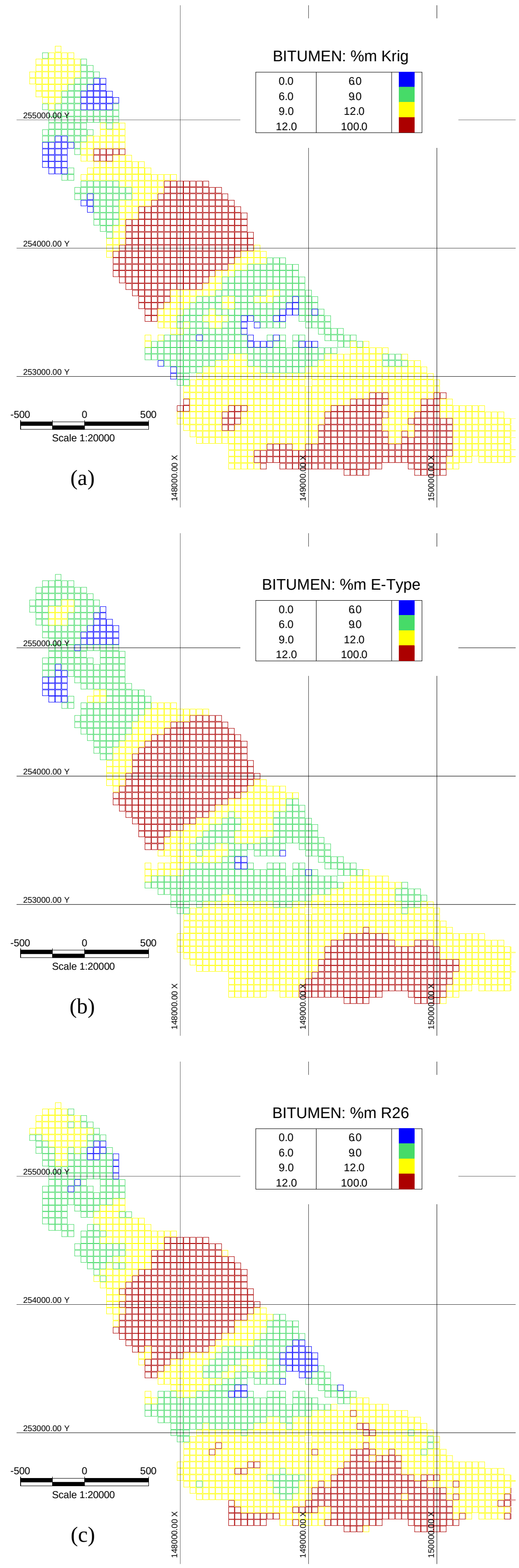

Figure 5: Plan view at 260m; (a) Kriged model, (b) E-type model, (c) realization 26.

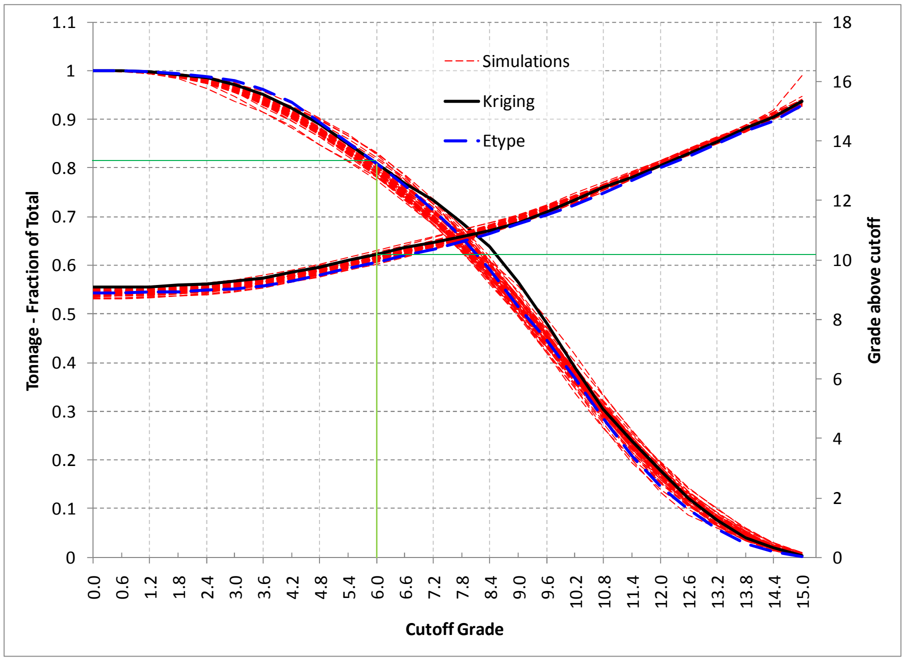

Figure 6: Grade tonnage curve of simulation realizations, kriged, and Etype block models

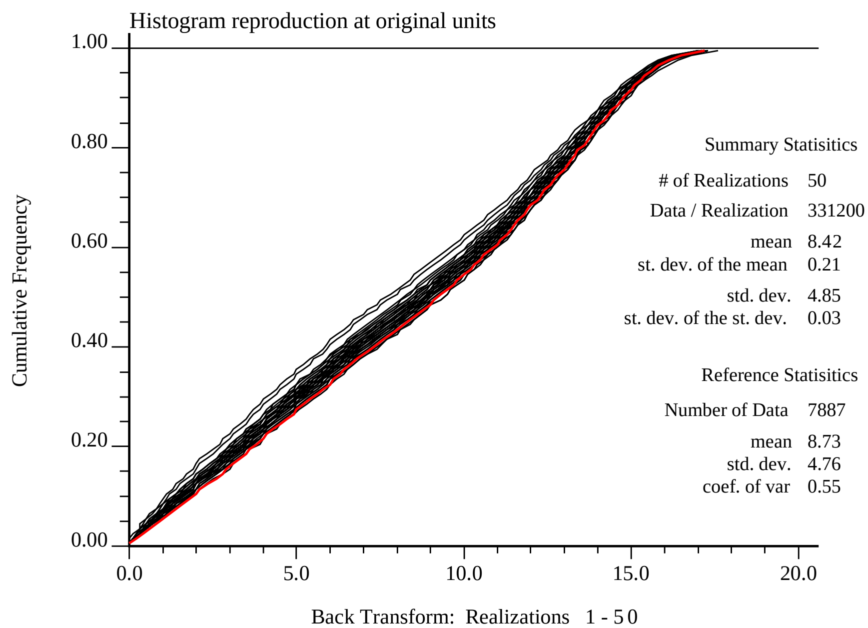

Figure 7: Histogram reproduction of simulation realizations (dashed lines) and histogram of original data (bold line)

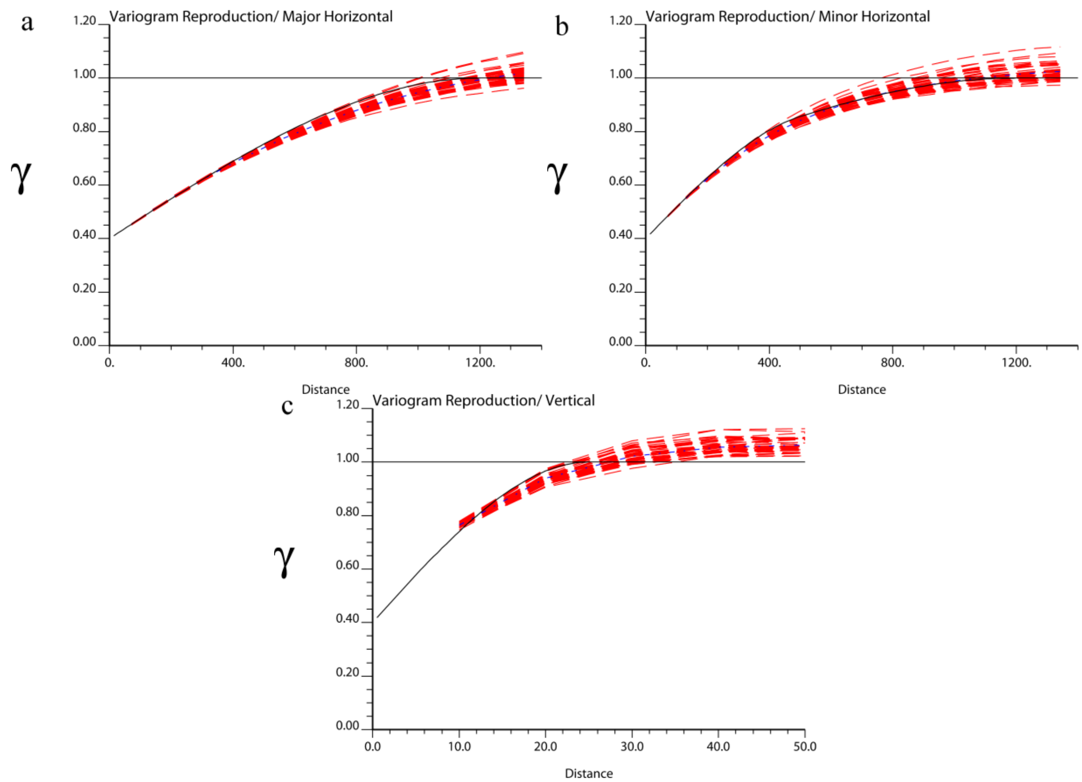

Figure 8: Variogram reproduction of simulation realizations (red dash lines) and reference variogram model (black line).

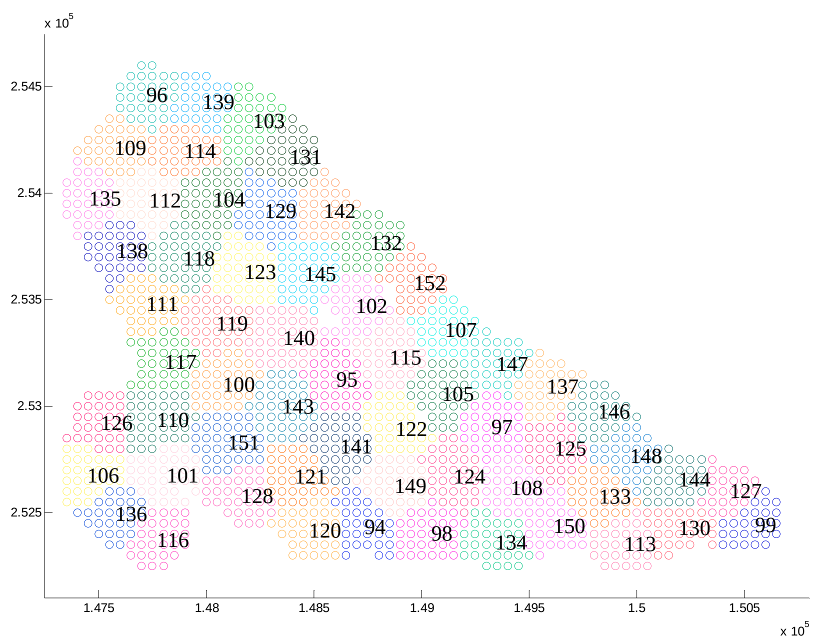

Figure 9: plan view of 280m with defined mining cuts.

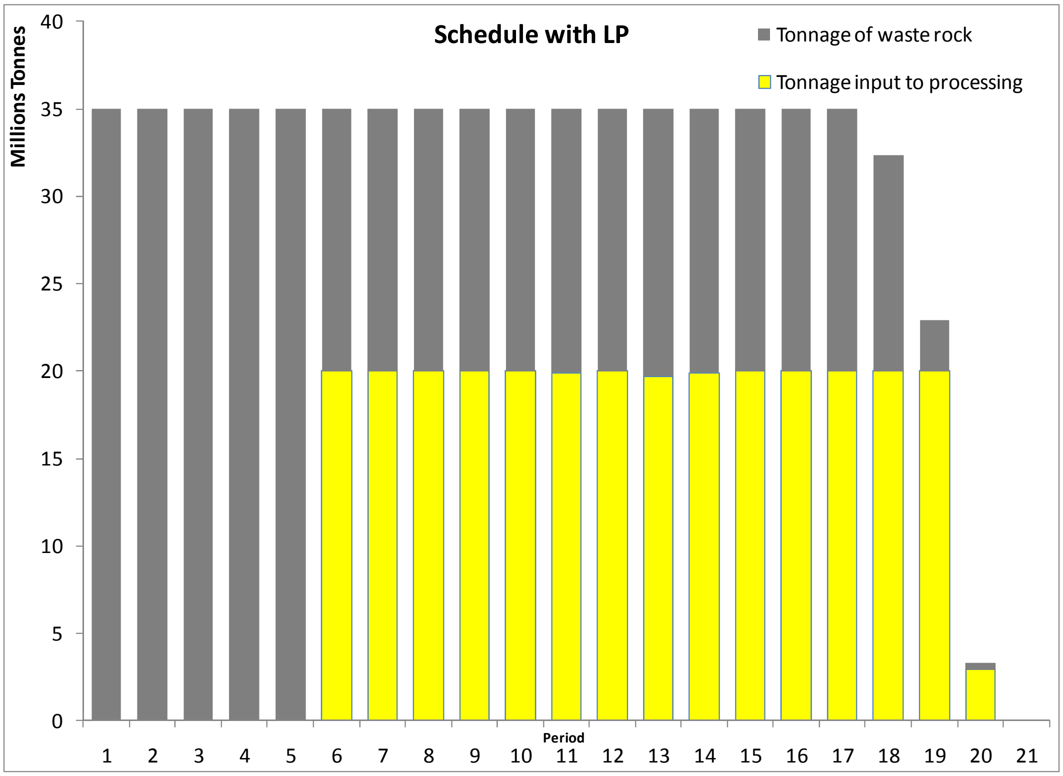

Figure 10: Production schedule based on the kriged block model.

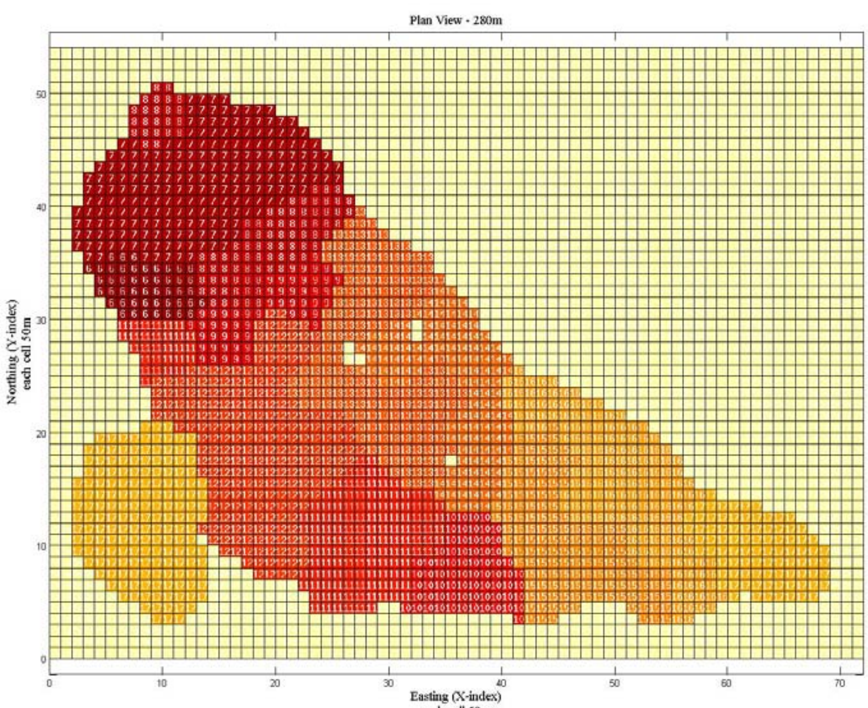

Figure 11: Plan view of 280m, schedule of extraction of blocks

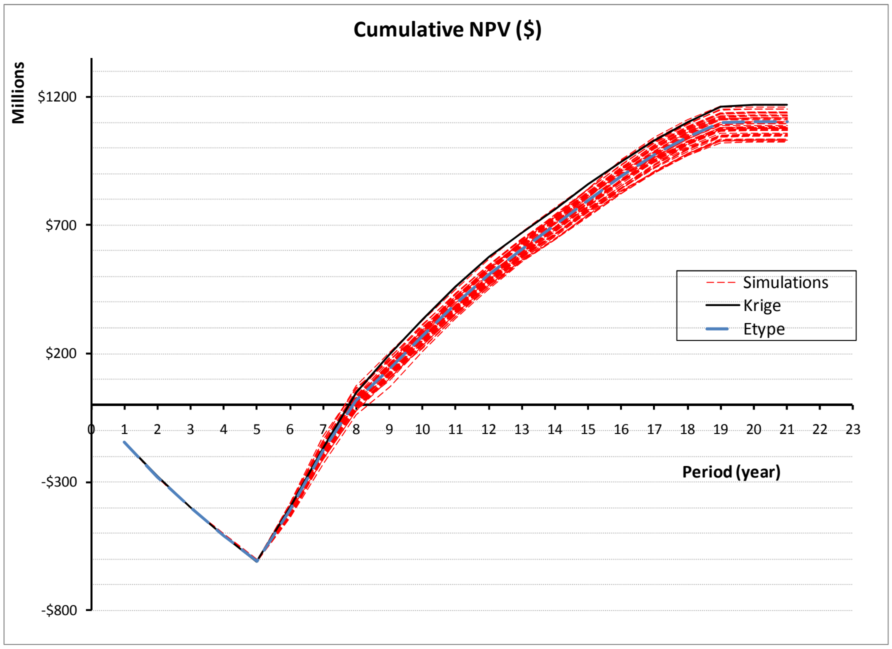

Figure 12: Cumulative discounted cash flow for 50 realizations (dashed lines) and kriged model (solid line).

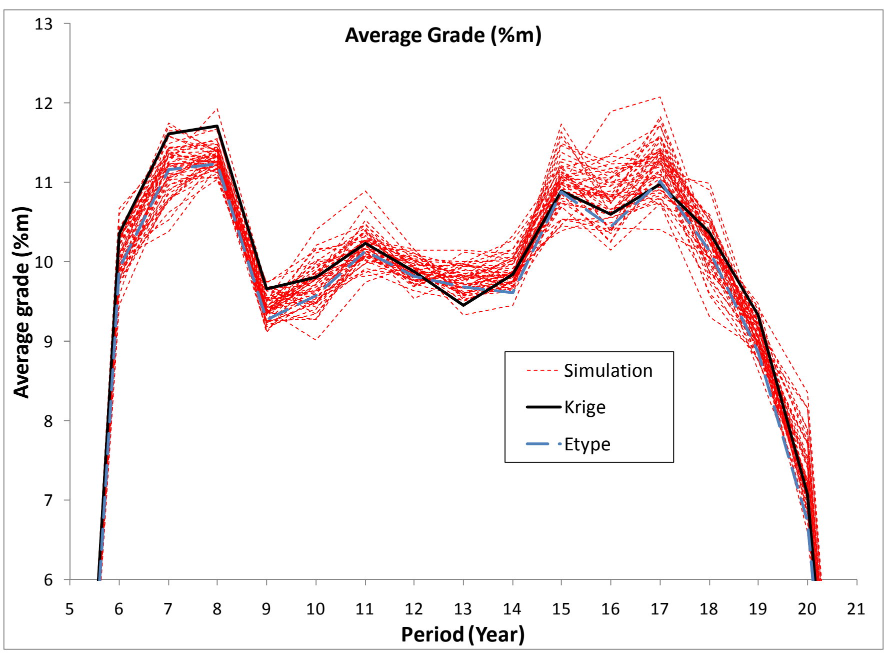

Figure 13: Head grade of simulation realizations, kriged, and E-type models.

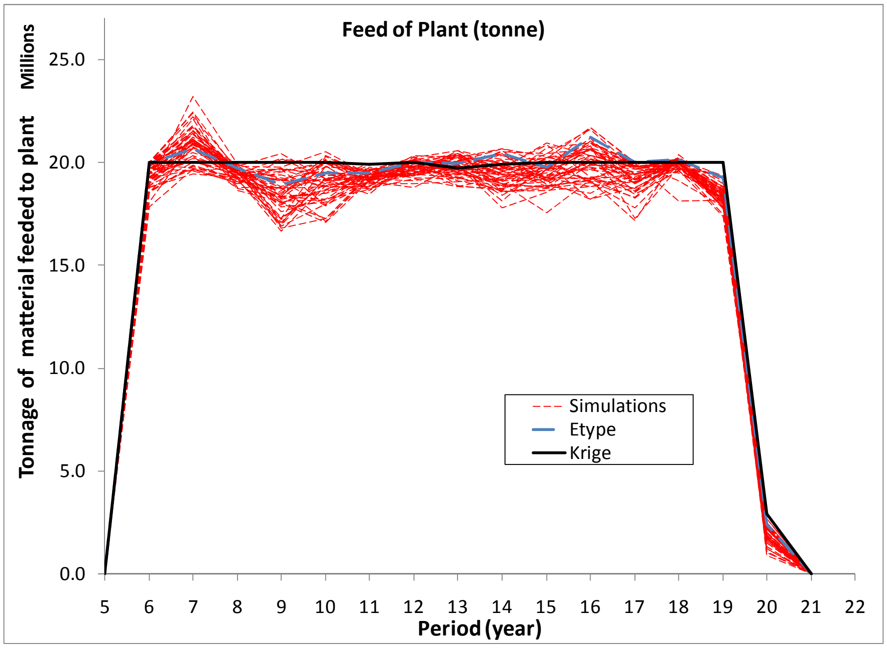

Figure 14: Plant feed (realizations dash lines), kriged model (solid line) and E-type (blue dash line).