Quantifying the Cost of Grade Uncertainty in Mine Planning

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Uncertainty is present because of lack of information. Conditional simulation algorithms are widely used to model grade uncertainty. This uncertainty has some negative effects on mine planning process and it is required to transfer and measure risk in the long-term production schedule. In this paper, the effect of grade uncertainty in processing plant and economic block value is presented and also a quantitative method is presented to calculate the discounted cost of uncertainty in a production schedule. An oil sand deposit is used to demonstrate the presented methods.

Introduction

Grade uncertainty is modeled by generating equal probable realizations using geostatistical conditional simulation techniques. For each block, a local distribution of grade can be generated with simulated values, which show the local uncertainty. Usually, average grade of a block is used to determine whether a block should be processed or not. Average grade is arithmetic mean of all simulated values or simply is the estimated grade value of the block using Kriging (Journel and Huijbregts, 1981). Cut-off grade is a critical threshold that any block with grade above this limit would be considered as ore and has an economical value and any material bellow cut-off grade will be dealt as waste with no economic value. Lane (1988) presented the fundamentals of cut-off grade calculation.

There are four different situations that may happen based on cut-off grade and local distribution of a block generated by n conditional simulations.

- 1. All n simulation values are bellow cut-off grade (Figure 1)
- 2. All n simulation values are above cut-off grade (Figure 2)
- 3. Not all off simulation values are above ore bellow the cut-off but average grade is bellow cut-off grade (Figure 3)
- 4. Not all off simulation values are above ore bellow the cut-off but average grade is above cut-off grade (Figure 4)

It is assumed that the number of realizations is sufficient to capture grade uncertainty with a reasonable level of statistical confidence. A synthetic case is assumed to demonstrate all four situations with lognormal distributions for grade of blocks. Four different mean and variances and a cut-grade 2% has been chosen. Figure 1 to Figure 4 show the Probability Density Function (PDF) at left and Cumulative Density Function (CDF) at right for all four situations.

For case 1 and 3, because the average grade of the blocks are below the cut-off, therefore these cases are considered as waste blocks (Figure 1 and Figure 3). Average grade of cases 2 and 3 are above the cut-off and these blocks will be treated and sent to the processing plant (Figure 2 and Figure 4). Based on n simulation values, there is a very small risk on decision that is made for cases 1 and 2 (Figure 1 and Figure 2). Block with distribution of case 3 has been chosen as waste because average grade is less than the cut-off grade. But there is 14.7 percent chance to get higher value than the cut-off grade. This means that there is 14.7 percent chance that this block would be considered as ore, but it was considered as waste. Let's assume an extraction schedule that is generated with average grade of blocks. At this case, this block with probability of 14.7 percent, may generate over production.

The same situation would happen to the case 4 (Figure 4). This block is classified as ore block because its average grade is higher than the cut-off grade. There is a chance of 15.2 percent that the grade of this block is below the cut-off grade. It means that with a block extraction schedule generated based on average grade, this block with a probability of 15.2 percent may be considered as waste. This will cause under production following the designed schedule.

Grade uncertainty and economic block value

The Economic Block Value (EBV) is presented by Eq. (1).

$$EBV_{n}^{t} = \begin{cases} o_{n} \times (\overline{g}_{n} \times p_{r} \times \text{price}^{t} - p_{c}^{t}) - (o_{n} + w_{n}) \times m_{c}^{t} & \text{if } \overline{g}_{n} \ge g_{cut} \\ -(o_{n} + w_{n}) \times m_{c}^{t} & \text{if } \overline{g}_{n} < g_{cut} \end{cases}$$

$$(1)$$

where o_n is the tonnage of ore, w_n is tonnage of waste, \overline{g}_n average grade of block n, p_r processing recovery, p_c^t processing cost, m_c^t mining cost, price^t is the present selling price of final product, g_{cut} is cut-off grade.

EBV is a positive value when the average grade of the block is above cut-off grade and equal to a negative mining cost for waste blocks (Figure 5).

To show the effect of grade uncertainty on EBV, a synthetic case is assumed. A block with a log normal distribution of grade with a mean and standard deviation of 2.2% and 0.5% respectively is simulated 10,000 times (Figure 6) .The cut-off grade is assumed to be 2%. Therefore, this block is considered as ore block because the average grade is above the cut-off grade.

EBV of this block is calculated for all 10,000 realizations where $p_r=100\%$, Price = 1\$, $p_c=0.5\$$ and $m_c=1.5\$$. The tonnage of this block is assumed 1 tonnes. Histogram of EBV is shown in Figure 7. 3744 times (37.44%) over 10,000 generated values, the block is assumed as waste because the simulated grade is less than cut-of grade. This shows with trimmed black column at Figure 7. 62.56% of the times, the simulated grade is above cut off-grade and assumed as ore (gray columns at the Figure 7). The average of EBV is -0.26 \$ and less than zero. It means that even for a block with average grade above cut-off grade, the average EBV in present of grade uncertainty may be less than zero and it is not economical to be processed.

Cost of uncertainty

The main objective of long-term mine planning is usually to maximize the net present economic value of a project subject to technical and other (e.g. environmental) constrains. Such an objective usually is modeled using optimization techniques. The goal is usually to find the sequence of extraction of blocks or mining-cuts to reach the maximum achievable net present value of the project. The input data into the production scheduling optimization, such as geological block model, grades, costs, prices, recoveries, and practical mining constraints are usually based on the best point estimates available at the time of optimization. Traditional mine planning techniques do not consider grade uncertainty and only one estimate of the block grade is used.

As it is shown in the previous section, the grade uncertainty might cause short falls and surpluses at the processing plant from the designed target production. This is because of making decisions only based on an estimated grade value for a block. Different estimation techniques are used to get average grade of block such as different Kriging method or Etype mean. If average grade of a block is less than the cut-off grade, then the block is classified as waste and vice versa. Therefore, uncertainty on the grade can cause miss classification. It means that, because the estimated grade is not certain, a block that has been classified as waste can be an ore block and vice versa. This miss classification is not considered at traditional mine planning methods which are using only a smooth block model, most of the time an average grade estimation such as Kriging or Etype is used.

The secondary objective seen increasingly in the literature is to take uncertainty into account. This fact that the input variables into the optimization model are uncertain, affects the optimization process. Recently some authors, such as Dimitrakopoulos and Ramazan (2008), present optimization algorithms which aim to maximize expected value of the target function, which is net present value (NPV), and minimizing the negative effect of uncertainty, which is called risk. These methods try to maximize NPV and minimize the risk of grade uncertainty by deferring the extraction of more uncertain blocks into the future and the effect of grade uncertainty will be reduced by gathering new information during mine life. The main idea is that uncertainty somehow costs money and should be deferred. The challenging question is the quantification of the cost of uncertainty. The cost of uncertainty has two main reasons:

- 1. For any shortfalls that may happen at processing plant, there is a missing profit. This profit could be achieved by feeding plant with full capacity.
- 2. On the other hand, assume that the processing plant is working with full capacity and there is enough ore to feed the plant for a while. At this moment truck and shovels are working to remove waste blocks as planned. Because of grade uncertainty and misclassification, a block that has been classified as waste is extracted and has a grade above cut-off grade. In real life, this block will not be extract and the schedule would be changed or most of the time there is a stockpile to send this extra ore to be used in the future. Not having a stockpile is very unlikely in real life. Let's assume a hypothetical case. In this particular situation, there are no stockpiles to store extra ore and the block should be extracted to follow exactly the same schedule. Therefore, this ore block should be sent to the waste dump and its revenue will be lost. This lost profit is also part of cost of uncertainty.

To quantify these two costs, a cost of not meeting the target production is presented in Eq. (2)

$$C_{t} = |P_{t} - \text{Target}_{t}| \times (\overline{g}_{t} \times P_{r} \times \text{Price} - P_{c})$$
(2)

Where C_t is the cost on not meeting the target production in period t, P_i is the input ore to the mill at period t, $Target_t$ is target production for period t, \overline{g}_t average grade of input ore at period t, P_r processing recovery, Price is the selling price of final product and P_c is the processing cost. By averaging C_t over all realization and discounting it over all the planning periods the discounted cost of uncertainty is present by Eq. (3)

$$C_{u} = \frac{1}{L} \sum_{t=1}^{T-1} \sum_{l=1}^{L} \left(\frac{C_{t}^{l}}{(1+i)^{t}} \right)$$
 (3)

Where L is the number of realizations, T is the number of periods and i is the discounting rate. Cost of uncertainty is calculated over all period except final period. Because any ore that is left for final period will be processed and will not exceed the target production. Therefore shortfall at final period is not because of grade uncertainty.

Case Study

An oil sands deposit in Fort McMurray, Alberta, Canada is used. Location of boreholes and histogram of data is presented at Figure 8 and Figure 9 respectively.

GSLIB (Deutsch and Journel, 1998) programs were used in this case study. Directional experimental variograms are calculated and fit using gamv and vmodel programs. The azimuths of major and minor directions are 50 and 140 degrees. Figure 10 shows the experimental and the fitted variogram models in major, minor and vertical directions.

KT3d is employed for Ordinary Kriging (OK) to estimate the bitumen grade (with no normal score transform) at each block location. Multiple realizations of the bitumen grade are generated using Sequential Gaussian Simulation (SGS) (Isaaks and Srivastava, 1989) at a very high resolution three-dimensional grid at the point scale. To get the average grade of a block, simple arithmetic averaging has been done between all simulated points inside the block. This step is called up scaling and program called blkavg was used at this stage. Etype mean was calculated using postsim program. Figure 11 illustrates the map of the bitumen grade for the Kriged, the E-type and realization 26 models at block scale. It is well-known that Kriging is conditionally biased (Isaaks, 2005) and on the other hand "there is no conditional bias of simulation when the simulation results are used correctly" (McLennan and Deutsch, 2004). Conditional biasness of Kriging can be reduced by tuning estimation parameter but it cannot be eliminated (Isaaks, 2005). Grade-Tonnage curve is the good tool to check the impact of Kriging biasness.

Figure 12Error! Reference source not found. shows the grade tonnage curve of simulation realization (dashed lines), Kriging (bold solid line) and Etype (bold dashed line). The conditional biasness of Kriging was tried to be minimized but still there are differences between Kriging and simulation results and that is because SGS is using Simple Kriging(SK). Also Etype is slightly different than Kriging;

Theoretically Etype model is identical with simple Kriging result at Gaussian space (Journel and Huijbregts, 1981).

Histogram and variogram reproduction are checked using gamsim and histpltsim respectively.

Figure 13 shows the histogram reproduction and Figure 14 show the variogram reproduction at major and minor horizontal and vertical directions. Generally Sequential Gaussian simulation would reproduce histogram and variogram of original data if it implements carefully. In this case reproduction of histogram and variograms are acceptable.

The ultimate pit limit design is carried out based on the Syncrude's costs in CAN\$/bbl of sweet blend for the third quarter of 2008 (Jaremko 2009). Price of oil was considered US\$45 with an exchange rate of 1.25:1 equal to CAN \$56.25/bbl SSB for the same time period. We assume that every two tonnes of oil sands with an average grade of 10% mass will produce one barrel of sweet blend, which is approximately 200 kg. We also assume a density of 2.16 tonne/m³ for oil sands, and a density of 2.1 tonne/m³ for waste material, including clay and sand.

Table 1 shows the pit design and production scheduling input parameters. The mining cost of of \$4.6/tonne and processing cost of \$0.5025/tonne is applied. Thirty three pit shells are generated using 49 fixed revenue factors ranging 0.1 to 2.5, based on the Kriged block model. The number of pit shells is reduced to 14 after applying the minimum mining width of 150 meters for the final pit and the intermediate pits. Table 2 summarizes the information related to the final pit limit based on Kriged block model at 6% bitumen cut-off grade. The minimum slope error, the average slope error and the maximum slope error respectively are: 0.0 degrees, 0.2 degrees, 0.4 degrees. The final pit limits was designed for Etype model and all the fifty simulation realizations with the exact same input variables.

The final pit based on Kriging block model was used at this stage. There are 14607 blocks inside the final pit. Using MATLAB (MathWorks Inc., 2007MathWorks Inc., 2007) c-mean clustering function, 1834 mining cut generated by aggregating blocks at the same level with similar grades. Kriging block model were used at LP optimization(Askari-Nasab and Awuah-Offeri, 2009). Two years of pre-stripping was considered to provide enough operating space and ore availability. No stockpile was defined and the target production was set to 36 million tonnes of ore per year with a mining capacity of 135 million tonnes per year. The interest rate is 10%. The mine life is 10 years. There are 653.61 million tonnes of material inside the final pit where 282.44 million tonnes is ore. The strip ratio is 1.31; also there are 37.4 million tonnes of ore with average grade less than cut-off grade and were consider as waste blocks.

The mixed integer programming was solved using TOMLAB CIPLEX (Holmström, 1989-2009) with a gap of 1%. Figure 15 shows the schedule generated by MILP and using Kriging block model. Gray and yellow bars are removed waste and ore materials respectively. The plan view and two cross sections of blocks and their extraction periods are shown at Figure 16. To capture the effect of grade uncertainty, Kriged value of blocks were replaced by simulated values and the same extraction schedule was followed. Any blocks had less simulated grade than cut-off grade was sent to waste dump even if it was consider as ore at Kriging block model and vice versa. Also After following the same schedule, at some realization, there are over produced ore that revenue of them has been count at NPV that is getting form that specific realization. This NPV is not correct; because is it impossible to process over produced ore where there is not stockpile. Therefore the revenue of these surplus ores must be removed. For this reason, there are two versions of results. One is the row results that surplus ore are not removed and revenue is counted and second version is the removed over produced ore and called "Cleaned Version". From Figure 17 to Figure 20 at the left, row results are shown and the cleaned version is at right. Figure 17 to Figure 19 show the effect of grade uncertainty on generated plan for cumulative NPV, head grade and feed of the plant respectively. Bold black line is Kriging, bold dashed blue line is Etype and dashed red lines are 50 conditional simulation results. Figure 17 the NPV of the cleaned realizations are less that row version. For example at row version there is a realization that generates more NPV than Kriged version but at cleaned one there is not any realization to exceed the NPV of Kriging. Figure 20 illustrates the box plot of input ore to the plant calculated with simulation realizations. Also yellow bars show the deviation from target production. As it is clear from this graph and Figure 19, there is probability to produce surplus ore and under produce at period 2 and 4 where only shortfall may happen at period 3. Also because of removing surplus ore at right graph, the deviation from target production is less than row result.

Table 3 shows the summary statistics of generated schedule for two versions. NPV of Kriging is 2461 million Dollars. Expected value of NPV is calculated from all 50 realizations. At above table the statistics shown the row results and at bottom table the results are for removed surplus ores. The average NPV for row and cleaned version respectively are 2335.4 and 2317.5, and are less than Kriged NPV. It is reasonable to have less expected NPV because the whole MILP algorithm was solved to maximize NPV of Kriging and the solution is optimized for Kriged block model. Table 4 shows the summery of statistics for cumulative discounted case flow at different periods. First two periods the cumulative discounted case flow is negative because of pre-striping at these periods and no extraction of ore.

The discounted cost of uncertainty is calculated based row results. Therefore the discounted Cost of uncertainty based on periods 3 to 9 is 178.9 Million Dollars.

For this case study, 861 over 1834 cuts are ore based on Kriging average grade compare to cutoff grade. 84 mining cuts of 861 ore cuts have less expected EBV than minimum acceptable EBV which is
calculated based on cut-off grade. Therefore, if average EBV is the criteria to choose a block to be
processed or not, then all of these 84 mining cuts should not be considered as ore cuts. This shows effect
of grade uncertainty on miss classification even with average grade above cut-off grade. The difference
summation of EBVs of these 86 mining cuts is 64 million Dollars.

Conclusions

In this paper the effect of grade uncertainty on four synthetic cases was illustrated. Four possible situations may happen based on estimated mean and a cut-off grade. Two cases may case shortfalls or extra ore production in generated schedule using only estimated value. This is because of grade uncertainty.

Also it has been shown that cut-off grade and one average value for a block are not good criteria to choose a block as ore or waste. There are some situations that the average grade of a block is above cut-off grade but expected dollar value of block is less than minimum threshold. Therefore average EBV of a block over realizations may be a better criterion to classify a block to be ore or waste.

The cost of not meeting the target production and cost of uncertainty were presented. Using all simulation realizations, the cost of uncertainty can be calculated. It is a good criterion to compare two different schedules. The Cost of Uncertainty is an average dollar value that by following a schedule over all realizations is imposed to the plant because of probability of not meeting the target production.

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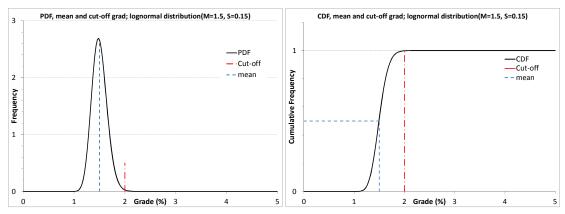


Figure 1. PDF (left) and CDF (right) for case 1, all n realizations and mean (dashed blue line) are less than cut-off grade (dashed red line).

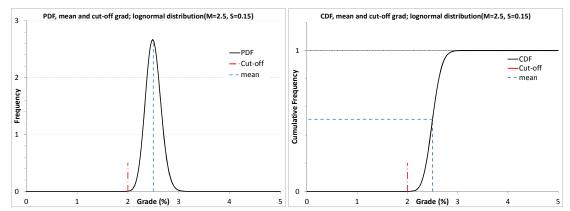


Figure 2. PDF (left) and CDF (right) for case 2, all n realizations and mean (dashed blue line) are higher than cut-off grade (dashed red line).

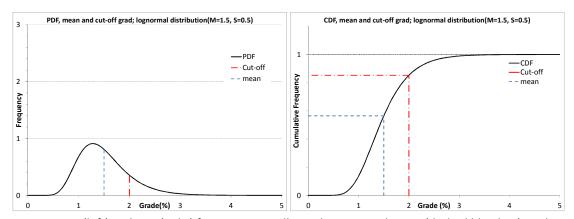


Figure 3. PDF (left) and CDF (right) for case 3, not all n realizations and mean (dashed blue line) are less than cut-off grade (dashed red line).

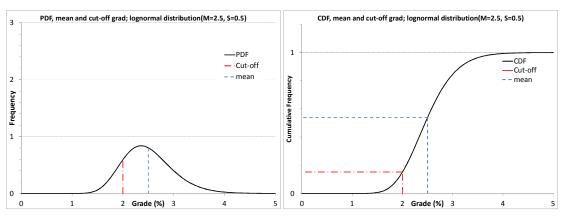


Figure 4. PDF (left) and CDF (right) for case 3, not all n realizations and mean (dashed blue line) are higher than cut-off grade (dashed red line).

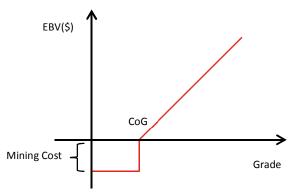


Figure 5. Cut-off grade and EBV.

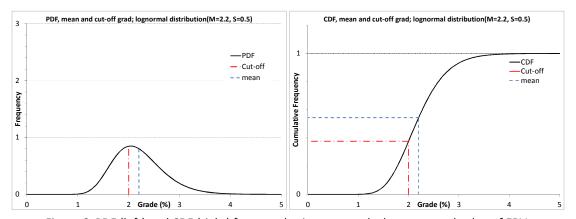


Figure 6. PDF (left) and CDF (right) for a synthetic case to calculate expected value of EBV.

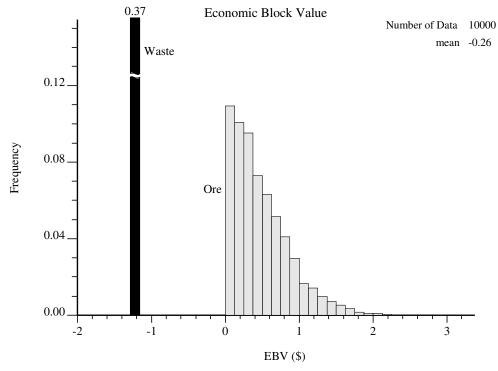


Figure 7. Histogram of EBV for a block with lognormal distribution, mean=2.2 and Std.dev=0.5.

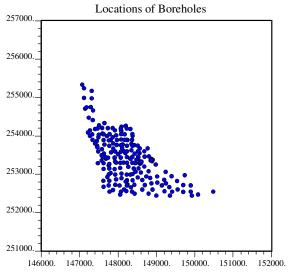


Figure 8. Location map of boreholes.

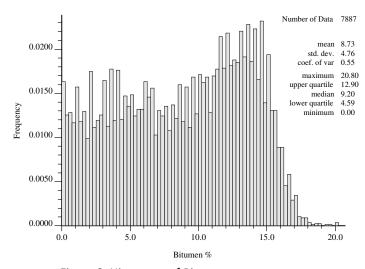


Figure 9. Histogram of Bitumen.

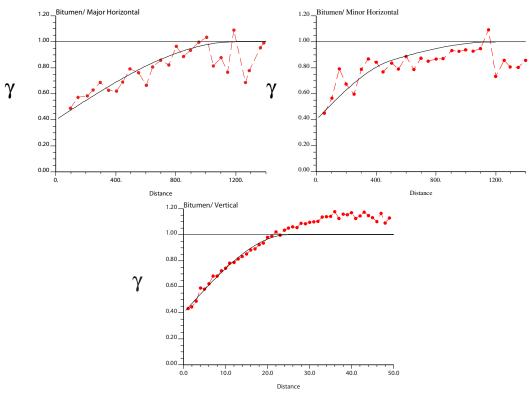


Figure 10. Experimental directional variograms (dots) and the fitted variogram models (solid lines)

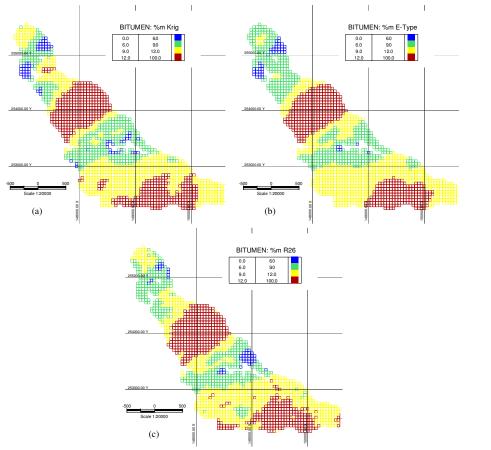
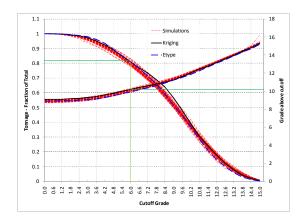


Figure 11. Plan view at 260m; (a) Kriged model, (b) E-type model, (c) realization 26.



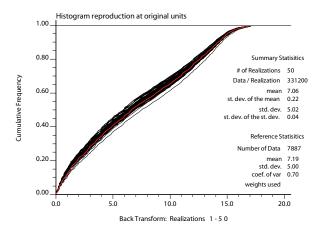


Figure 12. Grade tonnage curve of simulation realizations, kriged, and Etype block models

Figure 13. Histogram reproduction of simulation realizations (dashed lines) and histogram of original data (bold line)

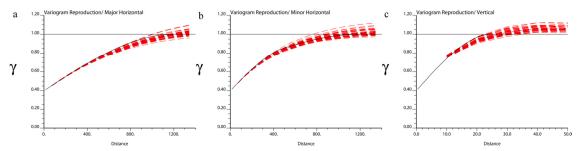


Figure 14. Variogram reproduction of simulation realizations (red dash lines) and reference variogram model (black line).

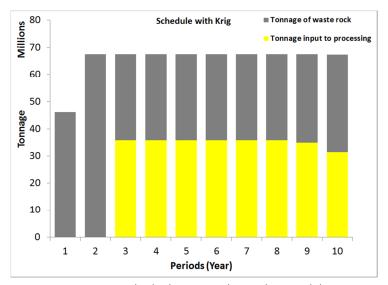


Figure 15. Schedual generated using krig model

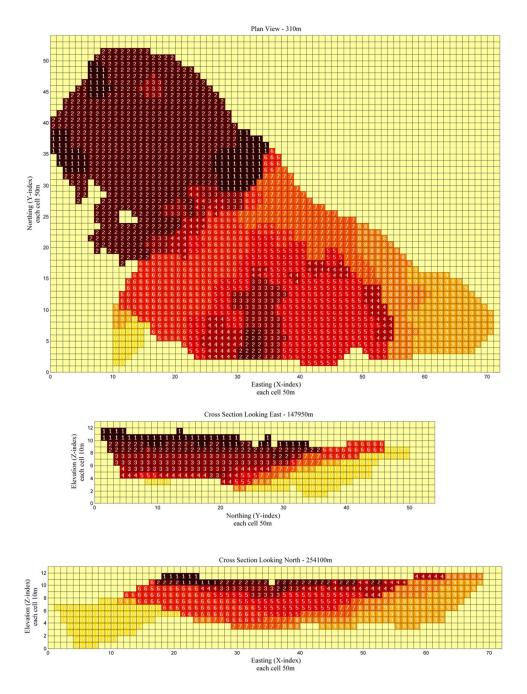


Figure 16. Plan view, cross section looking east and north for generated schedual

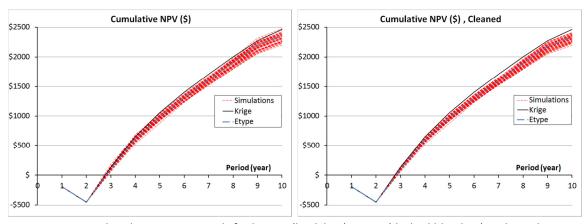


Figure 17. Cumulated NPV over periods for kriging (back line), etype (dashed blue line) and simulations (dash red line), surplus ore not removed at left and cleaned version at right

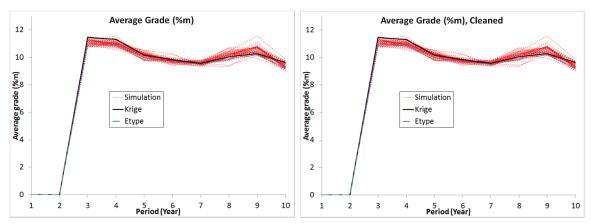


Figure 18. Input head grade to the plant over periods for kriging (back line), etype (dashed blue line) and simulations (dash red line), surplus ore not removed at left and cleaned version at right

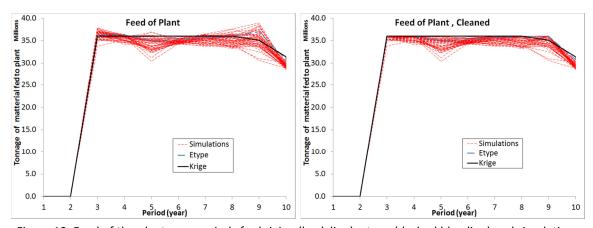


Figure 19. Feed of the plant over periods for kriging (back line), etype (dashed blue line) and simulations (dash red line), surplus ore not removed at left and cleaned version at right

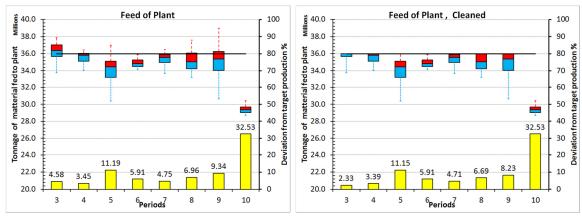


Figure 20. Boxplot and deviation from target production (yellow bars), calculated using simulation values, surplus ore not removed at left and cleaned version at right

Table 1. Final pit limit and mine planning input parameters.

Description	Value	Description	Value
Mining Cost (\$/tonne)	4.6	Processing Cost (\$/tonne)	0.5025
Cutoff grade (%mass bitumen)	6	Processing limit (M tonne/year)	36
Mining recovery fraction	0.88	Mining limit (M tonne/year)	67.5
Processing recovery factor	0.95	Overall slope (degrees)	20
Minimum mining width (m)	150	Pre-stripping (years)	2

Table 2. Material in the final pit using the kriged block model.

Description	Value
Total tonnage of material (M tonne)	653.61
Tonnage of ore (M tonne)	282.44
Tonnage of material below cutoff (M tonne)	37.4
Tonnage of waste (M tonne)	371.166
Bitumen recovered (M tonne)	23.29
Stripping ratio (waste:ore)	1.31

Table 3. Summary statistic of realization simulations when generated schedule with kriging is followed, surplus ore not removed at top and cleaned version at bottom

Row Version	Ore(MT)	STRO	Input Bitumen (MT)	Average %	NPV (M\$)
Mean	276.2	1.4	28.3	10.2	2335.4
Std. dev	3.9	0.0	0.5	0.1	64.2
Min	269.2	1.3	27.3	10.0	2201.9
Quartile 1	273.3	1.3	27.9	10.2	2285.1
Median	276.4	1.4	28.3	10.2	2332.2
Quartile 2	278.7	1.4	28.6	10.3	2385.8
Max	287.3	1.4	29.4	10.5	2473.8
Krig	282.4	1.3	29.1	10.3	2461.0
Etype	282.2	1.3	28.5	10.1	2360.8

Cleaned Version	Ore(MT)	STRO	Input Bitumen (MT)	Average %	NPV (M\$)
Mean	275.0	1.4	28.1	10.2	2317.5
Std. dev	3.1	0.0	0.4	0.1	56.0
Min	268.0	1.3	27.2	10.0	2195.8
Quartile 1	272.7	1.4	27.8	10.2	2267.7
Median	275.2	1.4	28.2	10.2	2322.7
Quartile 2	277.0	1.4	28.4	10.3	2363.0
Max	280.5	1.4	28.8	10.5	2415.6
Krig	282.4	1.3	29.1	10.3	2461.0
Etype	280.7	1.3	28.3	10.1	2341.5

Table 4. Summery statistics of Cumulative NPV at each period, surplus ore not removed at top and cleaned version at bottom

Period	1	2	3	4	5	6	7	8	9	10
Mean	-193.2	-449.8	112.4	603.3	975.7	1,303.9	1,600.6	1,893.0	2,175.6	2,335.4
Std. dev	0.0	0.0	24.6	32.7	36.1	38.9	42.6	51.7	62.9	64.2
Min	-193.2	-449.8	58.8	525.9	879.5	1,228.8	1,522.8	1,794.7	2,055.0	2,201.9
Quartile 1	-193.2	-449.8	95.4	578.6	947.2	1,280.9	1,565.7	1,849.1	2,124.9	2,285.1
Median	-193.2	-449.8	110.6	604.1	975.3	1,297.5	1,602.6	1,904.7	2,169.6	2,332.2
Quartile 2	-193.2	-449.8	129.9	620.1	1,002.2	1,337.7	1,638.8	1,935.7	2,230.3	2,385.8
Max	-193.2	-449.8	180.6	679.3	1,049.9	1,378.2	1,678.9	1,987.6	2,319.8	2,473.8
Krig	-193.2	-449.8	132.0	650.6	1,054.0	1,403.1	1,707.8	2,005.4	2,274.7	2,461.0
Etype	-193.2	-449.8	110.5	606.3	982.7	1,313.2	1,611.0	1,905.4	2,194.3	2,360.8
Period	1	2	3	4	5	6	7	8	9	10
Period Mean	1 -193.2	2 -449.8	3 102.2	4 592.2	5 964.0	6 1,292.2	7 1,588.4	8 1,879.1	9 2,157.7	10 2,317.5
Mean	-193.2	-449.8	102.2	592.2	964.0	1,292.2	1,588.4	1,879.1	2,157.7	2,317.5
Mean Std. dev	-193.2 0.0	-449.8 0.0	102.2 17.8	592.2 27.4	964.0 32.1	1,292.2 35.0	1,588.4 38.7	1,879.1 46.4	2,157.7 54.8	2,317.5 56.0
Mean Std. dev Min	-193.2 0.0 -193.2	-449.8 0.0 -449.8	102.2 17.8 58.8	592.2 27.4 525.8	964.0 32.1 879.5	1,292.2 35.0 1,228.8	1,588.4 38.7 1,520.7	1,879.1 46.4 1,792.7	2,157.7 54.8 2,048.9	2,317.5 56.0 2,195.8
Mean Std. dev Min Quartile 1	-193.2 0.0 -193.2 -193.2	-449.8 0.0 -449.8 -449.8	102.2 17.8 58.8 89.4	592.2 27.4 525.8 574.5	964.0 32.1 879.5 941.0	1,292.2 35.0 1,228.8 1,266.4	1,588.4 38.7 1,520.7 1,554.6	1,879.1 46.4 1,792.7 1,840.6	2,157.7 54.8 2,048.9 2,112.6	2,317.5 56.0 2,195.8 2,267.7
Mean Std. dev Min Quartile 1 Median	-193.2 0.0 -193.2 -193.2 -193.2	-449.8 0.0 -449.8 -449.8	102.2 17.8 58.8 89.4 102.1	592.2 27.4 525.8 574.5 593.6	964.0 32.1 879.5 941.0 962.3	1,292.2 35.0 1,228.8 1,266.4 1,290.7	1,588.4 38.7 1,520.7 1,554.6 1,585.8	1,879.1 46.4 1,792.7 1,840.6 1,889.9	2,157.7 54.8 2,048.9 2,112.6 2,161.6	2,317.5 56.0 2,195.8 2,267.7 2,322.7
Mean Std. dev Min Quartile 1 Median Quartile 2	-193.2 0.0 -193.2 -193.2 -193.2 -193.2	-449.8 0.0 -449.8 -449.8 -449.8	102.2 17.8 58.8 89.4 102.1 116.8	592.2 27.4 525.8 574.5 593.6 608.6	964.0 32.1 879.5 941.0 962.3 984.1	1,292.2 35.0 1,228.8 1,266.4 1,290.7 1,320.6	1,588.4 38.7 1,520.7 1,554.6 1,585.8 1,618.2	1,879.1 46.4 1,792.7 1,840.6 1,889.9 1,917.1	2,157.7 54.8 2,048.9 2,112.6 2,161.6 2,202.5	2,317.5 56.0 2,195.8 2,267.7 2,322.7 2,363.0