Conditioning Object Based Models with Gradient Based Optimization

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Object based modeling is a popular technique for generating complex training images for input to multiple point simulations because they are difficult to condition. In this paper, conditioning object based models is addressed by considering optimization techniques that are based on gradients. Gradients are determined as the mismatch between conditioning data and the object which is parameterized by a once (or twice) differentiable equation. In this paper, 2D channel objects (cross sections) are conditioned to wells to demonstrate the benefits and drawbacks of the method. In general, the benefits include fast optimization of objects. The drawbacks are the effect on the distribution of model parameters, specifically the radius of objects. When an object violates conditioning data there are two options to match the conditioning data: move the object, or reduce its size. These can be controlled independently or jointly. The effects of these actions are a change in the local proportions (if an object is moved) and a change in the distribution of object sizes (if the object is reduced in size).

Introduction

Many authors have applied object based modeling to a number of different geological settings. (Pyrcz and Deutsch 2004, Wen 2004, Pyrcz, Boisvert and Deutsch 2007, Deutsch and Tran 2002, Journel et al 1998, to name a few). Typically, these objects are parameterized by some function or geometric shape. Objects are placed in the model and gridded. The gridding is not necessary (see paper 114 in this report) but is convenient for geocellular based techniques such as kriging or simulation (see standard geostatistical texts for relevant techniques, i.e. Deutsch 2002).

The conditioning of these objects is often attempted by (1) iteratively with a place and test type methodology where objects are compared to conditioning data and rejected if they do not conform to the data (see paper 114 in this report) or (2) once the objects have been rasterized on a grid, the cells are modified to honor conditioning data (Pyrcz, Boisvert and Deutsch 2009). The limitation with (1) is that with reasonably dense data or noisy data it will take a large number of iterations to obtain suitable objects that account for all the conditioning data, if such objects are found at all. The limitation with (2) is that the shape of the objects is altered to fit the conditioning data; however, if done with an erosion/dilation strategy that considers the geometry of the object, the results can be reasonable (Pyrcz, Boisvert and Deutsch 2009).

Ideally, the objects would be 'moved' and 'scaled' in such a way as to preserve the integrity of the shape as well as honor the available conditioning data. In the proposed methodology the mismatch between the objects and the conditioning data is used as a gradient to intelligently 'move' and 'scale' the objects such that conditioning data is honored. This preliminary work is limited to 2D and a simple channel cross section is used for demonstration purposes. Extension to 3D and arbitrary geometries is discussed. Any object that can be defined by a once differentiable (gradient based optimization) or twice differentiable (newton method optimization) function can be considered. The complexity of the gradient will have some effect on the CPU requirements of the methodology.

Methodology

The first step in any object based methodology is to parameterize the objects of interested. For demonstration of the methodology the object selected will be a channel geometry defined by a horizontal top and semi-circular base (Figure 1). The object is then parameterized by a line (Eqn 1) and circle (Eqn 2). Note that any differentiable function could be used for the parameterization.

$$y = b \tag{1}$$

$$(x-a)^2 + (y-b)^2 = r^2 \quad \text{if } y<b \tag{2}$$

Any optimization technique requires the definition of an objective function which is to be minimized. In this case, the objective function is the violation of available conditioning data. The distance from the intersections of the wells with channel and nonchannel facies to the object is used as the criteria to minimize. If a channel does not violate the conditioning data then it does not contribute to the objective function. It would be reasonable to consider growing such channels until the channel boundaries match the conditioning data, while limiting the object size to a reasonable distribution; however, this is considered a future improvement once 3D considerations have been accounted for.

It is assumed that the facies conditioning data has been recoded to the same facies defined by the area in Equations 1 and 2 (Figure 1). Very noisy data should be preprocessed such that reasonable intervals to fit are obtained.

Before formulating the objective function distance, some nomenclature is required. The type of intersection is important. Consider the upper boundary between channel and nonchannle to be a type 1 intersection and the lower boundary between nonchannel and channel to be a type 2 intersections. All of the potential situations in which there could be violations are shown in Figure 2. There are two different distances that describe the level of mismatch between the channel and the conditioning data: d_1 and d_2. A violation of the upper intersection of a channel (type 1) results in a d_1 mismatch and a violation of the lower intersection of a channel (type 2) results in a d_2 mismatch. The objective function, *f(v)* where v is the spatial location of the intersection of the current object with the conditioning data (assumed to be a string), becomes the minimization of these distances (Equation 3). In fact, in the simple 2D case, optimization can proceed until the objective function is 0. A benefit of this formulation is that the optimal objective function value is known to be 0.0.

$$f(a,b,r,v\ \ ) = \sum_{i=1}^{n} d(a,b,r,v_i) \tag{3}$$

where $d(a,b,r,v_i)$ is a function that returns the distance between the channel at its current location (parameterized by Equations 1 and 2) and intersection *i* in the conditioning data (v_i).

To calculate the objective function and its gradients the calculation of *d* is required. When the violation occurs as a result of the upper surface (line, Equation 1, case b in Figure 2), the distance is simply the vertical distance between the channel top (line) and the type 1 intersection in the conditioning data (Equation 4).

$$d(a,b,r,v_i) = b - y_{vi,type1} \tag{4}$$

where $y_{vi,type1}$ is the y coordinate of the type 1 intersection in well *i*.

When the violation occurs throughout the entire channel thickness (line and circle, Equation 2, case a and e in Figure 2) the intersection of the well and the channel (y_{type2}) is required (Equation 5). The violation distance is then Equation 6 and 7.

$$y_{type2} = b - \sqrt{r^2 - (x_{well} - a)^2} \tag{5}$$

$$d(a,b,r,v_i) = b - y_{type2} \tag{6}$$

$$d(a,b,r,v_i) = \sqrt{r^2 - (x_{well} - a)^2} \tag{7}$$

When the violation occurs at the base of the channel (circle, Equation 2, case d in Figure 2) the intersection of the well and the channel (y_{type2}) is required (Equation 5). The violation distance is then Equation 8.

$$d(a,b,r,v_i) = y_{vi,type2} - b + \sqrt{r^2 - (x_{well} - a)^2} \tag{8}$$

Implementing a gradient based method requires the derivative of *f*, or the derivative of Equations 4, 7 and 8 depending on the type of violation that occurs. Derivatives of Equation 4 are:

$$\frac{df(a,b,r,v_i)}{da} = 0 \tag{9}$$

$$\frac{df(a,b,r,v_i)}{db} = 1 \tag{10}$$

$$\frac{df(a,b,r,v_i)}{dr} = 0 \tag{11}$$

Derivatives of Equation 7 are:

$$\frac{df(a,b,r,v_i)}{da} = -(x-a)(r^2 - (x_{well} - a)^2)^{-1/2} \tag{12}$$

$$\frac{df(a,b,r,v_i)}{db} = 0 \quad (13)$$

$$\frac{df(a,b,r,v_i)}{dr} = -r(r^2 - (x_{well} - a)^2)^{-1/2} \quad (14)$$

Derivatives of Equation 8 are:

$$\frac{df(a,b,r,v_i)}{da} = -(x - a)(r^2 - (x_{well} - a)^2)^{-1/2} \quad (15)$$

$$\frac{df(a,b,r,v_i)}{db} = 1 \quad (16)$$

$$\frac{df(a,b,r,v_i)}{dr} = -r(r^2 - (x_{well} - a)^2)^{-1/2} \quad (17)$$

There are three variables in this gradient, the a and b parameters control the location of the channel and the r parameter controls the size. In general, the portions of the gradient will tend to have the following predictable effects on the channel object:

***da* parameter**: with type 1 intersection violations the channel will be lowered, with type 2 intersection violations the channel will be raised.

***db* parameter**: the channel will be moved away from conditioning data to reduce the interval of the channel that violates the conditioning data.

***dr* parameter**: the channel will be reduced in size to reduce the interval of the channel that violates the conditioning data.

Implementation of gradient decent optimization involves generating initial a, b and r parameters and updating the values according to the gradient calculated above. The initial parameters are updated using Equation 18.

$$p = p + t\Delta p \quad (18)$$

Where p is the parameter of interest, Δp is the step orientation and t is a scalar step size, optimized using a golden section search. The step orientation is given by the negative gradient as determined by Equations 9-17:

$$\Delta p = -\nabla f(p) \quad (19)$$

Note that the gradient is also a sum over all the intersection types that are present for a given object. An object can intersect multiple wells; the gradients are simply summed for the object.

Example

The above methodology is implemented in Matlab for this trial case. The initial conditioning data are the intersections shown in Figure 1. A set of a, b and r parameters are drawn randomly from a user input distribution. This initial channel location is shown on Figure 3. The gradient is calculated depending on what type of violation is present (Figure 2). The a, b and r parameters are altered as per Equations 18 and 19. The evolution of three channels through the optimization process is shown in Figure 3. After each step, the gradient is recalculated (note that the type of violation can change) and the final channels are shown in Figures 3.

The process can be repeated *n* times and a library of objects that honor the conditioning data is built (Figure 4). Multiple realizations can be generated by selecting different objects from this library. A number of realizations are show in Figure 5. In this example, no effort was made to honor areal or vertical proportions; however, with the library of objects this could be easily accomplished. Moreover, only channels that match conditioning data have been shown. A large number of channels are generated that do not match conditioning data (i.e. in the empty areas of the model, Figure 4 right) these can be used to honor local proportions and input distributions of channel properties.

Discussion

The parameterization of the channel object in this example was quite simplistic. However, the extension to more complex shapes is intuitive while difficult. The only requirement is that the parameterization of the object (i.e. Equation 1 and 2) is differentiable. With the methodology outlined here, a large number of complex objects could be conditioned to well data.

One issue with the proposed methodology is that the placement of wells tends to be adjacent to nonchannel intersections because the optimization stops once the well is clear of violations, thus many channels are located adjacent to a well that has a nonchannel sample. Corrections can be implemented to discourage this result in the optimization.

Improvements

In general, the optimization process reduces the size of the objects so that conditioning data is honored. Once the channels have been conditioned, there is often some opportunity to 'grow' the object. Consider the optimized channels in Figure 3. Once optimized (size is likely reduced) the channels could be increased in size until the conditioning data is violated or the original size (radius) is honored.

Input distributions for the initial channel parameter selection are required. These are biasedly altered in the optimization process. A methodology for maintaining these input distributions is required. Likely, the initial starting parameters will have to be biased and channels selected from the resulting library in such a way as to honor the desired distributions.

Extremely dense and or noisy data will be difficult to fit with reasonably sized objects. This is a data issue and input conditioning data should be cleaned and scaled to the support of the objects.

Multiple facies were not considered. In the case of two facies there are two types of intersections (type 1 and type 2 in this paper). With multiple facies there will be more types to consider in the objective function. Ordering is an issue.

Conclusions

This preliminary work is promising. The movement of the channels through the optimization procedure is predictable and channels are altered in the 'correct' way to match conditioning data. Currently the CPU speed is fast because no grid is required, only channel intersections; however, with more complex objects and gradients in three dimensions, alternative optimization techniques may be required. Moreover, the addition of multiple facies will require additional object/data intersection calculations.

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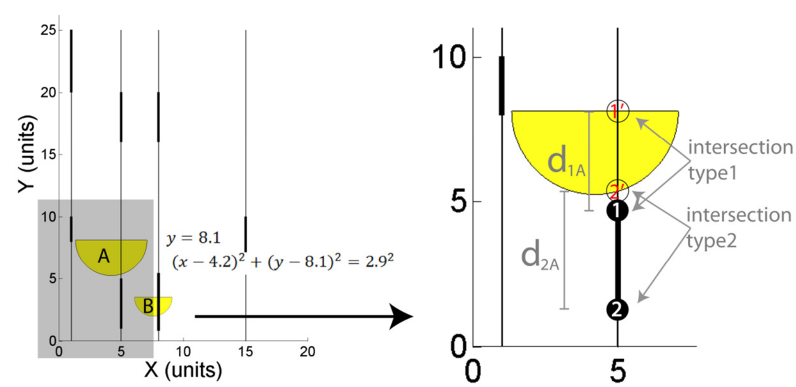

Figure 1: Left: Parameterization of the channel object. Well intersections are thick black segments=channel, thin segments = non channel. Right: distances used in the objective function definition.

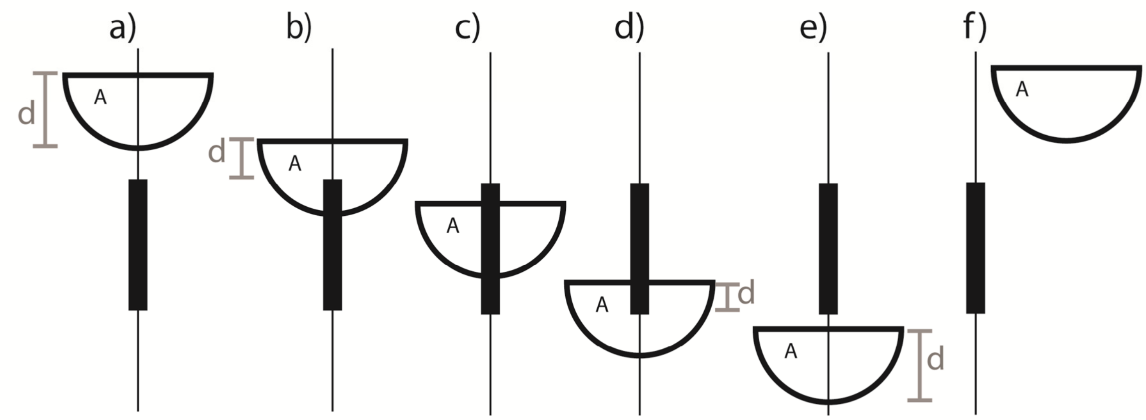

Figure 2: Types of intersections between channels and conditioning data shown as a vertical well (thin=nonchannel, thick=channel).

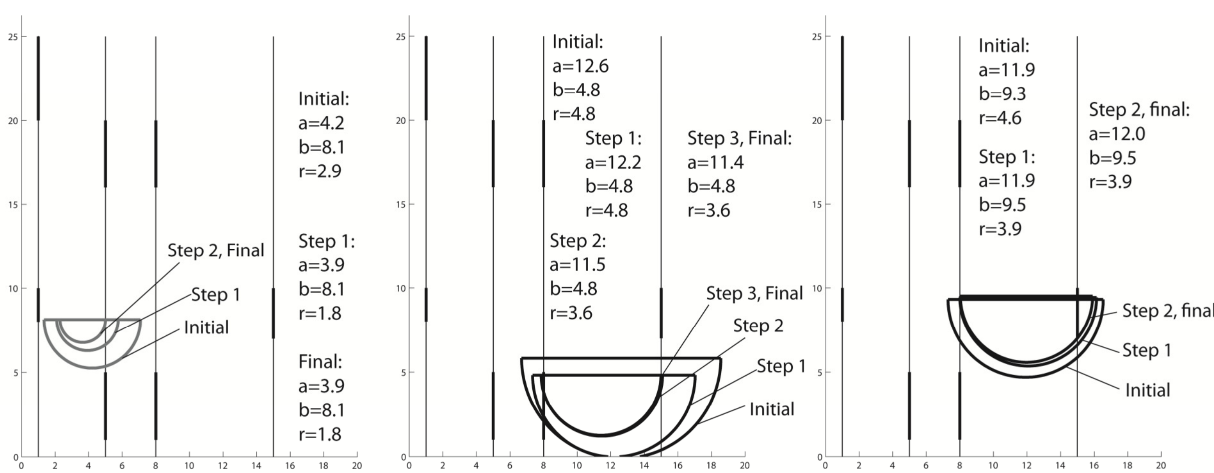

Figure 3: Three optimization runs for different cases. Left: Type a and e violation. Center: Channel violates two wells, type a, e and b violations. Right: Type d violation.

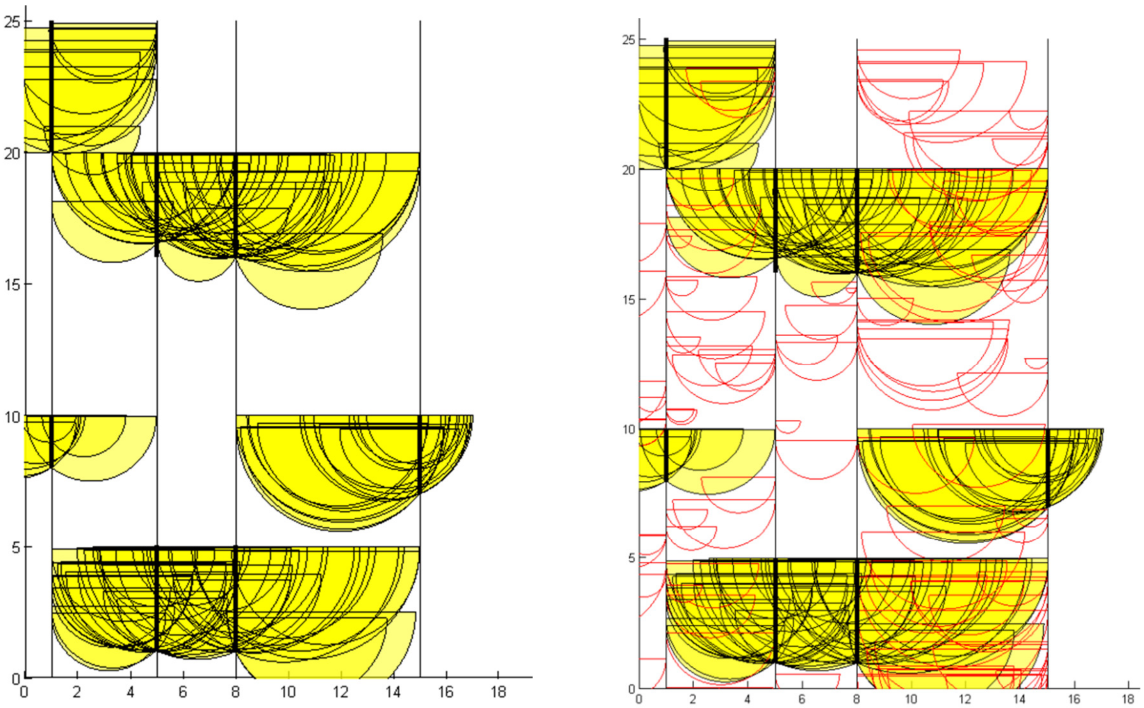

Figure 4: Library of channels. Left: Channels that intersect wells. Right: All channels.

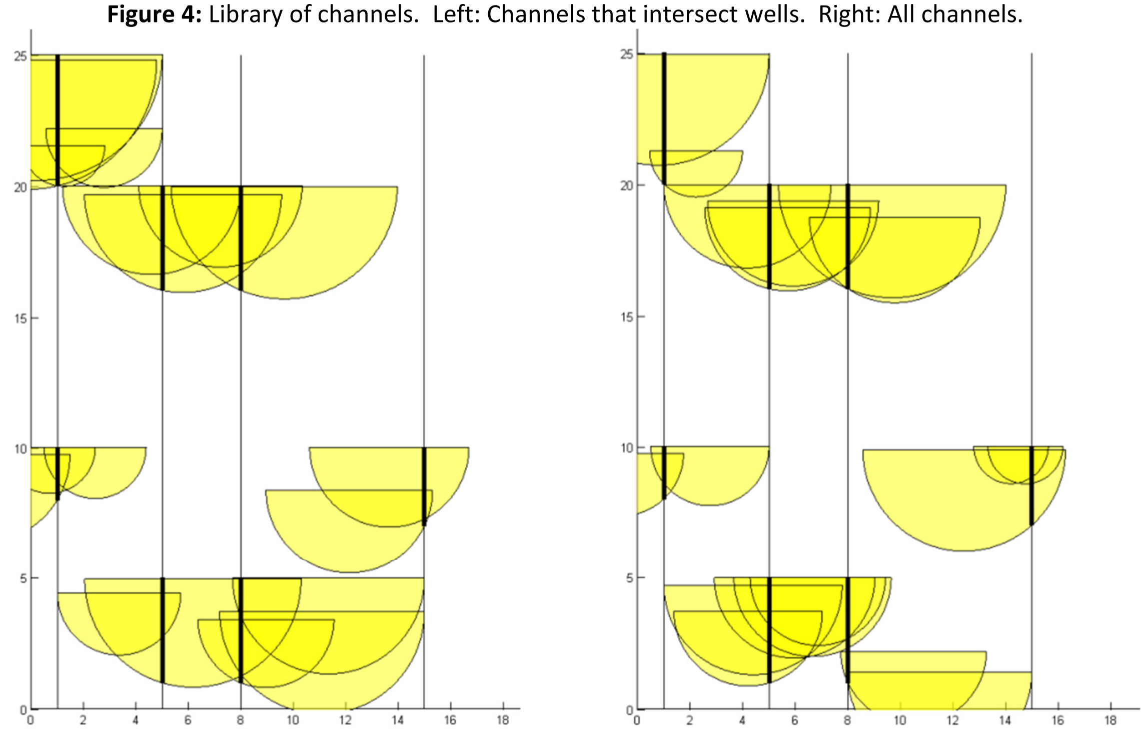

Figure 5: Two conditioned realizations (note the library shown in Figure 4 was not used to generate these realizations).