Generating Locally Varying Anisotropy Fields

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Recently many works have been aimed at non-stationary geostatistics. First order non-stationarity is fairly easy to incorporate into modeling using locally varying means or data transformations; second order non-stationarity is more elusive and often requires advanced techniques. One methodology is to incorporate a locally varying anisotropy (LVA) field. This field describes the magnitude and direction of anisotropy. The generation of this field is very deposit specific; however, some techniques exist that are useful. This paper summarizes these techniques and introduces two methodologies for LVA field generation from exhaustive data; one methodology uses gradients and one uses principle component analysis. Results for orientation determination compare favorably to the past moment of inertia technique. In addition to these LVA generation methodologies presented, the axial nature of the orientation vector describing LVA causes issues and is reviewed.

Introduction

The assumption of second order non-stationarity is inappropriate for many deposits in the mining and petroleum industry; note that first order non-stationarity will be ignored as there are standard techniques available (see McLennan 2008 for a review). Some reasonable techniques exist for incorporating second order non-stationarity and often require the specification of an LVA field (orientation and magnitude of anisotropy throughout the domain):

Kriging with LVA: Boisvert and Deutsch (2010): uses nonlinear paths to incorporate locally varying anisotropy (LVA field required).

Local anisotropy kriging (Stroet and Snepvangers, 2005): automatically determines local orientation (LVA field not required) but the data must display the features of interest.

Local search reorientation (Deutsch and Lewis, 1992; Xu, 1996; Sullivan et al. 2007, to name a few): local variograms and search are reoriented based on the LVA field (LVA field required).

MPS with locally varying statistics: (Caers and Zhang 2002) Multiple point statistical methods rely on stationary training images that have consistent orientations (Piotr 2009), incorporating LVA in MPS realizations is possible but requires an LVA field.

If the conditioning data is sufficiently dense to show the non-stationary features, the LVA field may not be required and LAK kriging could be applied. However, often the knowledge behind the LVA field is due to a geological interpretation of the area or an exhaustively sampled secondary data source. In such cases, the data would not display the non-stationary (second order) features; an LVA field can be constructed from the additional knowledge and imposed on the model to incorporate this additional information into the numerical model (see above techniques). Four types of data will be considered for LVA field generation:

1) **Point source data**: rarely, accurate sampling of the LVA field at the desired scale is available; however, it is possible. The simulation or estimation of LVA orientation and magnitude is not straightforward and is discussed in the context of axial data.
2) **Geological knowledge of the deposit**: often the style of mineralization can help guide the generation of the LVA field. This type of data is qualitative and requires quantification in the form of an LVA field. This is very deposit specific and three examples are provided to demonstrate suggested workflows.
3) **Exhaustive secondary data**: When a secondary variable that is related to the variable of interest has been measured or calculated exhaustively (such as an interpreted geophysical survey) it can be used to generate the LVA field. Perhaps this is the most difficult (yet most common) type of

data encountered for which LVA inference can be made. Techniques for incorporating this type of data are introduced and discussed.

This paper is focused on inferring the locally varying orientations of anisotropy. Assuming a constant magnitude is often realistic, when this is not the case a smoothly varying range of correlation can be estimated or simulated with traditional geostatistical tools. Future work will be conducted on locally varying magnitude inference. When considering the orientation of anisotropy it is important to understand that the data is not just circular (which can be decomposed into xy components and modeled), but is axial where the direction x is identical to the direction x+180°. This is an important consideration that is often overlooked and requires discussion.

This paper is organized as follows. First the axial nature of LVA orientation is discussed which leads to the estimation of an LVA field from point samples such as dipmeter measurements. Secondly, techniques for generating LVA fields from all three types of data are provided. Examples of LVA generation from geological interpretations include porphyry, gold and layered deposits. Techniques discussed for dealing with exhaustive data include PCA, gradients and the moment of inertia.

On the axial nature of orientation data

When considering LVA the data of interest is directional. Moreover, the orientation is said to be axial (or undirected) as the angle x is equivalent to –x. This is akin to assuming that the continuity of a deposit is the same in the strike and strike+180˚ orientations. This assumption need not be made; however, it is implicit in most geostatistical algorithms. The implications of this are that orientations of x and x+180˚ are identical. Typically when dealing with directional data a decomposition into the X and Y components of the vector is made (C=cosx and S=sinx) but this is only appropriate if the angles wrap on the unit circle, that is 0˚ is equivalent to 360˚. When considering axial data, such as LVA orientation, this standard transformation is not appropriate.

There are a number of distributions of interest when considering axial data (the interested reader is referred to Mardia and Jupp (2000) for more information). Consider the Fisher-Bingham distribution with pdf:

$$f(\mathbf{x};\boldsymbol{\mu},\kappa,\mathbf{A})=\frac{1}{\alpha(\kappa,\mathbf{A})}\exp(\kappa\boldsymbol{\mu}^T\mathbf{x}+\mathbf{x}^\mathbf{T}\mathbf{A}\mathbf{x})$$

Where **A** is a symmetric covariance matrix and $\mathbf{x}^\mathbf{T}\mathbf{x}=\mathbf{1}$ **.** If **A**=0 then the distribution reduces to the von Mises-Fisher density (Mardia and Jupp, 2000). If $\kappa=0$ then the distribution reduces to the Bingham density making the above distribution quite flexible for describing data on spheres.

Generating LVA fields from point source data

Dealing with axial data requires standardization of the observations such that $0\le \mathrm{x}\le 2\pi$ rather than $0\le \mathrm{x}\ \le \pi$ which is characteristic of axial data. This is required so that correct wrapping for axial data at 180˚ occurs rather than at 360˚ which is standard for directional data. Mardia and Jupp (2000) suggest simply doubling the data such that $0\le \mathrm{x}\ \le 2\pi$; in this way all the distributions and techniques that are appropriate for circular (2D) or spherical (3D) data can be applied to axial data. Data doubling is recommended when attempting to estimate orientation at an unsampled location when inferring orientation from point source data such as dipmeter measurements. After forcing the data to $0\le \mathrm{x}\ \le \pi$ (i.e. a strike of 270˚ is recoded to 90˚), these values are doubled and the standard transformation is applied:

$$C=Rcos(x) \quad \text{and} \quad S=Rsin(x)$$

Once the data has been transformed to circular coordinates (or spherical in 3D) C and S can be estimated (or simulated) using traditional techniques (kriging, SGS, etc). Care should be taken to reproduce the correct correlation between C and S; this correlation is very problem specific and may require cokriging or cosimulation if there is a strong relationship between C and S.

Generating LVA fields from geological knowledge

Often the most appropriate LVA fields are based on a detailed understanding of the deposit geology. Incorporating such LVA fields can improve numerical modeling by improving geological realism.

Unfortunately, the generation of such LVA fields is deposit specific and no single methodology will be appropriate for all deposits. Three examples of LVA fields will be provided to help guide interested readers in some of the available techniques for generating LVA fields from conceptual knowledge. The first example is a typical layered deposit where continuity follows along the orientation of a known or modeled surface. The second example is a folded gold deposit (see paper 104 in this report). The third example is a porphyry deposit (Boisvert and Deutsch 2011).

Generating LVA fields from surfaces is fairly straightforward. The dip and dip direction of the surface can be extracted with most commercial software and can be used as the strike and dip for the LVA field. In these cases the plunge of the LVA field is set to 0 (Figure 1). Note that this is different than a standard strategraphic transformation and is more flexible when considering multiple surfaces.

The LVA field for the second example, the gold deposit from paper 104 in this report, is created based on a geologist's interpretation of a quartz vein (Figure 2) and is generated by fitting three polylines to the limbs and hinge of the folded structure (Figure 2). Note that the deposit has been rotated such that strike is exactly North-South and the overall dip is 0°. For each East-West cross section, the intersection of these polylines (three points) is fit with a parabola. The slope of this parabola at each location of the model is used as the LVA field (Figure 3).

The third example considers the generation of a circular LVA field for the porphyry deposit detailed in Boisvert and Deutsch (2011) and is reproduced here for consistency. "The orientation of the LVA field is tangential to a circle with variable radius and position, within the barren core the anisotropy ratio is 1:1 and increases linearly to 10:1 through a transition zone" (Figure 4). This is typical of porphyry deposits and is a simple way to construct the LVA field. Each XY slice of the model is considered separately by defining the center location of the circle and the radius. The dip and plunge angles are assumed to be zero, a simple coordinate transformation can be applied if the deposit is dipping.

Generating LVA fields from exhaustive secondary data

Generating LVA fields from point data (discussed above) and from geological knowledge (discussed above) is very domain specific and is difficult to provide detailed guidance for; however, when there is an exhaustive secondary variable (i.e. geophysical survey or outcrops) additional tools are required to extract the orientation of LVA from the data. This assumes that the continuity of the variable of interest is identical (or highly related) to the exhaustively sampled secondary data. This discussion will be limited to 2D for visualization purposes. Consider the folded outcrop in Figure 5. The goal is to extract the local orientation from the image. The moment of inertia technique has been discussed in the past (Hassanpour and Deutsch 2008); however, results are mixed. When the shapes are well defined and the window size can be selected reasonably well, reasonable results are obtained (Figure 6). When the details of the features are less clear, as in this example, the MOI technique does not capture the underlying features of the image (Figure 7). Note, better results may be obtained with detailed tuning of the window size and other input parameters (Hassanpour and Deutsch 2008).

Two methodologies for generating orientation from the exhaustive data are presented. The first methodology is based on using the gradient of the image to determine orientation. The second methodology is based on PCA.

The first methodology (implementation based on Kovesi, 2000) determines the gradient at each location in the model by taking the difference between adjacent cells after applying a Gaussian kernel to smooth noise. The resulting orientation of the gradient provides the orientation at the given location.

The second methodology was developed by Feng (2003) for image and texture analysis. The methodology is fairly complex and requires a multi-scale pyramid decomposition of the image and uses the maximum likelihood estimate to determine the orientation (Feng, 2003).

The two methodologies are applied to the same folded deposit and compared (Figure 8). Note that the gradient method provides a measure of reliability for the orientation, orientations that are below a suggested limit of 0.5 are not shown.

Both the gradient and PCA methodologies perform well while the moment of inertia technique does not appear to capture the underlying features of the geology. Thus far the methodologies have been examined using an outcrop image but often seismic data is used to infer the orientation and contains noise. Adding Gaussian white noise to the image and comparing the resulting orientations

(Figures 9 and 10) to the orientations without noise (Figure 8) allows for an assessment of the sensitivity of the proposed methodologies to noise. At a signal to noise threshold of 10 the methodologies have difficulty reproducing the underlying features.

Both tested methodologies have promise for the determination of LVA orientation, even from data with significant noise. Visually, the PCA method appears less sensitive to noise, likely due to the hierarchical implementation of the methodology where the image is upscaled to a very coarse resolution and an optimal orientation across multiple scales is determined.

Conclusions

The main difficultly when dealing with axial data was reviewed and a 'doubling' of the data is the practical solution. Moreover, two new methodologies for determining LVA orientation were presented and generate reasonable results with the models tested. The main source of future work in this regard is the extension of the methodologies to 3D. The gradient based approach can be easily extended to 3D while the complex nature of the PCA implementation (Feng 2003) is more difficult.

References

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Xu W, 1996. Conditional curvilinear stochastic simulation using pixel-based algorithms. Mathematical Geology. 28(7):937-949.

Figures

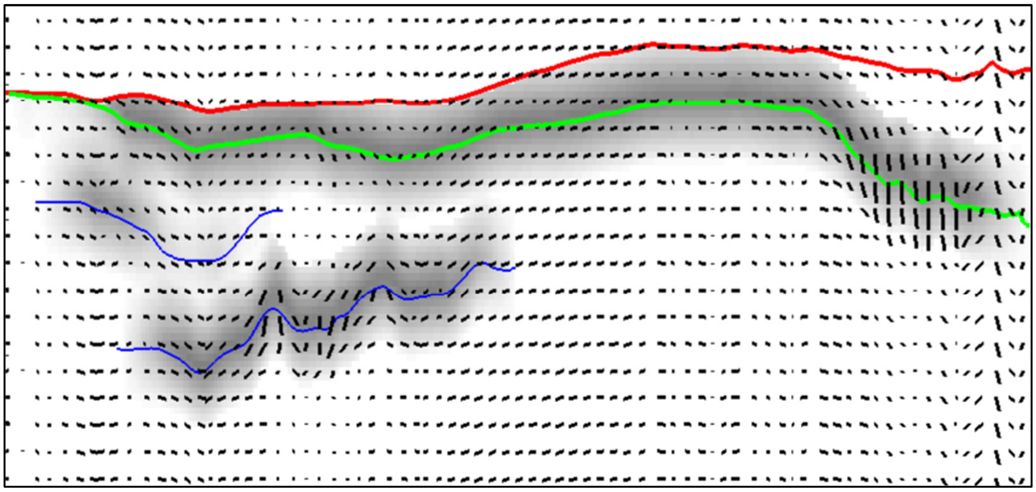

Figure 1: LVA field from surfaces. The intermediate surfaces only have influence around their respective areas. LVA orientation is a vertical average of all dips of each surface inversely weighted by the distance to the surface. Unit distances assumed. Note that some orientations seem off as the surface dips sharply into the page in some areas (2D slice of the 3D LVA field shown).

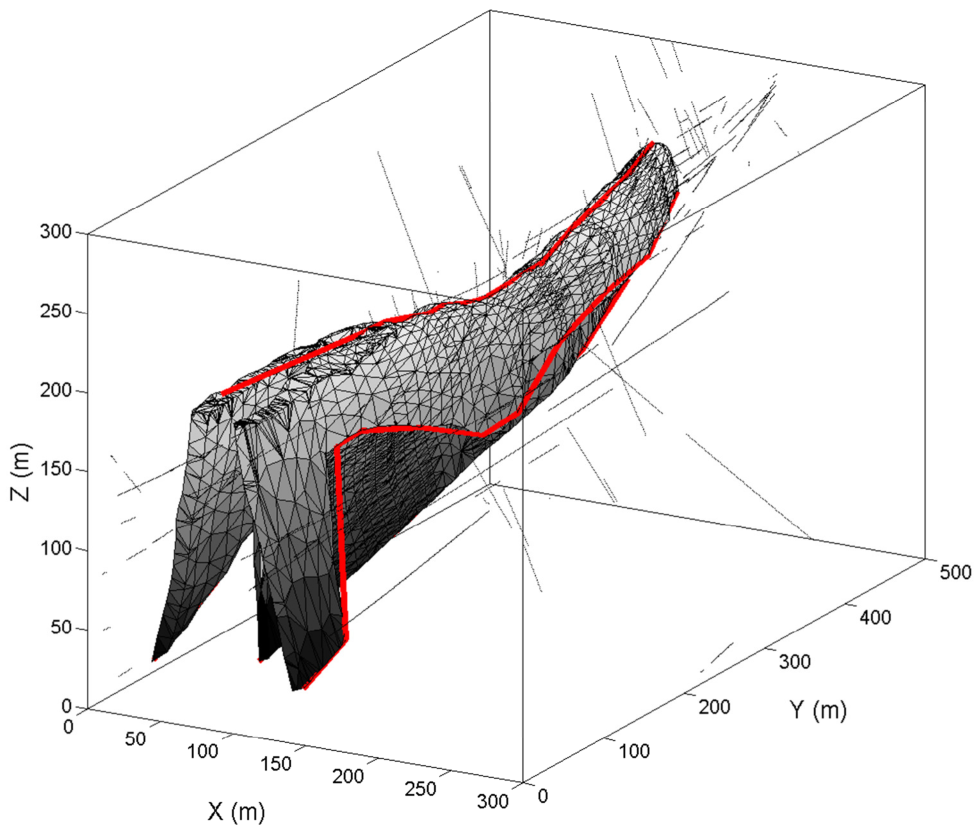

Figure 2: Newmont's deterministic solid. Three polylines fit to the limbs and hinge for LVA field generation.

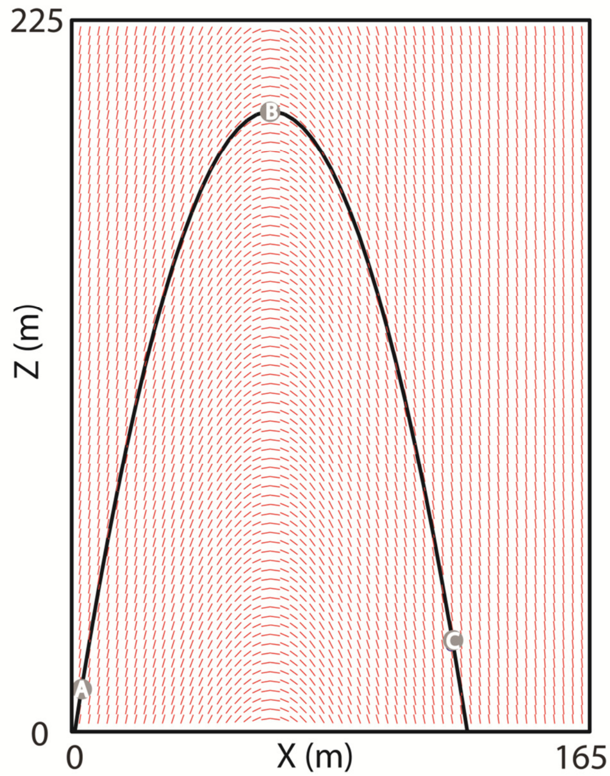

Figure 3: Typical cross section of the LVA field. Interpreted polylines from Figure 2 (gray intersections A, B, C) are fit with a parabola to determine the exhaustive LVA orientation.

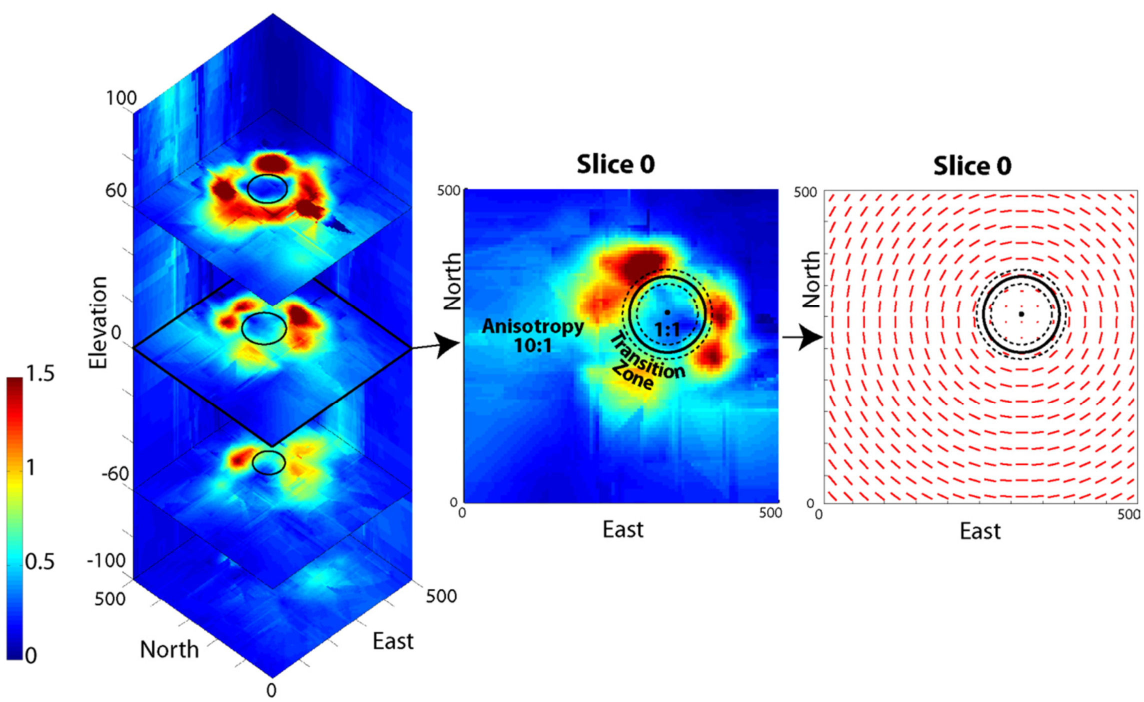

Figure 4: Left: Multiple slices of the block kriging map used to generate the LVA field. Middle: For each slice the LVA field is 1:1 inside the circle (defined manually for each slice) and 10:1 outside. Right: The LVA field. Length of the line is proportional to the anisotropy ratio.

Figure 5: Grayscal folded outcrop for generating LVA (from http://rst.gsfc.nasa.gov/Sect2/Sect2_1a.html).

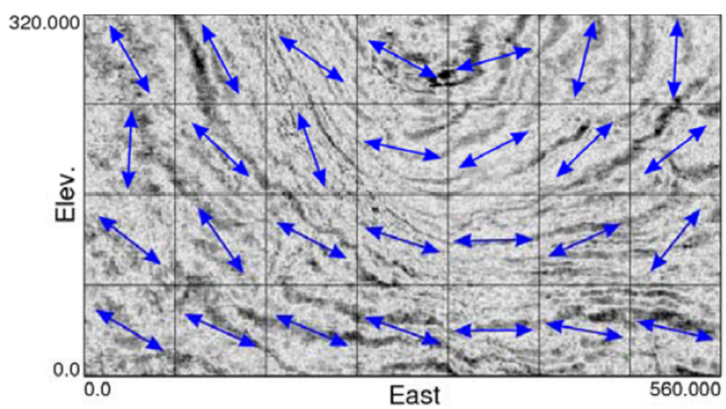

Figure 6: Using the moment of inertia to generate LVA oriention (Hassanpour and Deutsch 2008).

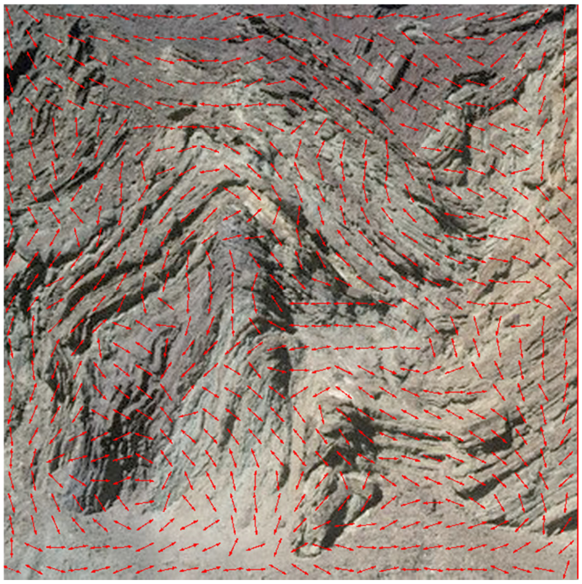

Figure 7: Using moment of inertia to generate LVA oriention for the outcrop.

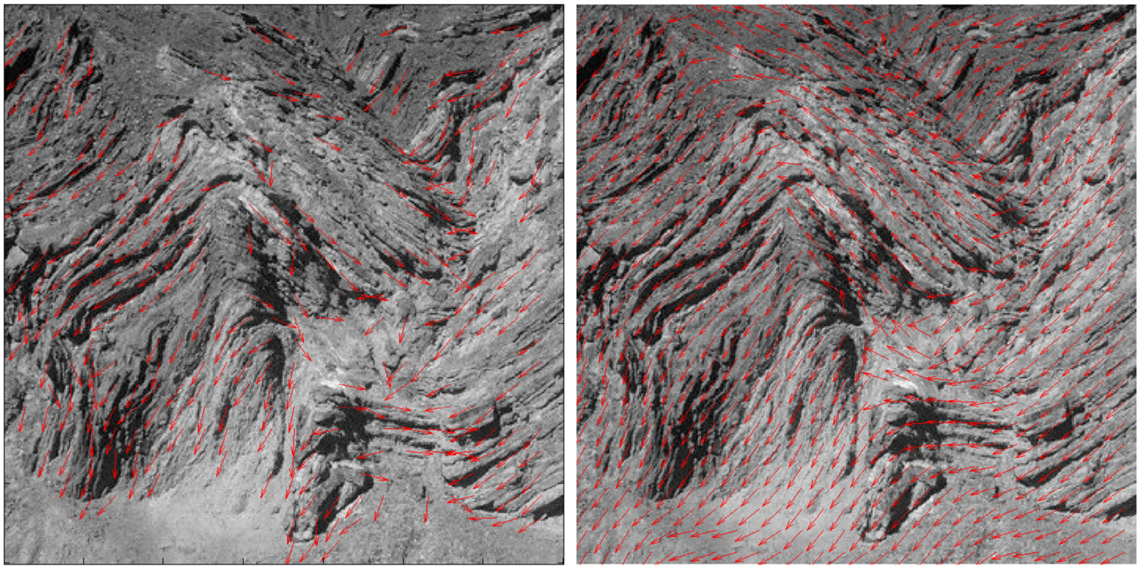

Figure 8: Using the gradient (left) and PCA (right) based methdoologies to generate the LVA oriention for the outcrop.

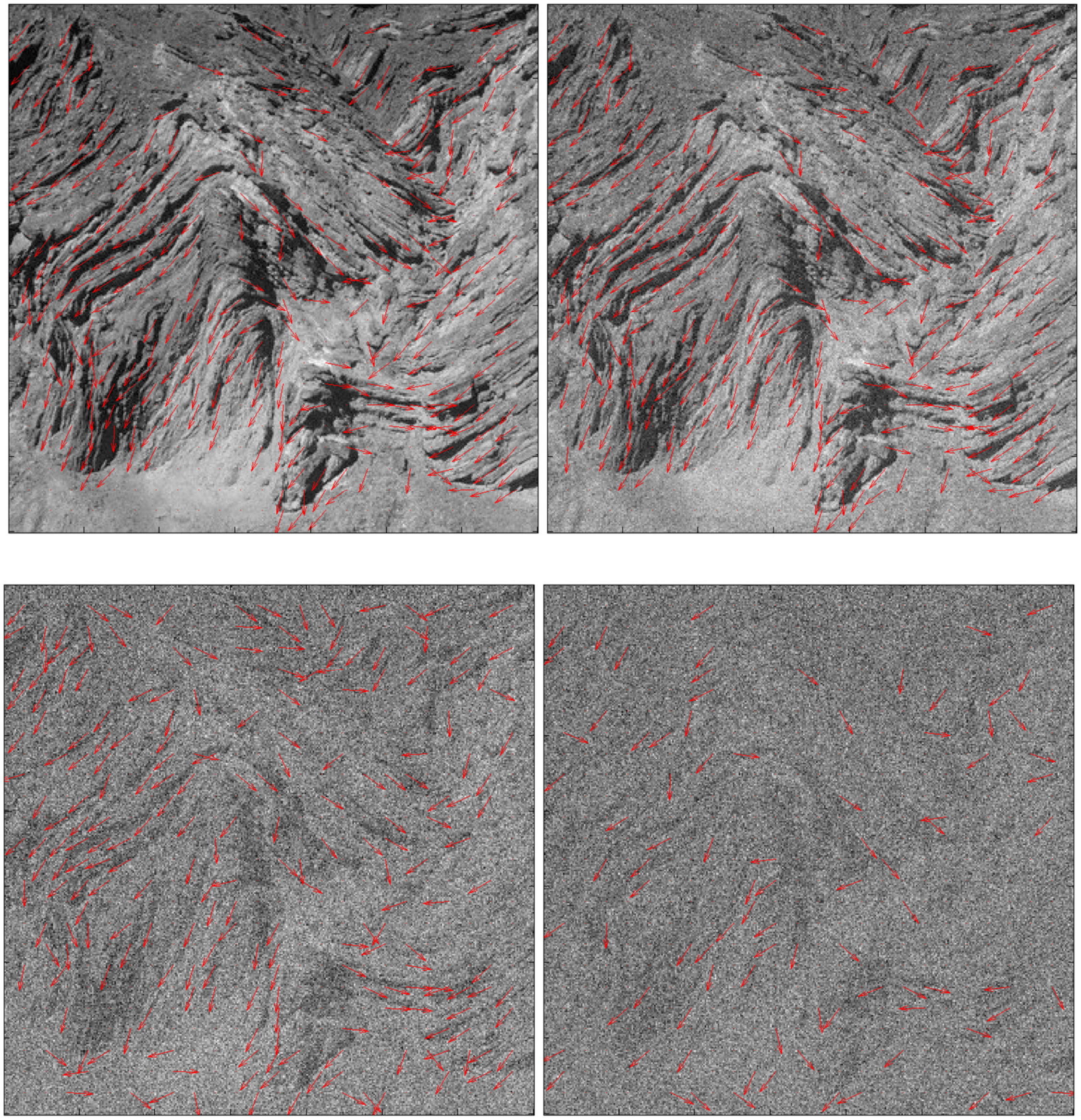

Figure 9: Gradient methodology adding Gaussian white noise to the image. From left to right, top to bottom, the signal to noise ratio is 50 25 10 5.

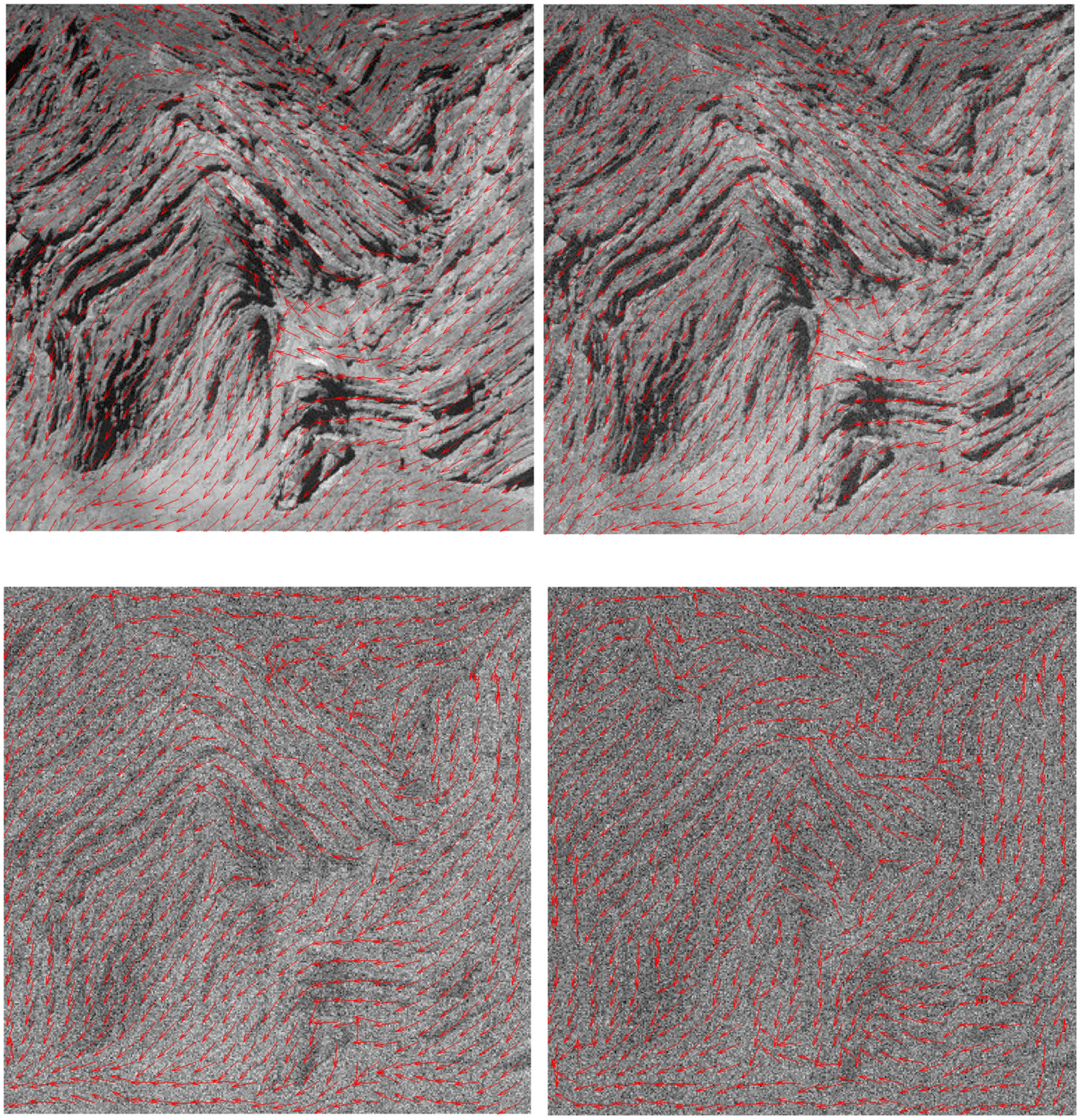

Figure 10: PCA methodology adding Gaussian white noise to the image. From left to right, top to bottom, the signal to noise ratio is 50 25 10 5.