Stochastic Distance Based Geological Boundary Modeling with Realistic Curvilinear Features

Jeff B. Boisvert

Obtaining accurate geological boundaries and assessing the uncertainty in these limits is critical for effective ore resource and reserve estimation. The uncertainty in the extents of an ore body can be the largest source of uncertainty in ore resource estimation when drilling is sparse. These limits are traditionally interpreted deterministically and it can be difficult to quantify uncertainty in the boundary and its impact on ore tonnage. The proposed methodology is to consider stochastic modeling of the ore boundary with a distance function recoding of the available data. This technique is modified to incorporate nonstationary modeling in the form of a locally varying anisotropy field used in kriging and sequential Gaussian simulation. Implementing locally varying anisotropy kriging retains the geologically realistic features of a deterministic model while allowing for a stochastic assessment of uncertainty. A case study of a gold deposit in Northern Canada is used to demonstrate the methodology. Locally varying anisotropy kriging generates realistic, anticlinal, geological boundary models and allows for an assessment of the uncertainty in the model.

Introduction

When considering resource estimation with a standard geostatistical block model, there are two main sources of uncertainty due to sparse sampling: (1) the uncertainty in the mineral grade distribution which controls the quality of ore found within each block of the model, and (2); identifying which blocks in the model are considered to belong to the ore deposit and which blocks are unmineralized waste. The boundaries between rock types define the stationary zones within which the data are considered to come from the same distribution (i.e. same mean, variance and spatial statistics). The grade distributions are modeled within each independent domain using kriging, inverse distance or change of support based strategies. The determination of the boundary separating ore and waste is a critical aspect in ore reserve estimation because incorrect assumptions of stationarity can lead to a significant bias in the final resource estimation model. Normally, it is not sufficient to model grades first and assume waste is determined by cutoff alone. Typically, a rock type model is generated prior to grade estimation to define stationary areas. The focus of the proposed methodology is to generate boundaries that are consistent with the geological knowledge of the ore body. Moreover, an important consideration in any boundary modeling approach is the ability to assess uncertainty in the boundary model. It should be noted that property modeling within the volumes defined by the modeled boundaries is beyond the scope of this work, the interested reader is referred to any standard geostatistical text for more information on property modeling (Cressie 1993; Isaaks and Srivastava 1989; Journel and Kyriakidis 2004; to name a few).

When drilling is dense, an accurate boundary model can be made deterministically because the uncertainty in the boundary is deemed sufficiently small and is ignored; however, when drilling is sparse the uncertainty in the ore/waste boundary is a significant factor in assessing resources and should be incorporated into numerical models. There are two general types of boundaries, hard and soft boundaries. Hard boundaries consider no transition zone between stationary areas, whereas soft boundaries contain a transition zone where the statistics change smoothly between the areas. Hard boundaries can be converted to soft boundaries by considering a mixture model between the two areas, thus, only hard boundaries will be considered. Hard boundary modeling is traditionally accomplished by manually digitizing the contact between zones on two dimensional slices of the deposit and interpolating between the slices; however, this typically does not allow for an objective assessment of the uncertainty in the boundary or when uncertainty is incorporated it requires the modeler to develop best, expected and worst case scenarios that are difficult and time consuming to construct. Stochastic models are an alternative to deterministic approaches and, by construction, allow for uncertainty assessment.

Indicator kriging or sequential indicator simulation (SIS) can be used when sufficient data is available and all geological features, such as folding, are well controlled by the conditioning data. Typically, in the exploration and pre-production phases of resource assessment, the spatial continuity of complex deposits is not captured with the available data spacing. In this situation, stochastic methods, such as SIS, rely heavily on the assumptions of stationarity and Gaussianity; these assumptions typically produce boundaries that are linear and do not conform to local geological features unless there is sufficiently dense data to control the models. The proposed methodology uses locally varying anisotropy (LVA) to incorporate complex geological geometries into numerical modeling. This is a balance between manual boundary inference where geological understanding dominates but uncertainty is difficult to quantify and stochastic methods where strong assumptions do not allow room for geological interpretations. The technique implemented is a distance based boundary model where the distance to the boundary is modeled exhaustively in the domain and zero contours define the boundary. This technique has been explored (Henrion Caumon and Cherpeau 2010; Jones Baerentzen and Sramek 2006; McLennan 2008) but previous works have relied heavily on the assumption of a global orientation of spatial continuity or sufficient conditioning data to control the nature of the boundaries. Under a decision of global stationarity, it is difficult to incorporate folding deposits or other nonlinear complex spatial features.

The paper is organized as follows. First, motivation for considering uncertainty in the ore deposit boundary is provided with a synthetic example. Manual deterministic boundary modeling is common in the mining industry and it is important to motivate the quantification of uncertainty in the boundary and its effect on ore tonnage. Second, the proposed methodology is described with a small example. A 2D geological cross section is modeled (1) deterministically (2) with the distance based approach and (3) with the proposed LVA distance based approach. Third, the LVA distance based methodology is demonstrated in 3D with a folded gold deposit in Northern Canada.

Motivation

To quantify the effect of boundary modeling on ore resource calculations, consider a lognormally distributed variable, which is common with variables such as grade. For simplicity, the deposit is assumed rectangular with a 20m thickness (Figure 1). The uncertainty due to the grade distribution can be determined by simulating 100 realizations using sequential Gaussian simulation (SGS) within the assumed boundary. The ore tonnage for each realization is calculated and the standard deviation of the set of realizations is used as a measure of the resource uncertainty within the deposit. Consider generating more realizations while varying the boundary within the limits defined by the outer drill holes (Figure 1) where the boundary holes do not intersect the deposit. It is assumed that the deposit limits defined by the upper and lower layers (i.e. the 20m thickness) has no uncertainty, considering uncertainty in the vertical dimension would only increase the importance of boundary modeling in the resulting analysis. The range of the variogram relative to the drill hole spacing is an important factor in assessing the overall uncertainty (Figure 2). The nugget effect is assumed to be zero and a single spherical structure is used. With this geometry, the boundary uncertainty is always higher than the uncertainty due to property modeling within the boundary (Figure 2 left). Note that this depends heavily on the shape of the deposit and how dense the delineation drill holes around the perimeter of the deposit are (Figure 2 right).

In a typical mine life, as mining progresses data spacing decreases and the uncertainty in the boundary also decreases; only with sparse data, relative to the spatial continuity of the deposit, is the uncertainty due to boundary modeling important (Figure 2 right). The intention of this example was to quantify the level of uncertainty resulting from unknown deposit limits and to motivate the importance of effective assessments of this uncertainty. When drilling is dense a single deterministic boundary model is sufficient as the uncertainty in the grade distribution within the stationary volume defined by the boundary dominates; however, boundary effects are critical when the drilling is sparse or the grade distribution within the deposit does not vary significantly (i.e. low coefficient of variation).

Methodology

Data typically available to model geological boundaries are measurements of the ore/waste contact in drill holes. Using the distance metric formulism, the drill hole data is recoded based on the distance from the sample to the nearest boundary intersection (Figure 3). The data within the ore deposit receive a negative distance and the data outside the deposit receive a positive distance. After the samples are coded based on the distance to the nearest sample, the values are interpolated for all unsampled locations and the ore/waste boundary is defined as the zero contour in 2D or isosurface in 3D (Figure 4). The nature of the underlying interpolation methodology controls the nature of the boundary. Past implementations of distance based boundary modeling use Euclidian distance to generate an isotropic or globally anisotropic boundary (Henrion Caumon and Cherpeau 2010; McLennan 2008), a review of techniques to determine the distance is provided in Jones, Baerentzen and Sramek (2006). The limitation with these implementations is the use of a globally stationary variogram which results in either (1) isotropic geological shapes, controlled somewhat by data conditioning or (2) if an anisotropic covariance function is used, geological shapes tend to be orientated along a single direction of continuity.

Consider a small 2D example where seven drill holes are used to delineate a deposit with three ore bodies. Assume that the ore bodies can be distinguished in the core and the respective intervals of ore are assigned to either ore body A, B or C. Using the distance function recoding and exhaustively interpolating the distance using an isotropic distance method (simple kriging), ore bodies B and C are relatively well reproduced because drilling has accurately delineated these objects (Figure 4 left); however; note the poor reproduction of ore body A when compared to the original geological interpretation (Figure 3). This poor reproduction is due to the limitation of considering an isotropic interpolation of the distance function. Incorporating a global direction of anisotropy (Figure 4 right) improves the connectivity of ore bodies B and C but ore body A remains disconnected although there is improvement over the isotropic case. There is some flexibility when kriging is used to estimate the distance function, the local mean can be adjusted to dilate the boundary and assess uncertainty (Figure 4 right); however, notice that the boundary dilates in a radial direction and never connects ore body A because of the local difference in anisotropy around the anticline. In reality, the orientation of geological features is often non-stationary and accounting for this in the distance function is important. Inverse distance methodologies would produce similar results as they also rely on the same Euclidian distance metric that underlies kriging.

The methodology proposed herein is to use anisotropic kriging (Boisvert and Deutsch 2010, 2011) to model the distance function. Geological features, such as anticlines, are stochastically incorporated into boundary modeling through an LVA field (Figure 5) that controls the local nature of the boundary, allowing for complex geometries in the resulting models (Figure 6). The orientation of the LVA field captures the local direction in which the deposit is most continuous (i.e. along the anticline for ore body A); this orientation is manually inferred from the geological interpretation of the ore body and is assumed to have a constant anisotropy ratio of 0.1. The anisotropy ratio is the ratio between the correlation range in the minor and major directions. Implementing a stochastic approach, such as the distance function, allows for an assessment of the uncertainty in the boundary (see case study) which is not possible with deterministic methods.

The LVA field is a critical input parameter and should be generated with consideration given to different geological interpretations. If there is uncertainty in the local geology, this should be carried through the analysis in the form of discrete scenarios with different LVA fields. The technique used for determining the LVA field is specific for each problem and type of data available; in this synthetic example it is possible to determine the orientation of the LVA field based on a visual inspection of the underlying deterministic interpretation. LVA field generation is discussed in more depth for the case study as a complex 3D LVA field is required.

The LVA field is incorporated into kriging following the methodology presented in Boisvert and Deutsch (2010, 2011). In short, the shortest anisotropic path using the Dijkstra algorithm (Dijkstra 1959) is used to determine the similarity (i.e. covariance) between data locations when building the kriging matrix. For example, Figure 5 shows the shortest path between locations A and B; the straight-line path does not correspond to the shortest distance due to the resulting anisotropic distance involved in traversing across cells perpendicular to the local orientation of anisotropy. This is only the case when assuming LVA. When considering an isotropic Euclidian distance or when anisotropy is constant as in most geostatistical algorithms, the shortest path is a linear segment connecting locations. Determining the shortest path would be a non-convex optimization problem where the anisotropic distance in a locally stationary area (i.e. one block of the model in Figure 5) is calculated as:

$$d = \sqrt{\left(\frac{d_{horiz\,max}}{a_{horiz\,max}}\right)^2 + \left(\frac{d_{horiz\,min}}{a_{horiz\,min}}\right)^2 + \left(\frac{d_{vertical}}{a_{vertical}}\right)^2}$$

where the anisotropy ratios are $a_1 = \frac{a_{horiz\,min}}{a_{horiz\,max}}$ and $a_2 = \frac{a_{vertical}}{a_{horiz\,max}}$

where the a parameters are the standardized ranges of correlation in a particular direction defined by the local orientation of the LVA field. Typically the major direction ($ahoriz_{max}$) is plotted with the minor ($ahoriz_{min}$) assumed perpendicular (Figure 5). The d parameters are the distance components in the principle directions with the variogram function assumed to have a standardized range of 1.0.

The distance function is exhaustively interpolated using kriging with the LVA field and the sign of the distance in each block of the model is used to determine an ore or waste classification. This is typically sufficient for traditional geostatistical approaches applied to ore reserve estimation; however, for visualization the boundary can be interpreted between blocks, in 2D the boundary corresponds to a polyline and standard contouring software can be used to determine the boundary. The extension to 3D results in an isosurface that must be interpolated between grid blocks (Bourke 1994).

The inclusion of soft secondary data can be accomplished by considering a locally varying mean with simple kriging. In areas where the secondary data indicate the location is likely inside the boundary, the local mean is decreased and areas where the secondary data indicate waste, the mean is increased. A calibration to drill hole data is required.

Case Study

The proposed LVA distance function methodology is applied to a subset of the Doris North deposit. The Doris deposit is part of the Hope Bay volcanic belt and is located approximately 170 km southwest of Cambridge Bay just south of Hope Bay in Nunavut, Canada (Carpenter et al 2003). The belt is an Archean greenstone volcanic belt (Carpenter et al 2003) with extensively studied local and regional geology; the interested reader is referred to the following references for additional geological information (Carpenter et al 2003; Fraser 1964; Gebert 1990, 1993; Sherlock et al 2003). Of interest to the proposed methodology is the anticlinal geometry of the deposit. A typical cross section of the deposit highlights the nonlinear features of the ore body (Figure 7). Any grade modeling of the deposit should include a geologically realistic boundary model of the mineralization found in the main quartz vein.

The available data, provided by Newmont Mining Corporation, consists of 667 drill holes with a naïve average gold grade of approximately 5g/t within the deterministic solid (no cutoff). A subarea of the Doris deposit (hinge zone on Figure 7) is used to demonstrate the methodology. A total of 59 drill holes penetrate the deposit within the modeling area. The modeling dimensions are 115m x 510m x 225m with 3m blocks (model size of 55x170x75) that cover a length of 510m along strike.

The methodology for generating the LVA field used in this example can be found in CCG paper 103 of this report and is shown in Figure 8.

The drill hole data available in the model area is recoded as per the distance function approach (Figure 3). With the LVA field provided in Figure 8 the boundary can be modeled (Figure 9). The volume of ore using the distance methodology with traditional kriging is quite low at 0.24Mm3 when compared to the deterministic estimate using the interpreted solid (paper 103) of 1.07 Mm3. This is due to the inability of traditional kriging to accurately account for the known geometry of the deposit resulting in poor limb reproduction (Figure 9) and significant volume underestimation. Incorporating LVA reduces this bias and results in a volume of 0.73Mm3. The bias remaining in kriging with LVA is due to the conditional bias of kriging and the large volume of waste relative to ore. This can be corrected using SGS; the resulting mean of 100 realization is 1.03 Mm3, 4% less than the deterministic solid.

The main benefit of incorporating LVA in boundary modeling is the ability to assess uncertainty. SGS is used to generate multiple realizations of the distance function (Figure 10). The resulting distributions of ore volume can be used to assess potential uncertainly in future mine plans developed for the area. The correct spatial structure of the deposit is reproduced and the global resources indicated by the deterministic solid are well respected (Figure 11).

The effect of anisotropy on the resulting models can be shown by examining how a slice of the model changes as the anisotropy ratio is increased (Figure 12). Without anisotropy (ratio = 1.0) the model creates circular regions around the intersections due to the linear weighting used in kriging. As the anisotropy ratio is increased from being isotropic to highly anisotropic (ratio = 0.1, 10 times more continuous along the anticline) the effect on the boundary model is increased conformance to the underlying anticlinal geometry (Figure 12).

Conclusions

As geostatistical techniques become common in standard mineral resource estimation applications, the incorporation of geologically realistic features into the model is becoming important. Practitioners are concerned that statistical models do not honor the known geological nature of the deposit under consideration. This is a valid concern if the end use of the models, such as mine design, is significantly affected by nonlinear spatial relationships. These geological features are not typically incorporated because it is difficult to statistically quantify complex geometries with standard two point geostatistical techniques such as kriging and SGS. The use of multiple point geostatistical techniques to handle non-linearities in geology are not yet common in mining. The proposed methodology can be used to incorporate complex geometries into numerical modeling when the LVA field can be inferred from an understanding of local geology or exhaustive secondary data. A reasonable LVA field can be made through consultation with expert geologists and important geological features incorporated into geostatistical modeling.

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Figures

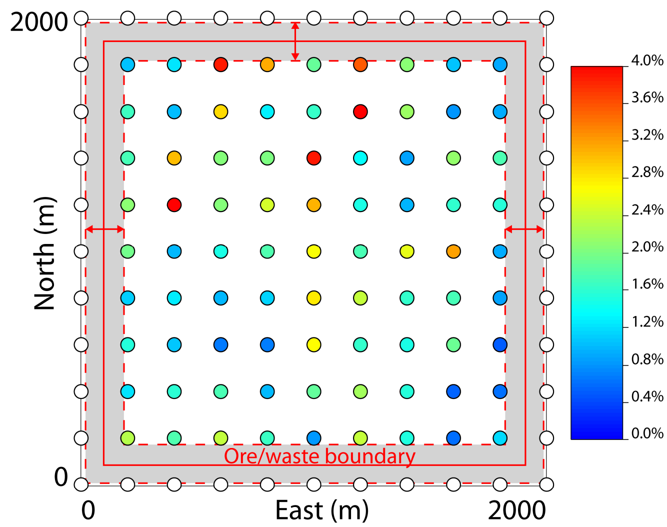

Figure 1: Drill hole spacing: 200m. Vertically averaged mineral grade shown. Outer drill holes do not intersect the deposit. The ore/waste boundary is located in the gray shaded region. Deposit shape assumed square.

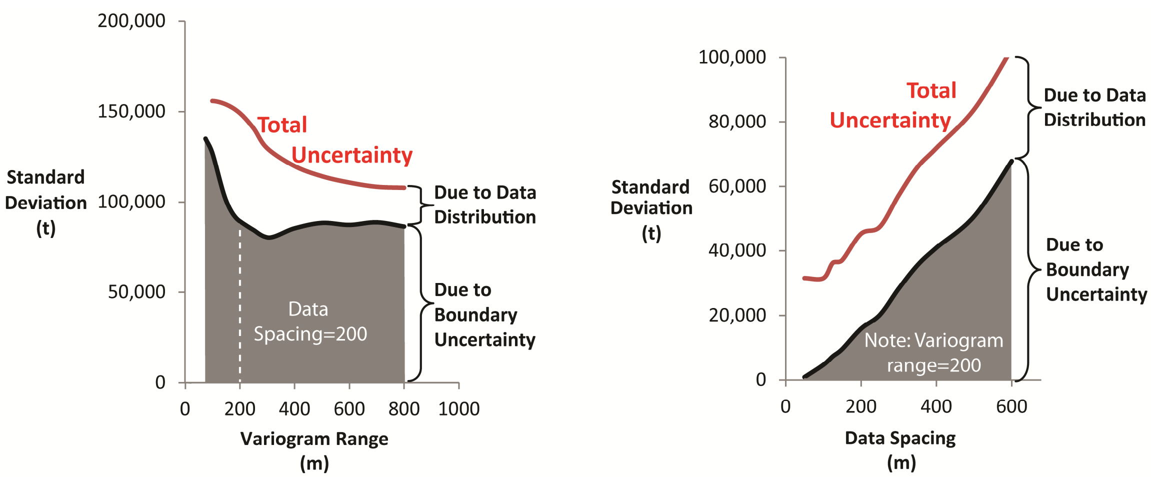

Figure 2: Proportion of the total uncertainty due to the uncertainty in the boundary, shown as a gray shaded region, and uncertainty in the data distribution, white area. Left: the variogram range for grade modeling is varied. Right: data spacing is varied.

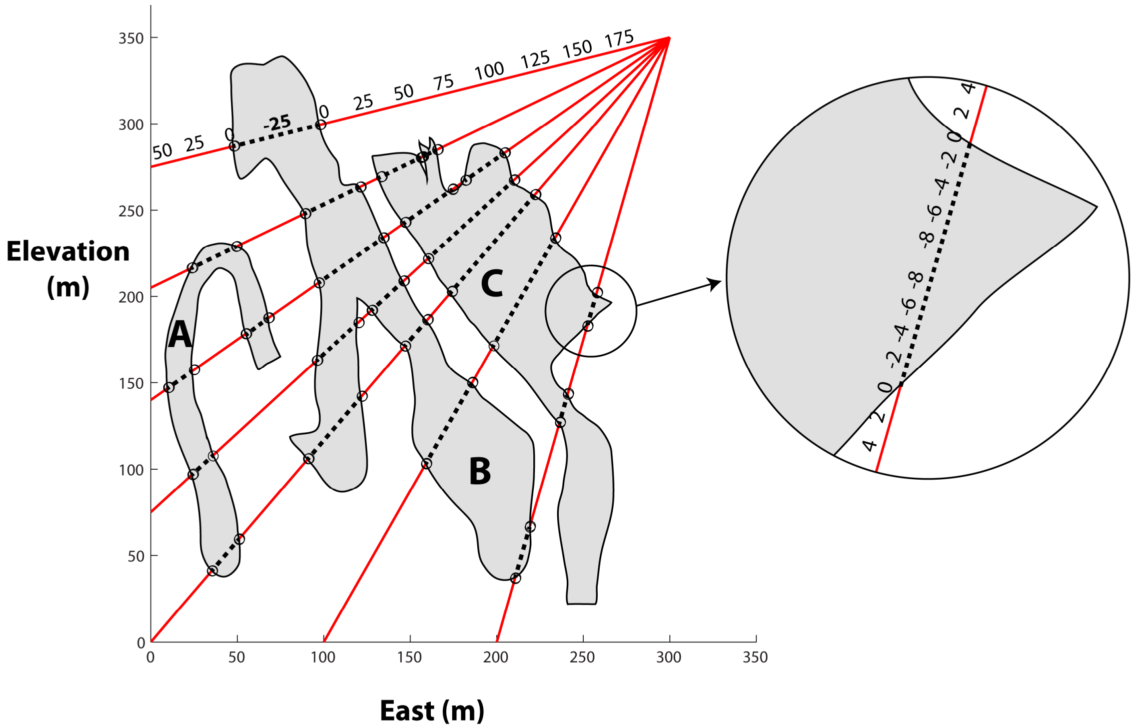

Figure 3: Folded deposit (modified from Frondel and Baum 1974), three disconnected bodies are evident, A, B and C. Synthetic drill holes are added by this author for illustrative purposes, only the geological interpretation of the ore body is taken from Frondel and Baum (1974). Each sample is coded based on the distance to the nearest contact (right). Distance is negative within the body of interest.

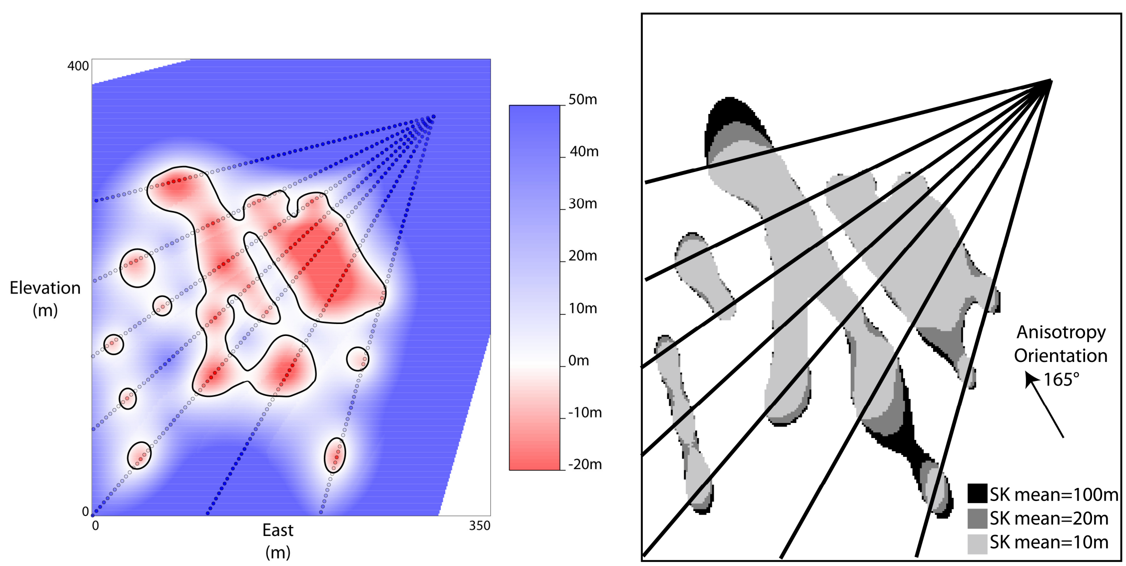

Figure 4: Left: Interpolating the distance value with isotropic kriging showing the modeled distance. Zero contours define the ore/waste boundaries. Right: Considering different mean values with anisotropic simple kriging, only the zero contours are shown.

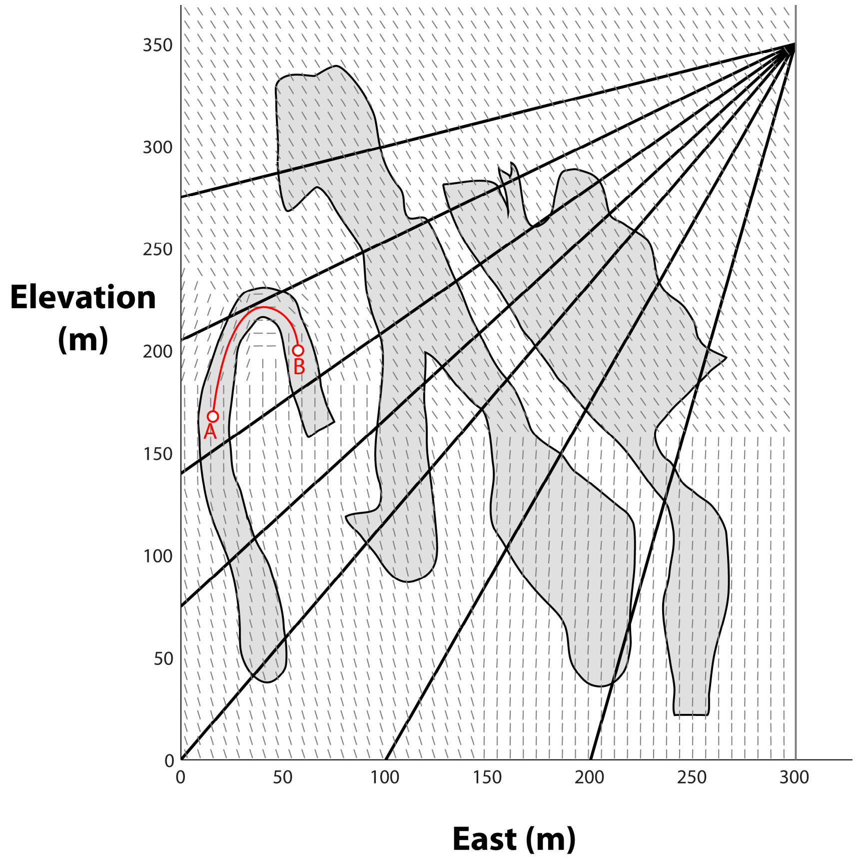

Figure 5: LVA field orientation showing the major direction of anisotropy. Magnitude of anisotropy is constant at 0.1.

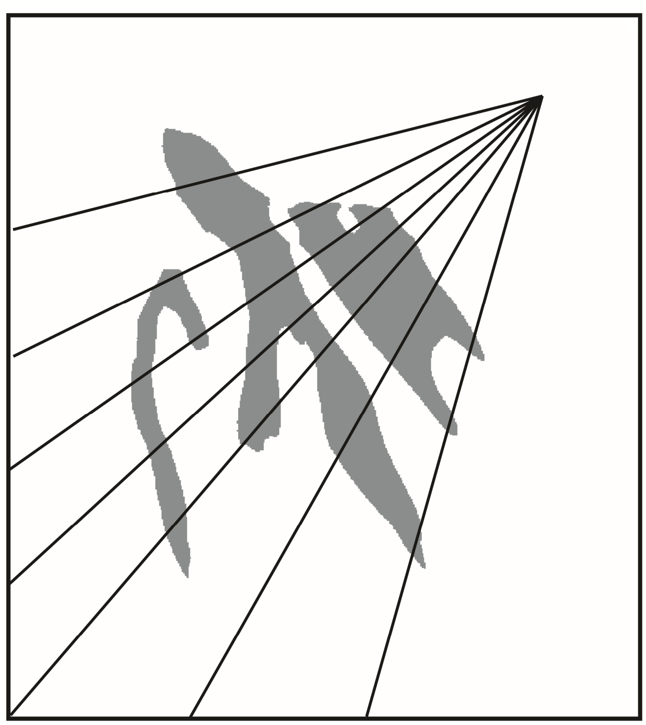

Figure 6: Zero contours considering LVA.

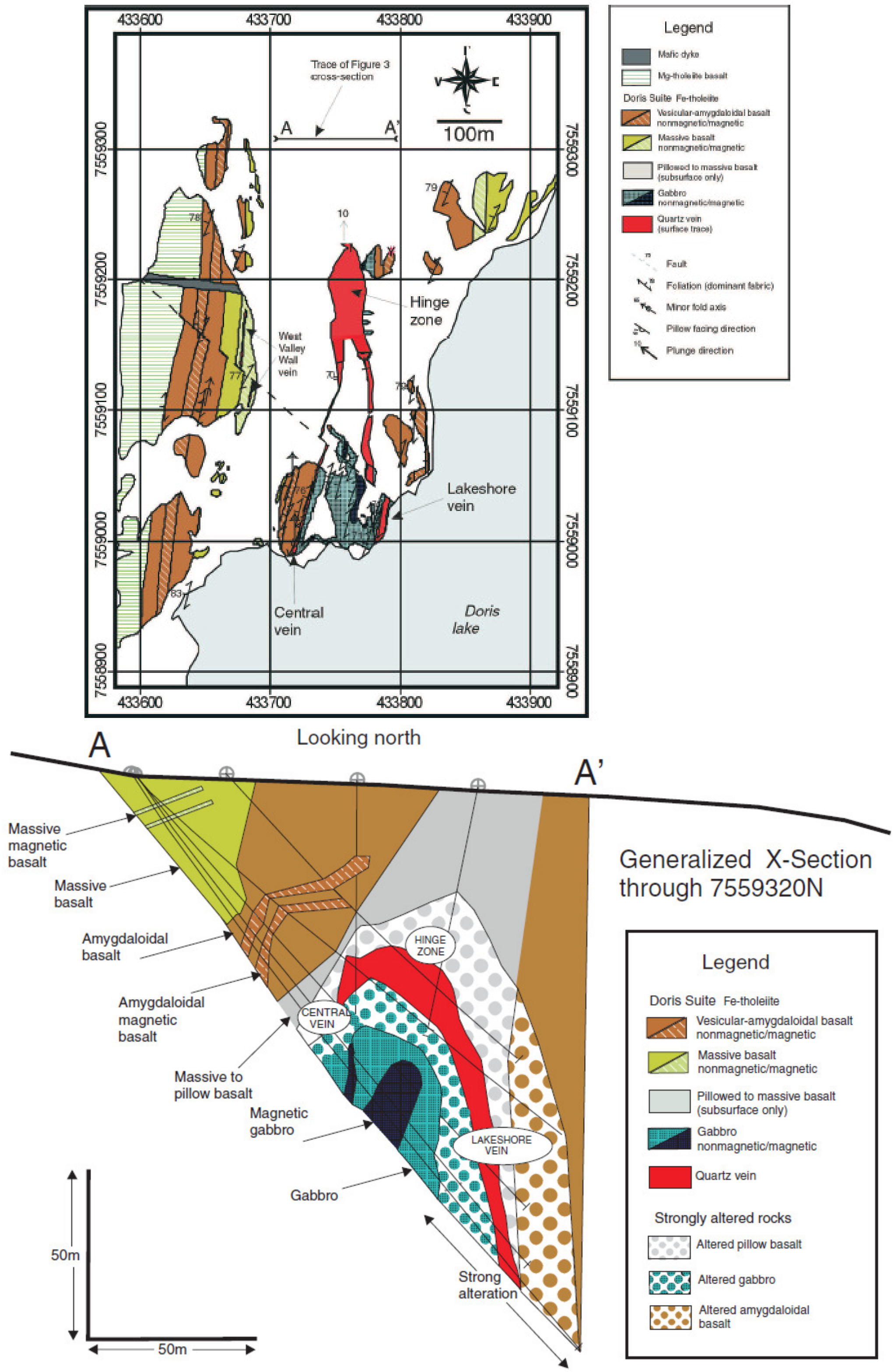

Figure 7: Above: Geometry of the deposit shown here as the central vein, hinge zone and lakeshore vein (Carpenter et al 2003). Below: typical cross section showing anticline (Carpenter et al 2003).

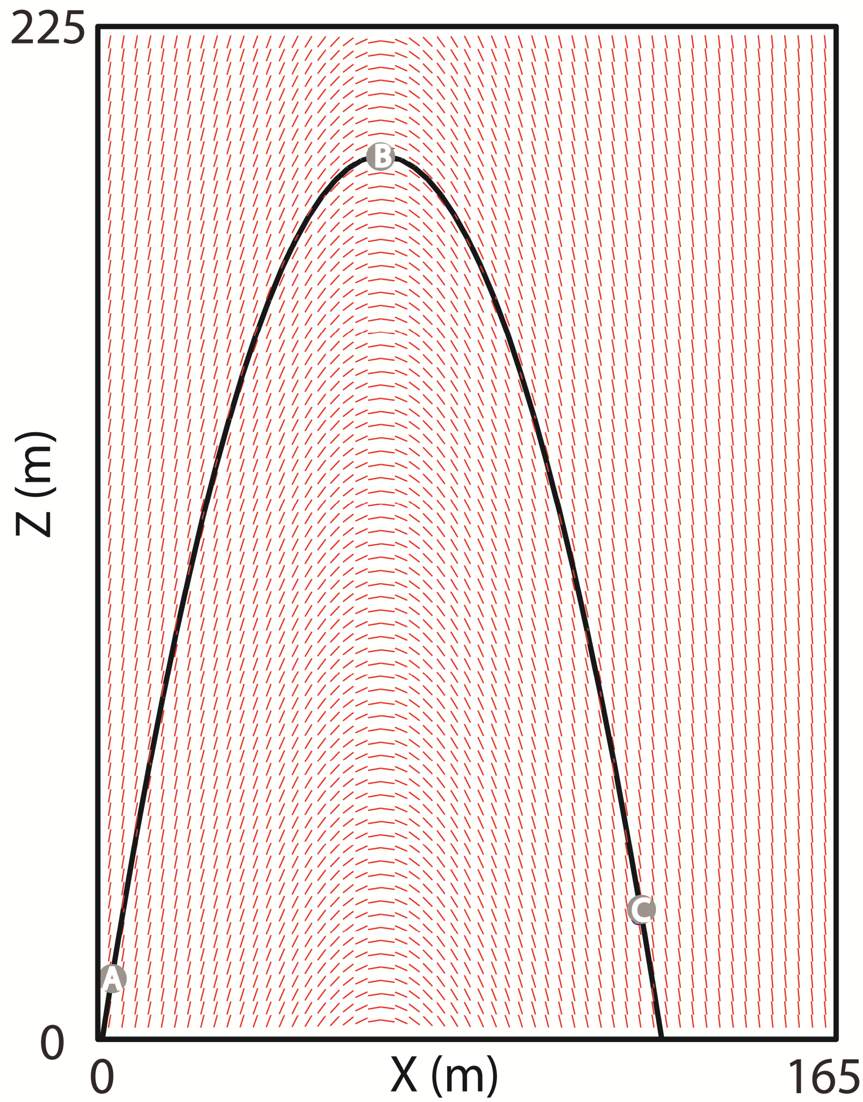

Figure 8: Typical cross section of the LVA field. Interpreted polylines from Figure 8 (gray intersections A, B, C) are fit with a parabola to determine the exhaustive LVA orientation.

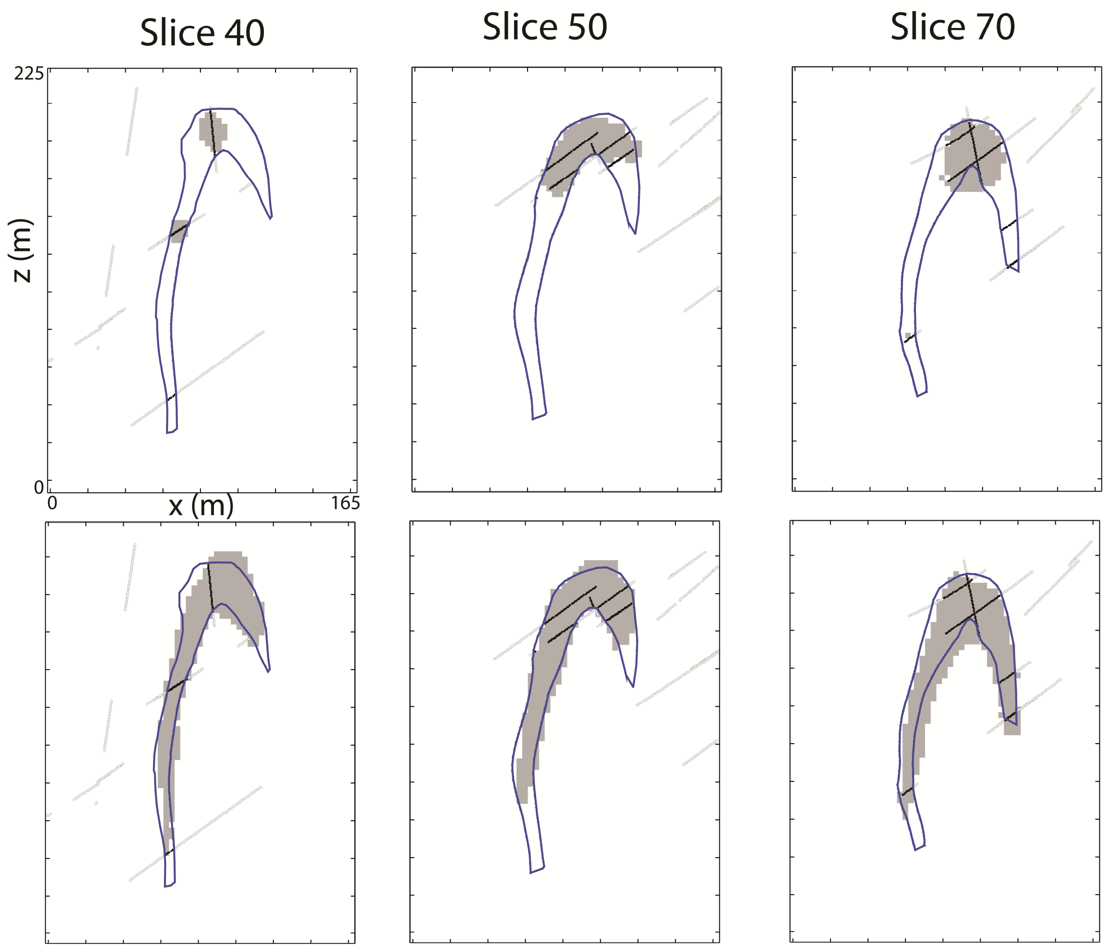

Figure 9: Above: Typical cross sections of the boundary generated with isotropic kriging. Below: Typical cross sections of the boundary generated with LVA. Outline of the deterministic model is shown for comparison.

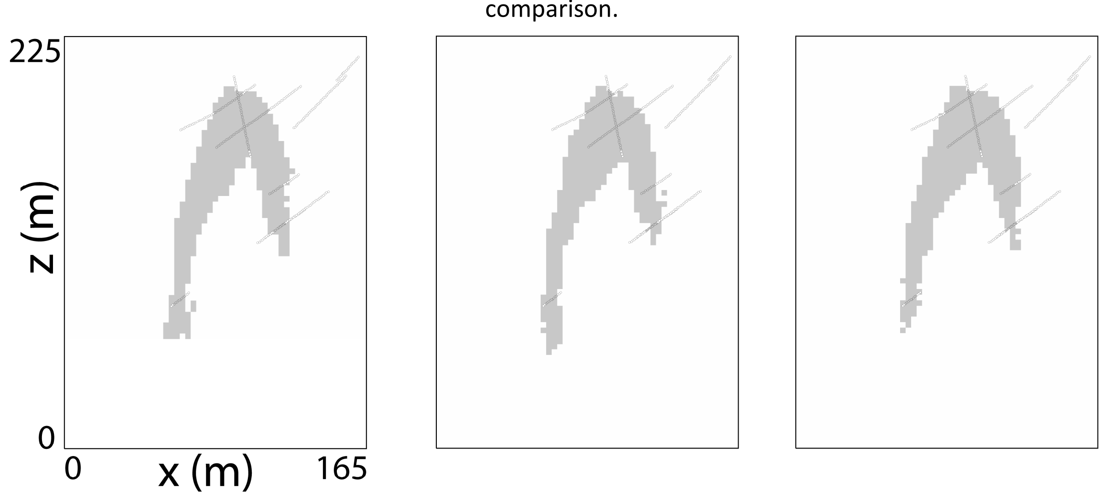

Figure 10: Three SGS realizations of the distance function for slice 70.

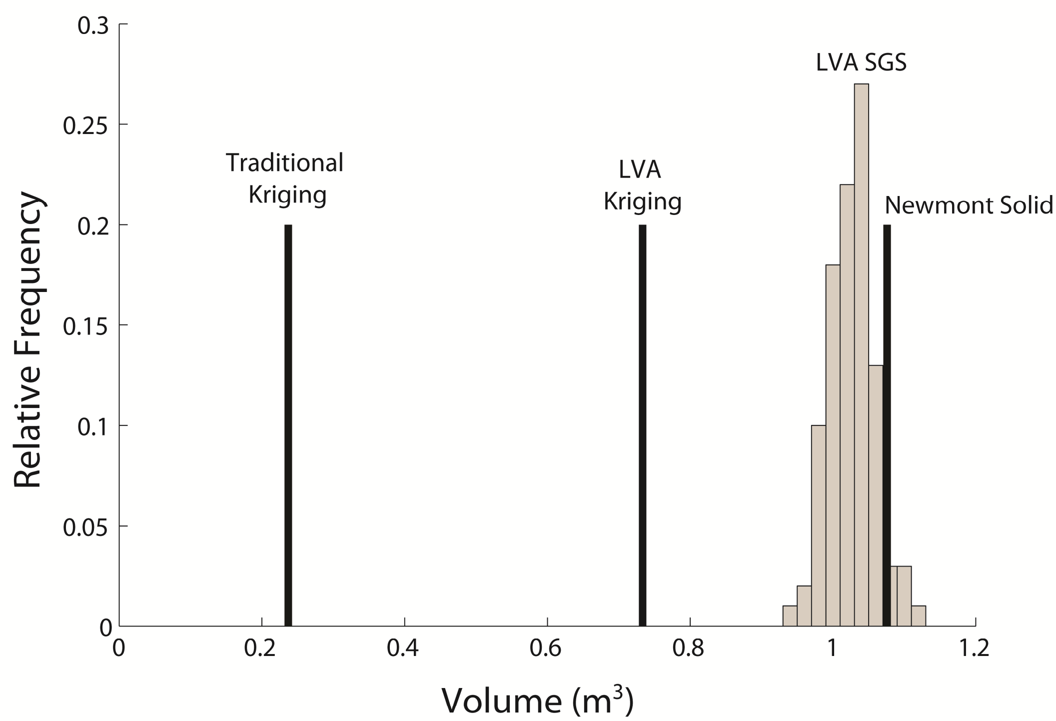

Figure 11: Histogram of SGS results using LVA. Volumes from the deterministic solid and kriging are also indicated.

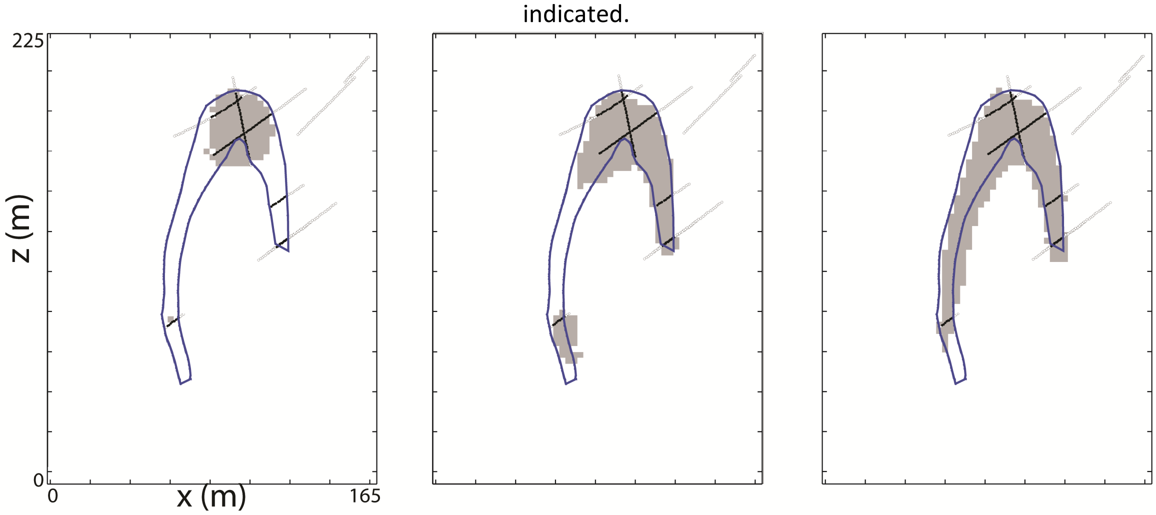

Figure 12: Kriging with different degrees of anisotropy. Left: anisotropy ratio of 1.0 (isotropic). Middle: anisotropy ratio of 0.5. Right: anisotropy ratio of 0.1.