Determination of Equivalent Elastic Moduli for Coupled Geomechanical-Flow Simulation of SAGD

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When characterizing stress sensitive reservoirs for reservoir performance prediction, considering flow simulation alone is insufficient; geomechanical and flow responses should be assessed concurrently. Geostatistical techniques are well developed and can be used to build heterogeneous models of reservoir properties as input to subsequent transfer functions such as flow and geomechanical simulators. In the case of conventional flow simulation, each geological realization consists of a structural model, facies model and petrophysical property models (porosity, permeability, saturation, etc.), which are used to solve the appropriate fluid flow equations. Rock mechanical properties play a similar role in geomechanical simulation as petrophysical properties play in fluid flow. Impact of heterogeneity consideration for geomechanical properties on coupled geomechanical flow simulation of SAGD process has been discussed by Khajeh et al. (2011) and they showed that to assess more accurate uncertainty analysis on flow and geomechanical responses of the SAGD process, considering realizations of the rock mechanical properties are required. The well documented Mechanical Earth Model (MEM) is a comprehensive geological model which can be used for coupled geomechanical-flow simulation of the SAGD process; however, homogenous rock mechanical properties are typically considered instead of stochastic models for computational reasons. The geomechanical response of a reservoir is sensitive to the values selected for geomechanical properties and it is practical to consider an equivalent homogeneous continuum such that geomechanical responses obtained from the homogenized model matches the geomechanical response of the truth (heterogeneous) model. Different analytical homogenization techniques are developed to determine equivalent elastic moduli (EEM), but the majority of these techniques consider specific configurations of facies and do not work well for complex spatial configurations such as the sand/shale sequences typical of the McMurray formation of Alberta-Canada. Considering sand geomechanical properties to be representative of the reservoir is a common approach. Moreover, a mixing rule averaging approach could also be used for determination of EEM. In this work, the accuracy of these various EEM methodologies are compared to EEM values obtained numerically by optimizing the geomechanical response of the reservoir. By knowing the vertical displacement profile (VDP) at the top of the reservoir for the fine scale (truth) model and the VDP's obtained from the listed techniques, the accuracy of considered EEM values is assessed. Sensitivity of change in VDP with respect to different operating conditions, type of elastic deformation and also spatial distribution of facies is examined.

Introduction

Canada has large heavy oil (oil sand) resources which are mostly deposited in the McMurray Formation in northeastern Alberta. Although a portion of oil sands can be recovered using surface mining technology, the majority are at a depth where in-situ recovery techniques, including Steam Assisted Gravity Drainage (SAGD), Cyclic Steam Stimulation (CSS) and Vapor Extraction (VAPEX), are being used to develop oil sands reserves. With the exception of Imperial Oil's Cold Lake CCS operations, SAGD has been proven to be the most effective.

Flow simulation is a reservoir characterization tool which helps reservoir engineers to history match production data and forecast reservoir performance. In most cases, considering flow simulation alone is insufficient for stress sensitive reservoirs and it is necessary to consider coupled geomechanical flow simulation. Oil sands in the McMurray Formation have in-situ interlocked fabric configurations, previous experimental studies by Dusseault and Morgenstern (1978), Agar et al. (1986), Kosar et al. (1987), Oldakowski (1994), Scott et al. (1994), Chalaturnyk (1996), Samieh and Wong (1996) and Touhidi-Baghini (1998) and numerical studies by Chalaturnyk (1996), Settari et al. (2001), Li (2006), Du and Wong (2007) and Azad and Chalaturnyk (2011) have shown that geomechanics can play a significant role in the SAGD process. Thus, conventional simulation techniques may provide less than optimum results for the response of the reservoir and coupled geomechanical flow simulation should be considered.

A geological model is one of the main inputs for the simulation process and geostatistical techniques are used widely to produce these models. Structural, facies and property models are included in each geostatistical realization. Petrophysical properties, i.e. porosity, permeability and saturations, are the only properties required for conventional flow simulation. Khajeh et al. (2011) shows that considering heterogeneous geomechanical properties has a significant effect on predicted reservoir performance; to obtain an accurate uncertainty analysis with respect to coupled geomechanical flow simulation of SAGD, geomechanical properties should be modeled stochastically. A comprehensive geological model consisting of petrophysical and rock mechanical properties as well as the in-situ stress state is termed a Mechanical Earth Model (MEM) and should be used for coupled geomechanical-flow simulation of SAGD.

To capture detailed geological information, models are generally built at a finer scale than is practical for flow simulation. Appropriate up-scaling techniques are required to generate coarser geological models that are suitable for simulation. The response (geomechanical or flow) of the upscaled reservoir model should be identical (within acceptable error) to the response obtained from the fine scale model. Significant research has been conducted into permeability up-scaling. Effective permeability up-scaling techniques can be divided in two categories; analytical (static) and numerical techniques (dynamic). Arithmetic, harmonic and geometric averaging are three types of analytical up-scaling. A generalization of these averaging techniques is Power Law averaging, developed by Deutsch (1989):

$$k_w = \left(\frac{1}{n}\sum_{i=1}^{n} {k_i}^{w}\right)^{1/w} \qquad \textbf{(1)}$$

In numerical techniques, effective permeability is calculated for a coarse block in such a manner that the flow response obtained for each block is the same as that obtained from the finer blocks. Dealing with coupled geomechanical-flow simulation, both the flow responses and geomechanical responses obtained from simulation of the coarse model should honour responses obtained from the fine scale model. In the same manner that permeability up-scaling occurs for flow modeling, rock mechanical properties should be up-scaled in such a manner that geomechanical responses match appropriately with geomechanical responses obtained from the fine scale heterogeneous model.

Up-scaling of geomechanical properties should be applied to both elastic and plastic properties; however, this work is restricted to elastic properties. Plastic deformation of the reservoir during SAGD will be considered in future works. Several analytical techniques have been developed to determine equivalent elastic media (EEM) of multi-facies materials. Mackenzie (1950) is one of the first authors who used a self-consistent model to determine EEM of media composed of three phases. Hashin (1955), Backus (1962), Hill (1965), Budiansky (1965) and Salamon (1968) developed other analytical formulations for EEM calculation. Although different assumptions are considered in these approaches, a common element is their consideration of a simplified configuration of phases (facies) which is typically a stratified configuration and may not be appropriate for complex facies configurations. In current coupled geomechanical flow simulation studies, homogenous geomechanical properties are generally considered for region under study. For the McMurray Formation, the predominance of sand facies usually results in the geomechanical properties of sand being chosen to represent the homogenized EEM of the formation. The challenge in this setting is that even the low percentile distribution of shale within the McMurray Formation may have a significant effect on the geomechanical response. Using a mixing rule technique is another approach which could be used to determine the homogenized EEM.

In this study the efficiency of existing analytical techniques for determination of EEM used for coupled geomechanical flow simulation of SAGD is investigated. For that purpose, three different shale/sand configurations are considered; (1) a layer cake model (2) spatially correlated models generated with sequential indicator simulation and (3) randomly distributed shale. The vertical displacement profile (VDP) for the top of the reservoir is considered as the geomechanical response for model comparisons. The VDP profile obtained from the fine scale models is compared to the VDP of the homogenized model in which EEM is obtained from (1) an analytical technique (2) using sand properties for the entire reservoir (3) a linear averaging mixture rule technique and (4) our proposed numerical optimization methodology where the difference in geomechanical response is minimized.

Sensitivity of the VDP with respect to SAGD operating conditions (i.e. injection pressure), nonlinearity consideration for elastic deformation and also spatial distribution of facies are discussed. Comparing the VDP of the proposed numerically homogenized media with other approaches used for determination of EEM shows that all techniques explored do not work appropriately for the determination of EEM and a new empirical correlation specific to the SAGD process is necessary.

Model description

To decrease boundary effects, the dimensions of a model considered for geomechanical analyses is usually 3 to 4 times larger than dimensions of the model considered for flow analysis. In addition to the common reservoir section between the two simulators, additional depth above and below the reservoir (overburden and underburden) and sideburden is considered in the geomechanical model (Figure 1). As the reservoir is the only section which is considered for coupled geomechanical-flow analysis, a coarser gird was considered for the regions surrounding the reservoir.

Fixed horizontal displacement for all sides of the model and fixed vertical displacements at the bottom of the model are considered. In-situ stress configuration (i.e. magnitudes and directions) has a significant impact on geomechanical response and affects the optimization of injection pressure to prevent cap-rock instability, the maximum dilatancy of the reservoir, and the selection of drilling direction to maximize SAGD performance. The magnitudes selected for minimum and maximum horizontal stresses, pore pressure and vertical stress are based on Collins (2002) and are given in Table 1.

Table 1. Initial stress, pore pressure and temperature for the case under study [After Collins (2002)]

Parameter	Value
Reservoir Depth	150 meter
σ_h / σ_v	1
σ_H / σ_v	1.5
Initial reservoir pressure	650 kPa
Initial reservoir temperature	12 °C

Sand/shale configuration models

In this study it is assumed that the shale volume percentile is 20% of the reservoir section. Three different cases for the sand/shale configuration are considered: a layer cake model; a spatially correlated model; a randomly distributed model. Sequential indicator simulation (SIS) as implemented in GSLIB (Deutsch, 1998) is used to build sand/shale sequences for spatially correlated and randomly distributed models (Figure 2).

Petrophysical and rock mechanical properties

Linear elastic deformation is considered for all model regions (Figure 1). Table 2 lists the properties considered for over, side and underburden. The reservoir section is the only region which is common between the two simulators. Different petrophysical (porosity, permeability and oil saturation) and elastic properties (bulk and shear modulus) are considered for sand and shale (Table 3). Additional rock mechanical properties are required for flow simulation and are summarized in Table 4. Parameters in Table 2 and Table 4 are selected based on previous studies (Chalaturnyk, 1996; Li, 2006).

Table 2. Grid Density information for the model under study

Zone	Parameter	Value
Overburden	Bulk Density (kg/m^3)	2200
	Bulk Modulus (MPa)	208
	Shear Modulus (MPa)	96.2
	Linear Thermal Expansion coefficient (°K^{-1})	2×10^{-5}
Sideburden	Bulk Density (kg/m^3)	2200
	Bulk Modulus (MPa)	620
	Shear Modulus (MPa)	286
	Linear Thermal Expansion coefficient (°K^{-1})	2×10^{-5}
Underburden	Bulk Density (kg/m^3)	2200
	Bulk Modulus (MPa)	4167
	Shear Modulus (MPa)	1923
	Linear Thermal Expansion coefficient (°K^{-1})	2×10^{-5}

Table 3. Petrophysical and elastic properties considered for sand and shale facies

Property	Sand	Shale
Bulk Modulus (MPa)	900	150
Shear Modulus (MPa)	415	69
Permeability (mD)	3000	1
Porosity (%)	0.3	0.01
Oil Saturation (%)	0.85	0.05

Table 4. Rock parameters used in flow simulator

Parameter	Value
Rock Compressibility (1/kPa)	5×10^{-6}
Rock Expansion Coefficient ($^{\circ}C^{-1}$)	3.84×10^{-5}
Rock Heat Capacity (kJ/kg°K)	1865
Rock Thermal Conductivity (W/m°K)	1.736

Analytical, mixing rule and numerical techniques used for EEM determination

The analytical approach developed by Budiansky (1965) was adopted for this study. In this approach the composite materials are assumed to be isotropic, elastic and spatial distributions of the phases are assumed such that, in general, the composite material is homogeneous and isotropic. Spatial distribution of the materials is not considered and EEM is a function of the initial elastic value and volume fraction of each material. The material is imagined to consist of contiguous, irregular grains of the constituent materials.

The averaged shear modulus in this approach is:

$$\frac{1}{G^*} = \frac{1}{G_N} + \sum_{i=1}^{N-1} \left(1 - \frac{G_i}{G_N}\right) \frac{c_i}{G^* + \beta^*(G_i - G^*)} \tag{2}$$

c_i is the volume fraction of each material and β^* is:

$$\beta^* = \frac{2(4 - 5\nu^*)}{15(1 - \nu^*)} \tag{3}$$

In Eq. 3 ν^* is the poisson's ratio of composite material. Shale and sand are assumed to have the same poison ratio, thus, ν^* is 0.3 for the composite material as well.

The averaged shear modulus using a linear averaging technique (mixing rule) is obtained from:

$$G^* = \sum_{i=1}^{N} c_i G_i \tag{4}$$

To obtain EEM numerically, an error function, which is representative of the difference between the truth (fine scale model) VDP response and VDP obtained from the homogenized value, is defined as follows:

$$Error = \sum_{i=1}^{n} (x_i - x_i^*)^2 \tag{5}$$

where x_i is the value of vertical displacement along the top of reservoir using a homogenized EEM and x_i^* is the value of vertical displacement on the top of the reservoir obtained from the fine scale model. The numerically determined EEM is the value which results in the minimum error. A Golden section search approach was used to find EEM numerically. Golden section search is a technique for finding the extremum (minimum or maximum) of a function by successively narrowing the range of values inside which the extremum is known to exist. Golden section search was introduced by Kiefer (1953).

Results

Layer Cake Model. Consider an injection pressure of 1500 kPa, the resulting VDP at the top of reservoir after 1000 day is obtained (Figure 3). The VDP obtained from the numerical EEM, the Budiansky approach, the mixing rule approach and assuming sand properties are compared (Figure 3 and Table 5). Using sand properties as EEM, which is common in industry, results in the largest mismatch. Although choosing the analytical approach results in a lower error in comparison with the other techniques, the results obtained from all three alternative approaches underestimate the displacement obtained from the fine scale (truth) model. The numerical approach provides a reasonable estimate of the overall VDP even if some local accuracy is lost.

Table 5. homogenized EEM values obtained from different approaches-Layer Cake Model

Approach used to obtain EEM	Homogenized Bulk Modulus [MPa]
Sand Properties	900
Mixing Rule	750
Analytical Technique	617
Numerical Technique	**500**

In SAGD the driving forces which result in stress-strain redistribution are due to temperature and pressure changes. Variation in operating conditions result in variations in steam chamber propagation and associated temperature and pressure profiles. In addition, consideration of elastic parameters as a function of mean or minimum effective stress, i.e. nonlinear elasticity, is another factor which has a significant effect on the geomechanical response of a reservoir. None of the EEM approaches discussed consider these two important effects; however, the numerical approach used to obtain an homogenized EEM value is based on VDP obtained from the fine scale model and a change in VDP due to operating conditions and effective stresses will have an effect on the upscaled EEM value. In the following sections, sensitivity of VDP with respect to injection pressure and non-linear elasticity is investigated.

Sensitivity of VDP to Injection Pressure. By changing injection pressure from 1500 kPa to 3000 kPa the sensitivity of VDP to injection pressure is assessed (Figure 4).

Sensitivity of VDP with respect to Non-linear Elasticity Consideration. Chalaturnyk (1996) and Li (2006) proposed empirical correlations for the variation in elastic parameters as a function of minimum effective stress. Equations 6 and 7 are suggested by Li (2006) and Chalaturnyk (1996) respectively. The correlation suggested by Chalaturnyk (1996) is used for updating Young Modulus as a function of minimum effective stress to determine the sensitivity of VDP (Figure 5).
Significant changes in VDP are obtained when changing operating conditions (Figure 4) and nonlinear elasticity (Figure 5), while none of these factors is considered in the determination of EEM in conventional homogenization techniques used for calculating EEM.

$$E = 950\, P_a\, \left(\frac{\sigma_3'}{P_a}\right)^{0.5} \tag{6}$$

$$E = E_0\, (\sigma_3')^{0.875} \tag{7}$$

Spatial Correlated Model. Except for initial petrophysical and elastic properties, all conditions mentioned for the layer-cake model are considered for the spatially correlated model. Figure 2-b is considered for the facies model and petrophysical and elastic properties from Table 3 are assigned to the sand and shale facies. Figure 6 illustrates the VDP for the simulations conducted on the spatially correlated model. EEM values using different approaches are summarized (Table 6).

Table 6. homogenized EEM values obtained from different approaches-Spatially Correlated Model

Approach	Homogenized Bulk Modulus [MPa]
Sand Properties	900
Mixing Rule	750
Analytical Technique	617
Numerical Technique	**560**

Similar to the layer cake model, there is significant changes between the numerically determined EEM and the values obtained from the other approaches. It is important to note that the over predictions of EEM, ranging from 10% to 60% (relative to the value determined from the error minimization numerical technique) are not as large as the over predictions from the layer cake model, suggesting that inclusion of facies distributions in an EEM methodology is warranted. Also, the shape of the VDP, as predicted by the fine scale model, is very poorly captured by all equivalent media techniques due to the nonlinear response of VDP to the spatial location of the shale facies (Figure 2-b).

Sensitivity of VDP with respect to Injection Pressure. Figure 7 shows the sensitivity of the VDP of the spatially correlated model with respect to steam injection pressure changes.

Sensitivity of VDP with respect to Non-linear Elasticity Consideration. Figure 8 shows the same graph as Figure 5 but for spatially correlated model. Again considerable differences in VDPs can be seen by changing injection pressure and considering non-linear elastic deformation.
The nugget effect and variogram anisotropy ratio (horizontal to vertical variogram range), used in SIS when generating the facies realizations, are two important parameters which effect the spatial distribution of facies. A nugget effect of zero and an anisotropy ratio of fifty was considered for Figure 2-b. In the next section it will be shown that not only by considering the same nugget and anisotropy ratio different realizations result in different VDP, but also there will be significant change in VDP as a result of considering different nugget and anisotropy ratios.

Sensitivity of VDP with respect to the Spatial Distribution facies. Figure 9 shows 4 facies models with the same nugget and anisotropy ratio considered for Figure 2-b. Figure 10 shows VDP of these 4 realizations. The red solid line is VDP of the model shown in Figure 2-b. Injection pressure is considered to be 3000 kPa. Clearly, for each set of nugget effect-anisotropy ratio, different realizations result in significantly different VDPs (Figure 10). In Figure 11, five additional realizations (beyond the base case of N=0, AR=50) are generated by varying the nugget effect and anisotropy ratio. Figure 12 shows variation in displacement profile as a result of the variation in variogram parameters considered for facies modeling. Figure 10-b is the same as Figure 2-b and the results obtained for this model are shown by the red solid line in Figure 11. Injection pressure was considered to be 3000 kPa.

Changes in VDP as a result of change in spatial distribution of sand/shale facies could be interpreted as change in steam chamber shape and change in the position of pressure and temperature fronts as a result of change in spatial distribution of facies. As a result of the low permeability and porosity of shale, it behaves as a barrier in the thermal recovery processes. Temperature and pressure changes are the mechanisms which cause stress-strain redistribution and, therefore, changes the geomechanical response of the formation, as measured by VDP, for different realizations. So it should be expected that different spatial distributions result in different VDP's. The key outcome from these simulations, however, is that EEM techniques that do not somehow honor these facies distributions will not reflect these variations in VDP and can have an impact on issues such as cap rock integrity assessments; considering facies proportions alone in the determination of EEM is insufficient.

Randomly Distributed Model. In Figure 13 the VDP of the heterogeneous fine scale model (Figure 2-c) is shown as a black solid line. VDPs obtained from different techniques are also shown. The red solid line is related to the case in which the EEM value is obtained numerically. In Table 7, EEM values obtained from each different technique are summarized with predicted EEM values in comparison to the numerically determined value ranging from -5% to 39%.

Table 7. homogenized EEM values obtained from different approaches-Randomly Distributed Model

Approach used to obtain EEM	Homogenized Bulk Modulus [MPa]
Sand Properties	900
Mixing Rule	750
Analytical Technique	617
Numerical Technique	**650**

In comparison to the other two facies configuration, (layer cake and spatially correlated models) the EEM (and VDP) obtained from the analytical technique is very similar to the one obtained numerically. It can be concluded that for randomly distributed heterogeneous models (pure nugget effect), the analytical technique used here can be used for the determination of homogenized EEM; however, the geological reality of this facies configuration is questionable. Using analytical techniques to obtain homogenized EEM means that it has been assumed that there is no spatial correlation in facies configuration and the facies are randomly distributed.

Figure 14 shows sensitivity of VDP respect to injection pressure. Figure 15 shows sensitivity of VDP respect to non-linearity consideration for elastic deformation. The same conclusions made for the two previous facies configurations can be observed for the randomly distributed model as well. The VDP is sensitive to operation conditions and type of deformation consideration.

Discussion and Conclusions

In this study, efficiency of existing approaches to define homogenized EEM considered for coupled geomechanical flow simulation process was assessed. For that purpose three different sand/shale configurations were considered and EEM from three different approaches (the analytical technique, using sand properties and the mixing rule) were compared. The best and most precise EEM (in comparison to the fine scale truth model) was obtained numerically and the corresponding VDP of all of these four approaches were compared. Generally it can be concluded that:

- Geomechanical responses, which were the VDP's along the top of the reservoir, obtained from homogenized models in which EEM are calculated using standard approaches are in poor agreement with fine scale (truth model) response predictions.
- Using sand properties as homogenized media is the most typical technique in current industrial SAGD projects dealing with coupled geomechanical-flow simulation. In all three facies configurations considered it was shown that using sand properties as homogenized media results in VDP's that depart the most from the fine scale model response.
- The analytical homogenization technique appeared to be applicable to cases where no spatial correlation was considered for facies modeling. It means that by using analytical techniques for determination of EEM spatial correlation in facies distribution has been ignored and random distribution of sand/shale facies is (perhaps unknowingly) assumed for many studies.
- For the three analytical EEM approaches considered in this study, none were capable of capturing the effect of variation in the VDP as a result of changing operating conditions, non-linear elastic deformation and the spatial distribution of facies. However, it was shown that there is considerable change in VDP as a result of the change in any of these mentioned parameters.

Future work

Developing new correlations for defining EEM which could be used for coupled geomechanical-flow simulation of SAGD is important to allow for more rigorous, large scale simulations. This correlation should be obtained based on numerical studies and should be a function of parameters which have an effect on the geomechanical response of reservoirs. Some of the main parameters which have effect on VDP was discussed in this study are, operating conditions, non-linear elasticity deformation and the spatial distribution of facies. Variation in inherent properties of each facies is another parameter which should be considered beside facies spatial distribution. In addition to these parameters, the effect of time and change in VDP in different time step should be investigated as well.

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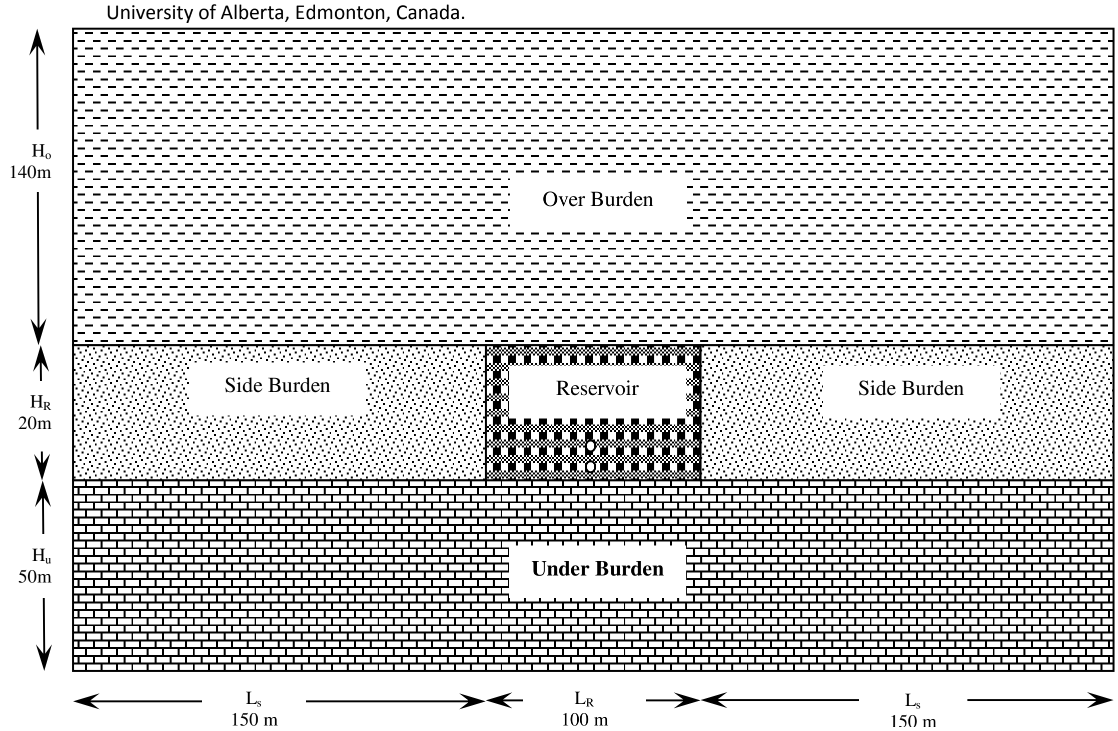

Figure 1. Model description and dimensions used for this study

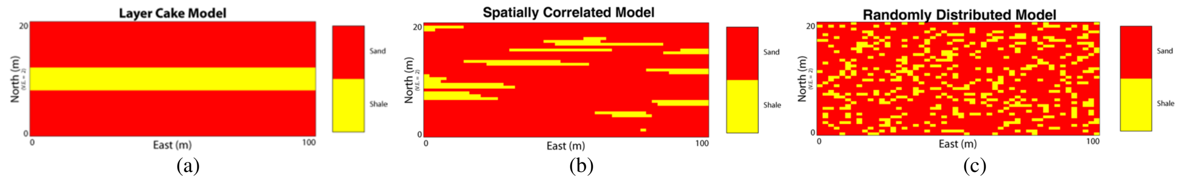

Figure 2. a) Layer cake, b) Spatially correlated and c) Randomly distributed sand/shale configurations

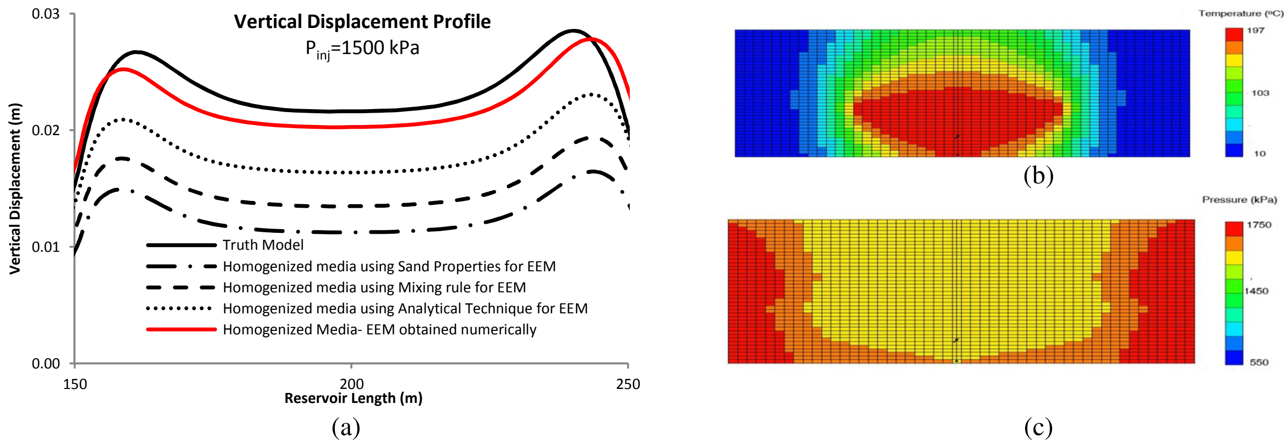

Figure 3. Layer Cake model. a) VDP's of the heterogeneous fine scale (truth) model and homogenized media using different approaches to obtain EEM, b)Temperature profile and c) Pressure profile.

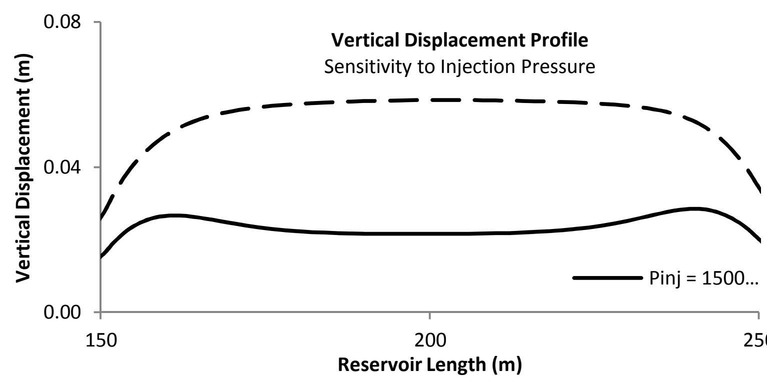

Figure 4. Sensitivity of VDP respect to Injection Pressure- Layer Cake Model

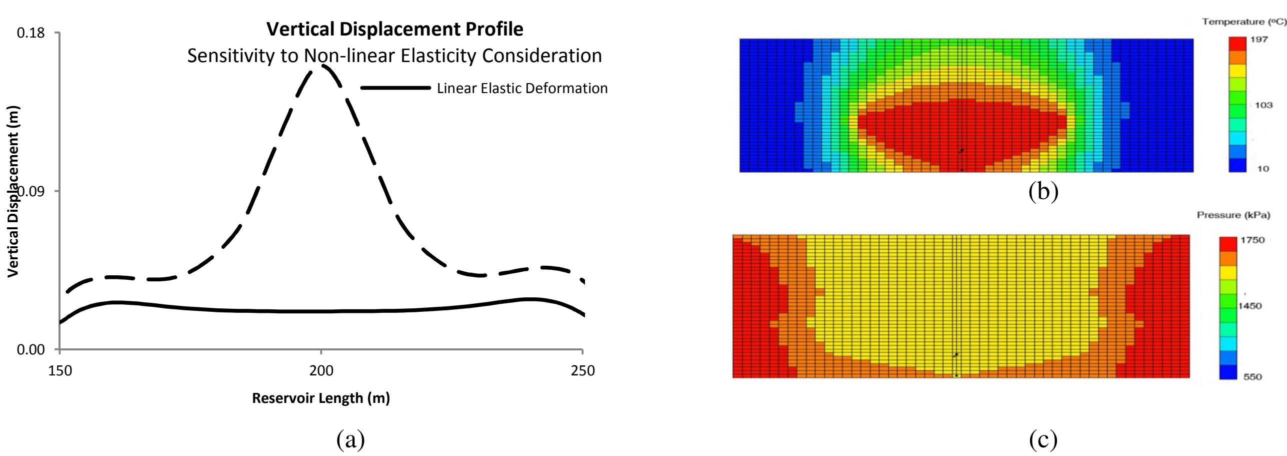

Figure 5. a) Sensitivity of VDP respect to non-linear elasticity consideration in Layer Cake Model, b)Temperature profile and c) Pressure profile

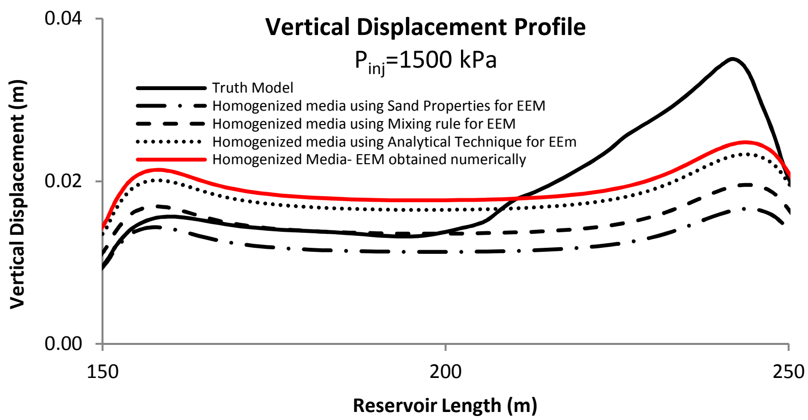

Figure 6. VDP's of heterogeneous (truth) model and homogenized media using different approaches to obtain EEM- Spatially Correlated Model

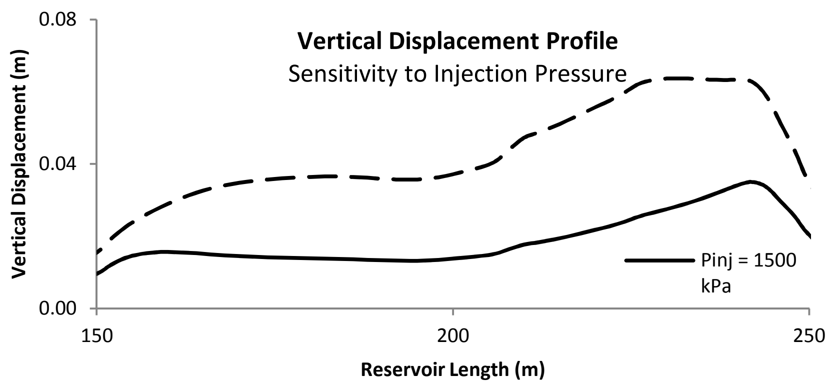

Figure 7. Sensitivity of VDP respect to Injection Pressure- Spatially Correlated Model

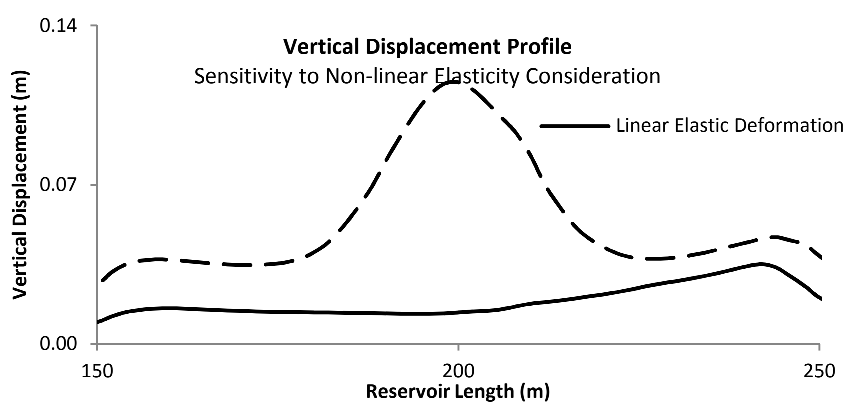

Figure 8. Sensitivity of VDP respect to non-linear elasticity consideration- Spatially Correlated Model

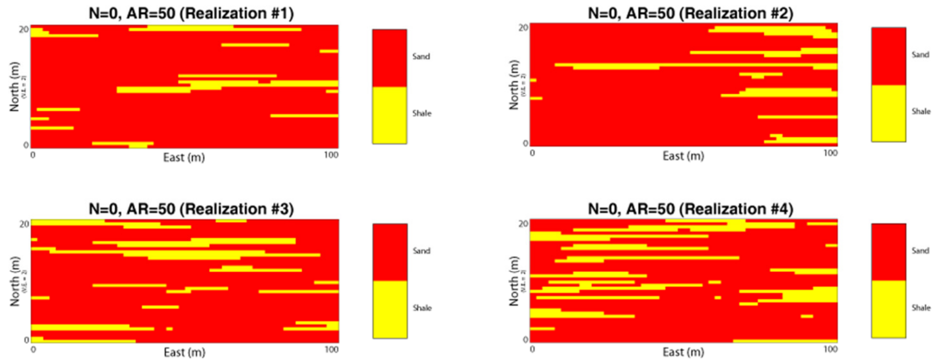

Figure 9. Four different Facies models by considering Nugget effect (N)=0 and Anisotropy Ratio (AR)=50

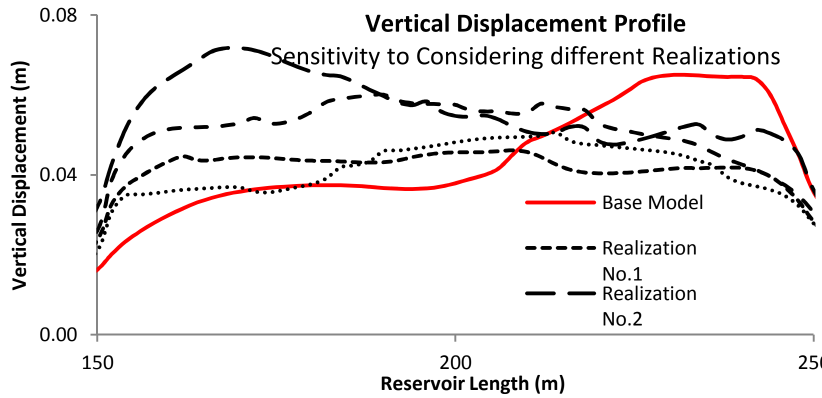

Figure 10. Variation in VDP by considering different realization for by considering Nugget Effect (N)=0 and Anisotropy Ratio (AR)=50

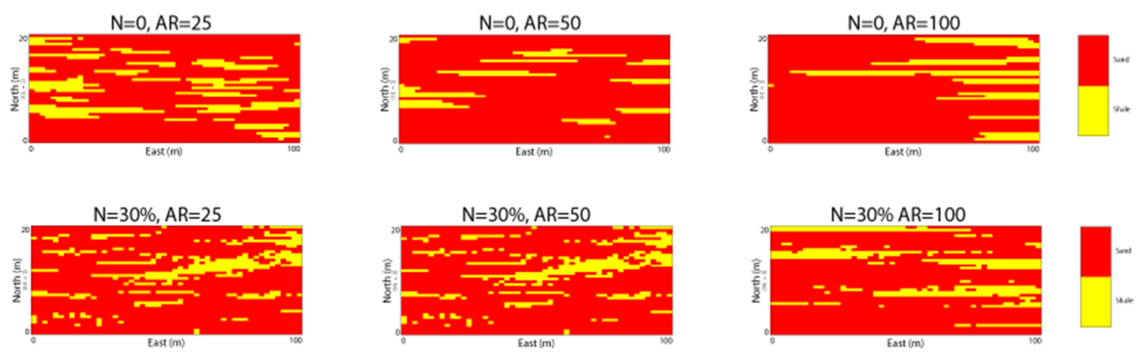

Figure 11. Different Facies Models as a result of the nugget effect (N) and Anisotropy ratio (AR), a)N=0-AR=25, b) Base Case: N=0-AR=50, c)N=0-AR=100, d)N=0.3-AR=25, e)N=0.3-AR=50 and f)N=0.3-AR=100

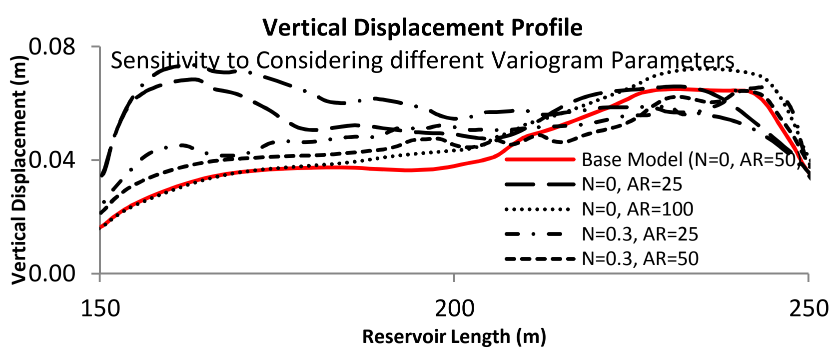

Figure 12. Variation in VDP by considering different values for Nugget Effect(N) and Anisotropy Ratio (AR)

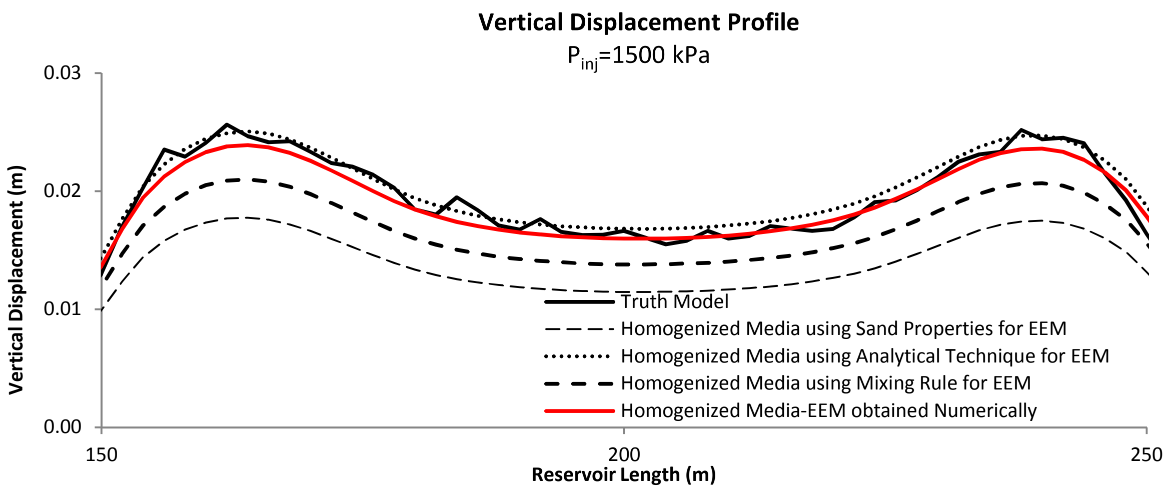

Figure 13. VDP's of heterogeneous (truth) model and homogenized media using different approaches to obtain EEM-Randomly Distributed Model

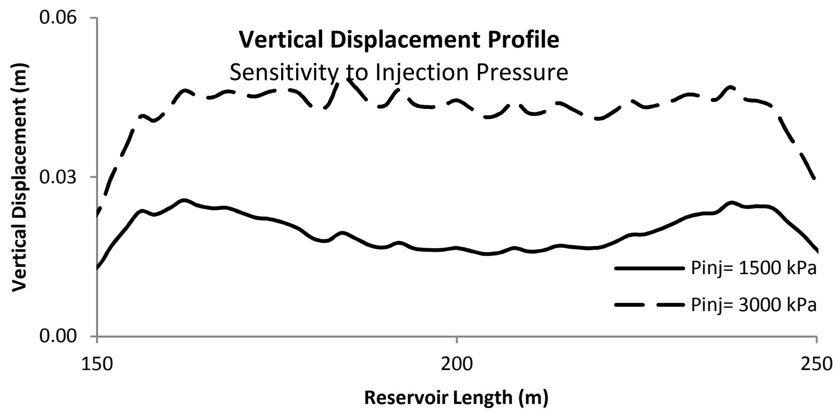

Figure 14. Sensitivity of VDP respect to Injection Pressure- Randomly Distributed Model

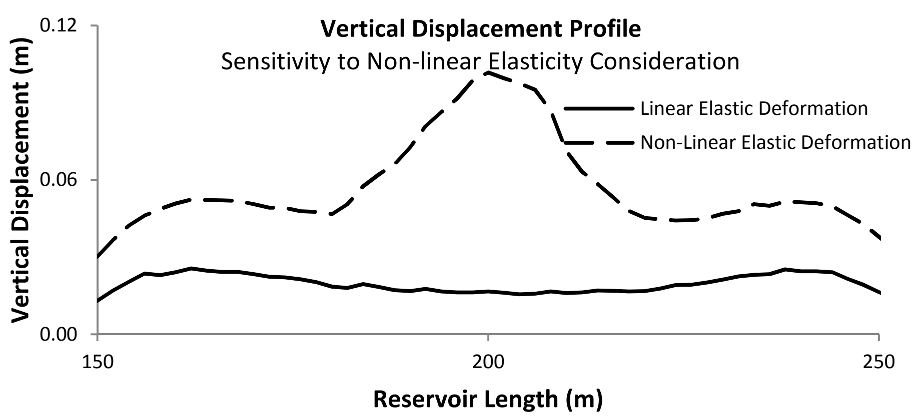

Figure 15. Sensitivity of VDP respect to non-linear elasticity consideration- Randomly Distributed Model