Developments Toward Multiscale Modeling

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Scale is an important issue in reservoir modeling. Often array of data are available for reservoir modeling. Eventually using all available data will reduce the level of uncertainty in reservoir models. But the thing is, data collected from a certain scale are not representative for any scale other than what they were collected at. Data scale must be taken into account when integrating them into numerical reservoir model. Integrating data from these wide ranges of scales into the reservoir model is a complex task. It is complex task because data measured at different scales reflect different degrees of heterogeneity and can have different degrees of accuracy. Also data from different scales tend to be of different variables, Therefore understanding how variables are correlated at different locations is important task in developing the theory of multi-scale modeling. This paper presents in very general way the attempts to develop a methodology for multi-scale reservoir heterogeneity and uncertainty modeling. The goal is a reservoir model that reproduces the multi-scale data in a way that encounters no artifacts, no biases and handles numerical features of geological data such as nonlinearity and the proportional effect

Introduction

Predicting future reservoir performance is an important goal of reservoir flow models. Performance forecasting permits optimization of the economic recovery of the oil and gas resources. Reservoir simulation is an established approach to forecast the performance of a reservoir for a particular development strategy. Data is expensive and sparse. Geostatistical models are used with the available data to build numerical models for reservoir simulation. Petroleum reservoirs are heterogeneous. Reservoir properties such as facies, porosity, permeability, faults, fractures and fluid saturations vary in space. The heterogeneity comes from variability in the depositional environment and subsequent events such as compaction, solution and cementation. An important goal of geostatistics is to build numerical models of heterogeneity that can be used in flow simulation. A central premise of geostatistics is to represent realistic spatial variability. Flow simulation is more reliable using geostatistical models that take into account heterogeneity. Historical geological models built using different techniques such as inverse distance led to less accurate flow forecasting.

Scale is an important issue in reservoir modeling. The aim is to describe a reservoir volume of 10^5-10^7 cubic meters of rock with few data. The data are gathered from different sources often at a much smaller scale. Accounting for the data scale is essential for accurate forecasting. For example, porosity values may be determined from cores or well logs that have significantly different scale than the grid blocks in flow simulation. The difference in scale should be accounted for when assigning properties to flow simulation grid blocks of an even larger scale. **Table1** shows some of the available measurements at different scales. Geostatistical models can be produced at different scales. The resulting models should be consistent when upscaled or downscaled: however, they will not be if the models are constructed by conventional techniques. **Figure1** illustrates the upscaling and downscaling concept. The scale is in cubic metres. There have been attempts to construct scale consistent models. Several methods for multi-scale modeling are available including conventional techniques such as cokriging, sequential gaussian simulation with block kriging and bayesian updating of point kriging.

Direct simulation is a recent proposal. The direct simulation proposal is difficult to implement because of practical problems such as the proportional effect. High valued areas often show more variability than low valued areas. The proportional effect is a natural phenomenon; it is a fundamental fact that needs to be dealt with. The proportional effect can be seen on the variogram and in the prediction of local uncertainties. Relative variograms can be used to address the issue of the proportional effect on the variograms; however, there is no clear methodology on how to tackle the proportional effect issue in the prediction of local uncertainties. Transferring the data to Gaussian units mitigates the proportional effect issue, however, multi-scale data cannot be transferred directly to Gaussian units as data from different scale do not average linearly which can lead to biases and inconsistencies in the results. A common practice is to perform multi-scale modeling with direct simulation techniques, that is, using the data in their original units. This practice can handle the difference in scale, but the proportional effect issue still exists as direct simulation techniques assume that the variance is independent of the mean, while in reality the variance is indeed a function of the mean. A consequence of this assumption is that uncertainty in low valued areas is overestimated and uncertainty in high valued areas is underestimated. This paper presents in very general way the attempts to develop a methodology for multi-scale reservoir heterogeneity and uncertainty modeling. Understanding how variables are correlated at different locations is an important task in developing the theory of multi-scale modeling. determining the relation between different volumes at the same location with examples will be presented.

Vision on how multi-scale modeling could proceed

Assume that we have data from three different scales, seismic, well log and core samples, the probability density function (PDF) can be can be established. **Figure 2** shows a hypothetical PDF sketch for three different types of data. The red PDF represent data collected from large scale (seismic), The yellow PDF represent data collected from a smaller scale (well log) and the blue PDF represent data collected from small scale (core smaples).

The aim is to develop a methodology that can provide mapping of point variable Z to the Gaussian variable Y and vice-versa for different scales. **Figure 3** shows a hypothetical sketch of the target chart. **Figure 4** shows a flow chart of how developing a methodology for multi-scale reservoir heterogeneity and uncertainty modeling can be achieved. First a scale dependent transform methodology for data at different scales coming from variety of sources has to be devolved. Then a methodology to simultaneously process all the transformed data has to be developed. Co kriging and/or block kriging under multi-Gaussian model can be used to develop a methodology to simultaneously process all the transformed data. The output of this process is the prediction of a conditional mean and conditional variance at unsampled location. At this stage a conditional distribution at unsampled location can be generated in Gaussian units, it then can be back transformed to obtain a scale dependent conditional distribution in original units which has no artifacts, no biases and handles numerical features of geological data such as nonlinearity and the proportional effect.

Relation between different volumes at same location

Understanding the relation between different volumes is important. If the marginal distribution of two volumes is said to be Gaussian and the bivariate distribution is Gaussian then the relationship between them is fully captured by the correlation coefficient. The correlation coefficient can be theoretically calculated, we should be able to verify Theory numerically .

Theory

Knowing how two volumes at two different locations correlate with each other will enable us to predict the volume at the location of interest. The Covariance equation can be written as follow:

$$C(h)=E\left\{\left[Z(u)-\mu(u)\right]\left[Z(u+h)-\mu(u+h)\right]\right\}$$

Where
C(h)= The Covariance
Z(u)= Variable value at location u , Z(u+h)= Variable at location u+h
μ(u)= Variable mean at location u , μ(u+h)=Variable mean at location u+h
In Geostatistics the Variogram is usually used instead of Covariance to measure the spatial dependence between variables. The Variogram equation can be written as follow:

$$2\gamma(h)=E\left\{\left[Z(u)-Z(u+h)\right]^2\right\}$$

The above equation shows that there is no mean required for Variogram computations while it is needed for Covariance computations. For that reason in Geostatistics it is the preferred method to measure the spatial dependency between variables.
To develop the theory of multi-scale modeling we choose to work under the Multivariate Gaussian frame work.
The Multivariate Gaussian distribution equation can be written as follow:

$$f_X(x)=\frac{1}{(2\pi)^{k/2}\left|\sum\right|^{1/2}}\exp\left(-\frac{1}{2}(x-\mu)'\sum^{-1}(x-\mu)\right)$$

where:
μ is the 1Xn vector of mean values and $\sum$ is the nxn matrix of covariances.

The above equation shows that the multivariate Gaussian distribution is defined by the covariance not the variogram That's why it is important to understand and compute the covariances of data at different scales.

Correlation coefficient between points and blocks is equal to

$$\rho_{xy} = \frac{Cov\{X,Y\}}{\sqrt{Var\{X\}Var\{Y\}}}$$

Covariance between points and blocks is equal to

$$Cov\{X,Y\} = E\{X \bullet Y\} - E\{X\}E\{Y\}$$

$E\{X\}, E\{Y\}$ are the mean value for points and block respectively,

they are equal to zero when standardization is assumed

$$Cov\{X,Y\} = E\{X \bullet Y\}$$

Consider X_k to be points values and Y to be block values

Block values are just the average of points values

$$Y = \frac{1}{n}\sum_{i=1}^{n} X_i$$

$$Cov\{X,Y\} = E\left\{X_k \bullet \frac{1}{n}\sum_{i=1}^{n} X_i\right\} = \frac{1}{n}\sum_{i=1}^{n} E\{X_k X_i\} = \frac{1}{n}\sum_{i=1}^{n} C_{ik}$$

Points variance is equal to one

$$Var\{X\} = 1$$

Block Variance is equal to

$$Var\{Y\} = E\{Y^2\} - m^2 = E\left\{\frac{1}{n^2}\right\}\sum_{i=1}^{n}\sum_{j=1}^{n}\{X_i Y_j\} = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}$$

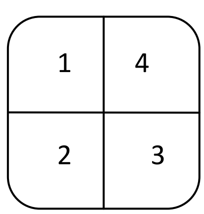
The correlation coefficient between points and blocks can bewritten as

$$\rho_{xy} = \frac{C_{ij}}{\sqrt{C_{ij}}}$$

where: Var{X}= point vriance=1, Var{Y}= Block variance, X_k= point value (the one you decide to keep), Y= block value, and m=mean (equals to zero if standardization is assumed)

Example (From Theory)

The aim of this example is to calculate the correlation between two volumes at the same location from theory and verify the results from practice. Let's consider a 2 x 2 block and calculate the covariance between the points and the block values using a spherical model with a nugget effect equal to 0.1 and a range of 32



The block value would be the average of the points values.

$$X = \frac{Y_1 + Y_2 + Y_3 + Y_4}{4}$$

$$Cov\{X, Y_1\} = E\left\{\frac{1}{4}(Y_1 + Y_2 + Y_3 + Y_4) \bullet Y_1\right\}$$

$$= \frac{1}{4}\left(E\{Y_1^2\} + E\{Y_1Y_2\} + E\{Y_1Y_3\} + E\{Y_1Y_4\}\right)$$

$$= \frac{1}{4}\left(1 + 2C(h=1) + C\left(h=\sqrt{2}\right)\right)$$

for spherical model

$$\gamma(h) = C_0 + C_1\left[\frac{3}{2}\frac{h}{a} - \frac{1}{2}\left(\frac{h}{a}\right)^3\right]$$

$$\gamma(1) = 0.1 + 1\left[\frac{3}{2}\frac{1}{32} - \frac{1}{2}\left(\frac{1}{32}\right)^3\right] = 0.147$$

$$\gamma\left(\sqrt{2}\right) = 0.1 + 1\left[\frac{3}{2}\frac{\sqrt{2}}{32} - \frac{1}{2}\left(\frac{\sqrt{2}}{32}\right)^3\right] = 0.166$$

$$Cov\{X, Y_1\} = \frac{1}{4}\left(1 + 2(1 - 0.147) + (1 - 0.166)\right) = 0.885$$

$$Var\{X\} = 1$$

$$Var\{Y\} = E\{Y^2\} - m^2 = E\left\{\frac{1}{n^2}\right\}\sum_{i=1}^{n}\sum_{j=1}^{n}\{X_iY_j\} = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}C_{ij}$$

$$Cov\{X, Y\} = E\{X \bullet Y\} - E\{X\}E\{Y\}, Y = \frac{1}{n}\sum_{i=1}^{n}X_i$$

$$Cov\{X, Y\} = E\left\{X_k \bullet \frac{1}{n}\sum_{i=1}^{n}X_i\right\} = \frac{1}{n}\sum_{i=1}^{n}E\{X_kX_i\} = \frac{1}{n}\sum_{i=1}^{n}C_{ik}$$

$$\rho_{xy} = \frac{Cov\{X, Y\}}{\sqrt{Var\{X\}Var\{Y\}}} = \frac{C_{ky}}{\sqrt{C_{ky}}} = \frac{0.885}{\sqrt{0.885}} = 0.9407$$

Example (from Practice)

Given a domain size 512 by 512, block averaging 2 by 2, variogram model = Spherical with range equal to 32 and nugget effect equals to 0.1, Figures 5, 6,7 shows points distribution, 2x2 block averaging distribution and a points - block scattered plot respectively. The scattered plot illustrate a strong correlation between points and block values with correlation coefficient equal to 0.939 which is almost identical to the value obtained from theory, Figures 8,9 shows 3x3 block averaging distribution and its point-block scatter plot.

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Table 1. Measurements at different scales.

Type	Level	Measurement Scale	Measurements
Micro	Pore	~Millimetre	Pore geometry Grain size Mineralogy
Macro	Core	~ Centimetre	K,kr,Ø,Pc Wetability Saturation
Mega	Grid block	~Metre	Logs Single well tracer
Giga	Interwell	~Kilometre	Well test Surface seismic Interwell tracer test

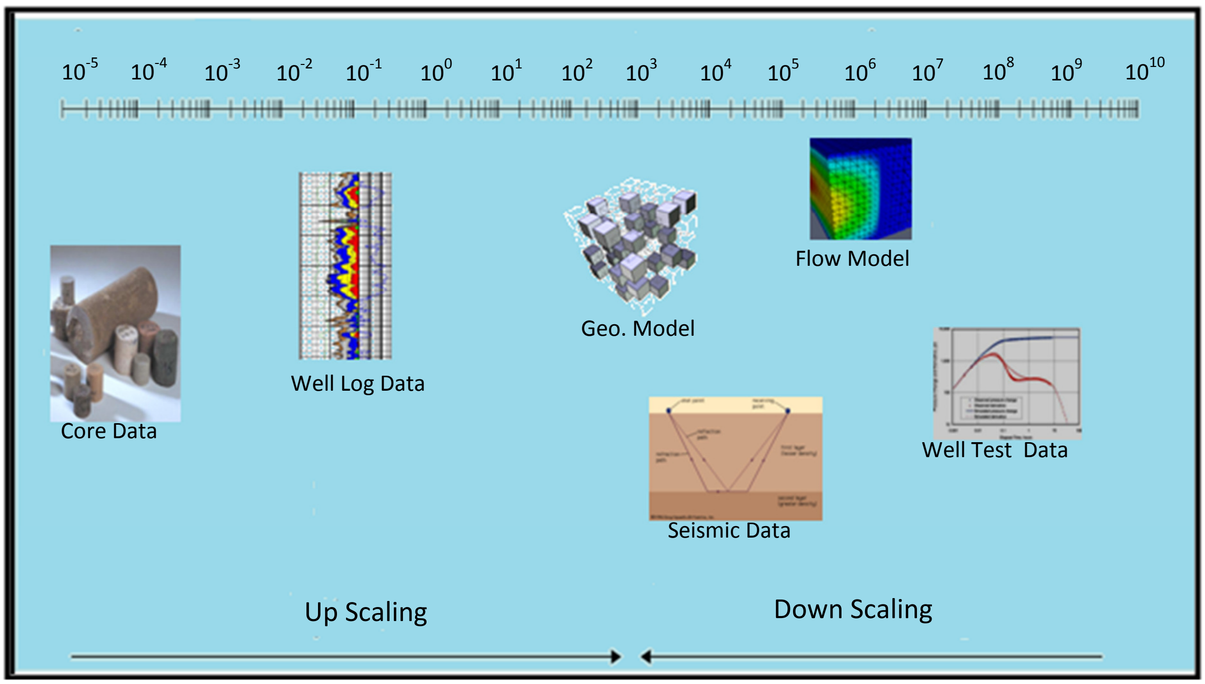

Figure 1. Upscaling , downscaling concept.

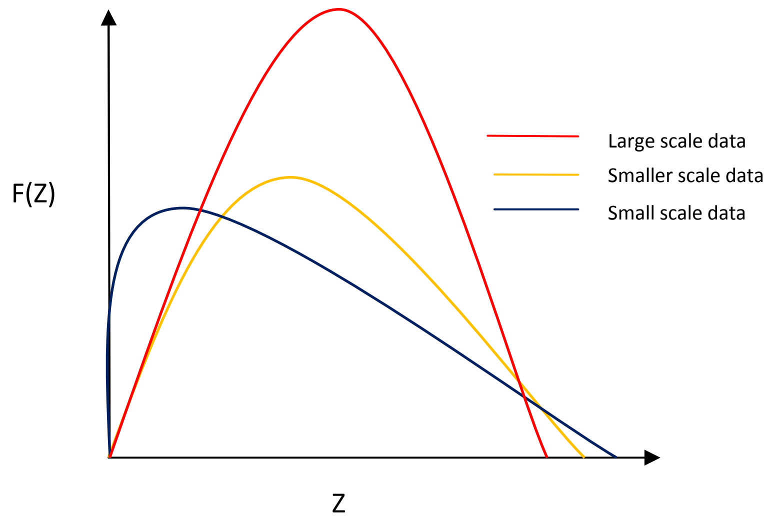

Figure 2. Hypothetical distributions for data from different scales.

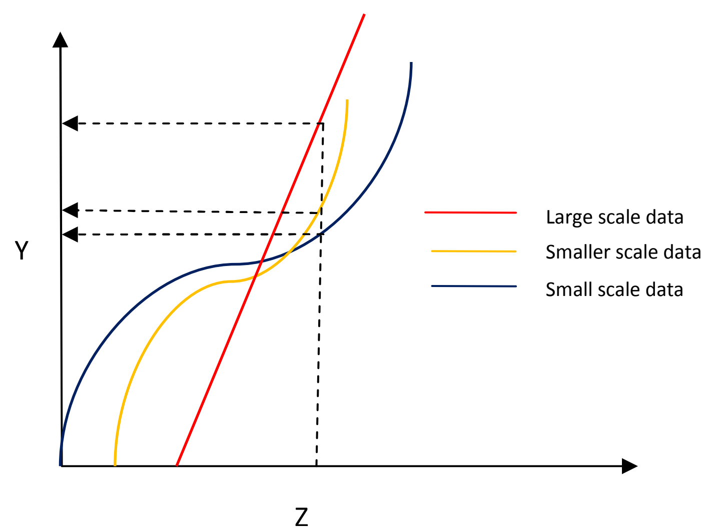

Figure 3. Mapping of point variable Z to the Gaussian variable Y

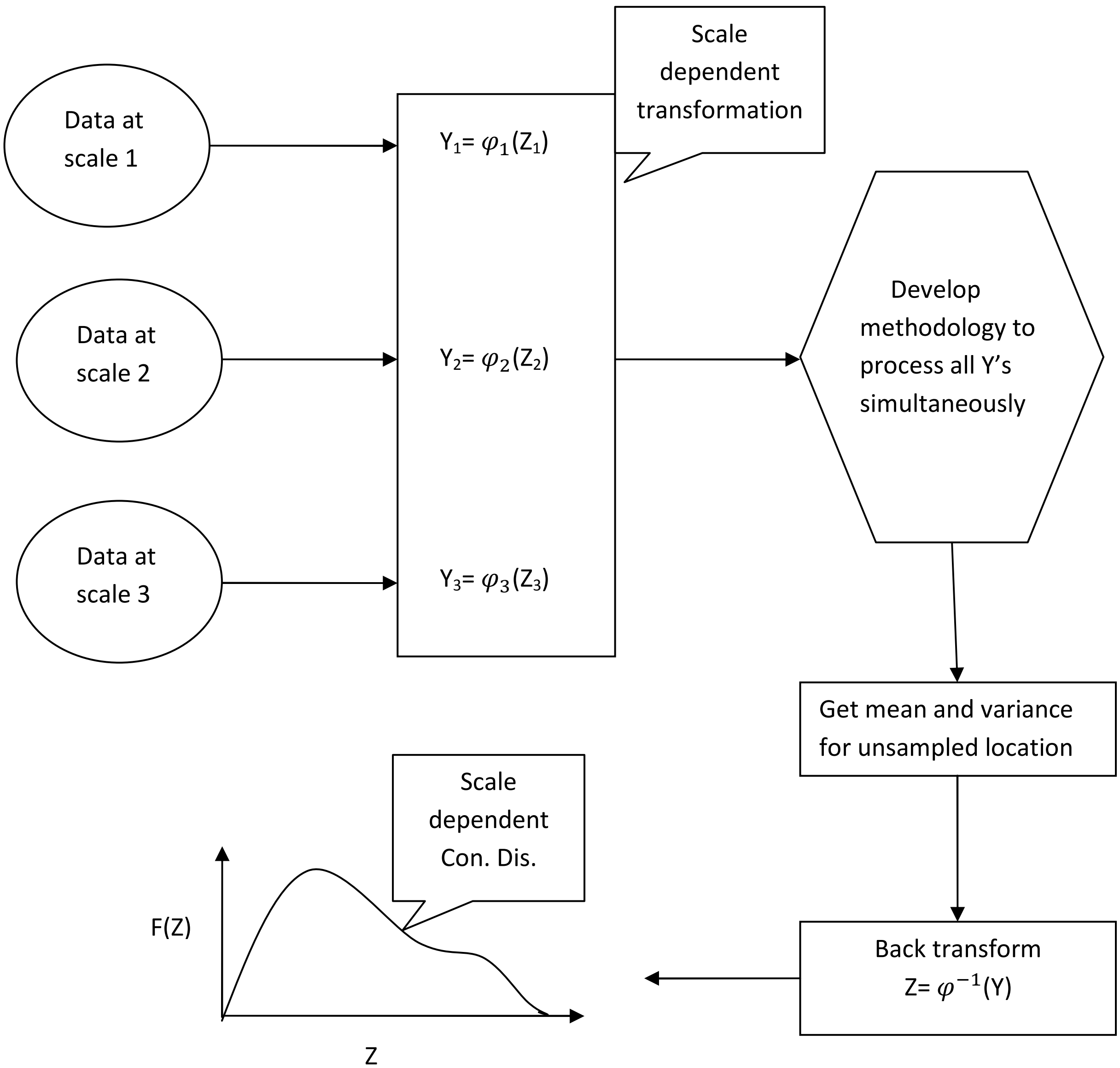

Figure 4. flow chart of developing a methodology for multi-scale reservoir heterogeneity and uncertainty modeling.

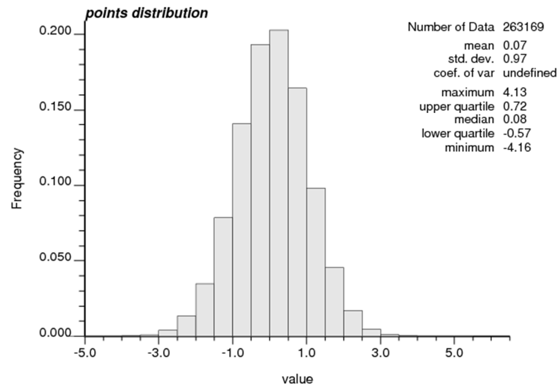

Figure 5. Point scale distribution

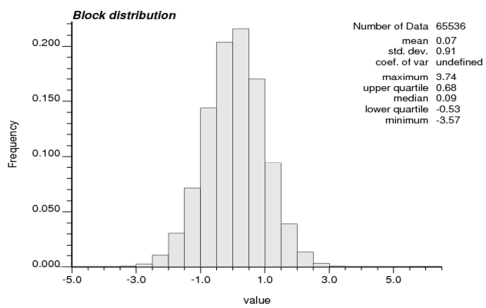

Figure 6. 2x2 block averaging distribution

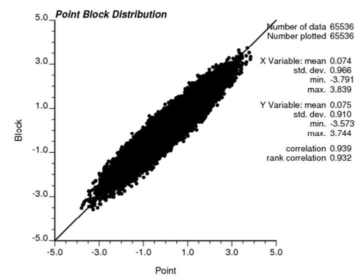

Figure 7. points-blocks scattered plot

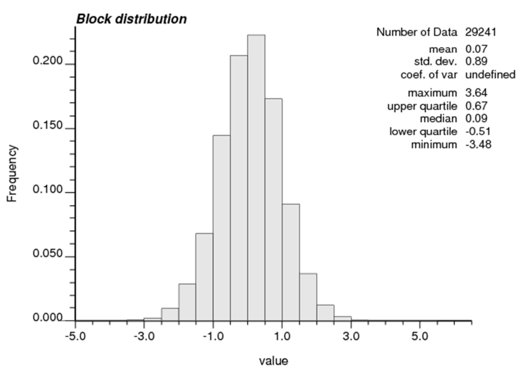

Figure 8. 3x3 block distribution

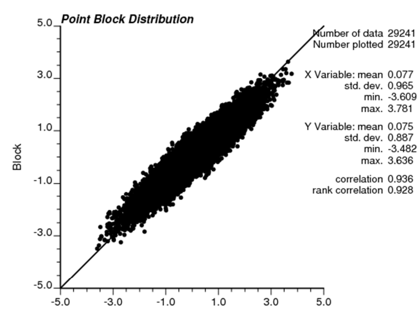

Figure 9. points-blocks scattered plot