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Linear Programming model for Long-term Mine Planning in the presence of Grade Uncertainty

Behrang Koushavand, Hooman Askari-Nasab and Clayton V. Deutsch

The optimality of an open pit production scheduling problem is affected dramatically by grade uncertainty. Recent research initiatives have attempted to consider the effect of grade uncertainty on production schedules. These methods either are aimed to minimize the risk using grade uncertainty or to maximize the net present value (NPV) without taking into account grade uncertainty explicitly. Another major problem in open pit production scheduling is the size of the optimization problem. The mathematical programming formulation of real size long-term open pit production schedules are beyond the capacity of current hardware and optimization software. In this paper a mathematical programming formulism is presented to find a sequence in which ore and waste blocks should be removed from a predefined open pit outline and their respective destinations, over the life of mine, so that the net present value of the operation is maximized and the deviations from the annual target ore production is minimized in the presence of grade uncertainty. Two main methods are presented (1) without a stockpile and (2) with a stockpile. The new parameters that are controlling the uncertainty part of the optimization are studied. At the end an oil sand deposit at north of Alberta is used to generate an optimum schedule.

Introduction

Mine planning is the process of finding a feasible block extraction schedule that maximizes net present value (NPV) and is one of the critical processes in mining engineering. Also, there are some technical, financial and environmental constraints that should be considered. Whittle (1989) defined open pit mine planning as "Specifying the sequence of blocks extraction from the mine to give the highest NPV, subject to variety of production, grade blending and pit slope constraints".

The uncertainty of ore grade may cause some shortfalls in the designed production and discrepancies between planning expectations and actual production (Koushavand and Askari-Nasab, 2009; Osanloo et al., 2008; Vallee, 2000). Therefore using only one block model would not be optimal. Different authors present methodologies to transfer grade uncertainty and show the impact of uncertainty at mine planning.

Dowd (1994) proposed a risk based algorithm in surface mine planning. Ravenscroft (1992) and Koushavand and Askari-Nasab (2009) used conditional simulated orebodies to show the impact of grade uncertainty on production scheduling. Dowd (1994) and Ravenscroft (Ravenscroft, 1992) used stochastic orebody models sequentially in traditional optimization methods. However the sequential process cannot produce an optimal schedule considering uncertainty.

Godoy and Dimitrakopoulos (2003) and Leite and Dimitrakopoulos (2007) present a new risk inclusive Long Term Production Plan(LTPP) approach based on simulated annealing. A multistage heuristic framework is presented to generate a final schedule, which considers geological uncertainty as to minimize the risk of deviations from production targets. A basic input to this framework is a set of equally probable scenarios of the orebody, generated by conditional simulation. They report significant improvement on NPV in the presence of uncertainty. Heuristic methods do not guarantee the optimality of the results. Also these techniques sometimes are very complex and there are many parameters needed to be chosen carefully to get reasonable results.

Dimitrakopoulos and Ramazan (2004) proposed a probabilistic method for long-term mine planning based on linear programming. This method uses probabilities of being above or below a cut-off to deal with uncertainty. The LP model is used to minimize the deviation from target production. This method does not directly and explicitly account for grade uncertainty and may not maximize the NPV.

Leite and Dimitrakopoulos (2007) present a technique that generates an optimal schedule for each realization. Afterwards, simulated annealing is used to generate a single schedule based on all schedules, such that the deviation from target production is minimized. For each conditional simulation, an optimum schedule is generated. Using simulated annealing, a single schedule is generated based on all schedules such that deviation from target production is minimized. The main drawback of this method is that it does not necessarily find the optimum solution.

Dimitrakopoulos and Ramazan (2008) present a linear integer programming (LIP) model to generate the optimal production schedule. Equally probable simulated block models were used as input. This model has a penalty function that is the cost of deviation from the target production and is calculated from geological risk discount rate (GDR) that is the discounted unit cost of deviation from a target production. They use linear programming to maximize a new function that is NPV minus penalty costs. With this method, it is not clear how to define the GDR parameter. For different GDR values there are different optimal solutions. In the presented model, mixed integer programming has been used. A variable is defined for each block. Adding constraints increases the complexity and CPU time to solve the optimization. This method is not tractable with a real case study. Also there is not stockpile defined at this method.

Background

Geological characteristics of each point (grade) are assigned using available estimation techniques. Kriging (Deutsch and Journel, 1998a; Goovaerts, 1997) is the most common estimation method used in industry; however, Kriging results do not capture uncertainty and may leading to conditional biased reserve estimates(Isaaks, 2005). Also, Mine plans that are generated based on one input block model fail to quantify the geological uncertainty and its impact on the future cash flows and production targets.

Geostatistical simulation algorithms are widely used to quantify and assess uncertainty. The generated realizations are equally probable and represent possible outcomes (Deutsch and Journel, 1998b; Goovaerts, 1997; Journel and Huijbregts, 1981). Choosing one of these realizations will not be objective to fair uncertainty assessment. Also, generated final pit limit and production schedule based on one block model would not necessarily be the optimum one. Therefore to get robust and optimum long-term production planning (LTPP), a sufficient number of realizations should be used simultaneously.

Mine planning is a process that defines a sequence of extraction of blocks with the objective of net present value (NPV) maximization. The mine production scheduling can be formulated as an optimization problem. NPV is the discounted revenue that discounted cost has been deducted from it:

$$\text{discountedprofit} = \text{discounted revenue} - \text{discounted costs} \tag{1}$$

Askari-Nasab and Awuah-Offei (2009) have presented the objective functions of the LP formulations that maximize the net present value of the mining operation. It is needed to define a clear concept of economic block value based on ore parcels which could be mined selectively. The profit from mining a block depends on the value of the block and the costs incurred in mining and processing. The cost of mining a block is a function of its spatial location, which characterizes how deep the block is located relative to the surface and how far it is relative to its final dump. The spatial factor can be applied as a mining cost adjustment factor for each block according to its location to the surface. The discounted profit from block is equal to the discounted revenue generated by selling the final product contained in block n minus all the discounted costs involved in extracting block, this is presented at Eq.(1) The discounted cost can rewrite as Eq. (2) and Eq. (3):

$$v_n^t = o_n \times \left(g_n \times r^t \times \mathrm{P}^t - cp^t \right) \tag{2}$$

$$q_n^t = \left(o_n + w_n \right) \times cm^t \tag{3}$$

Where n is the id number of block, v_n^t and q_n^t are discounted revenue and cost of extraction from block n at period t respectively. o_n and w_n are the tonnage of ore and waste for block n, cp^t and cm^t are cost of processing and mining at period t per ton respectively. r^t is processing recovery, P^t is the price of final product. If there is more than one valuable element in the final product, the revenue of the block will be added up for each element. Also if there are contaminants that are to be processed and eliminated from final product the cost of processing will be deducted from revenue of that block.

The objective function is to maximize the summation profit (Eq. (1)) of all blocks at all periods with two separate decision variables for each block at each period. First, q_n^t , is the portion of the block n to be extracted at period t and second, z_n^t, is the portion of block n to be processed (if it is ore) at period t. Therefore the mathematical form of the optimal mining schedule is presented in Eq. (4):

$$Max \quad \sum_{t=1}^{T}\sum_{n=1}^{N}\left(v_n^t \times z_n^t - q_n^t \times y_n^t\right) \tag{4}$$

Subject to:

$$gl^t \le \frac{\sum_{n=1}^{N} g_n \times o_n \times z_n^t}{\sum_{n=1}^{N} o_n \times z_n^t} \le gu^t \qquad \forall t = 1,2,\ldots,T \tag{5}$$

$$pl^t \le \sum_{n=1}^{N} o_n \times z_n^t \le pu^t \qquad \forall t = 1,2,\ldots,T \tag{6}$$

$$ml^t \le \sum_{n=1}^{N}\left(o_n + w_n\right)\times z_n^t \le mu^t \qquad \forall t = 1,2,\ldots,T \tag{7}$$

$$z_n^t \le y_n^t \qquad \forall t = 1,2,\ldots,T \ , \ n = 1,2,\ldots,N \tag{8}$$

$$a_n^t - \sum_{i=1}^{t} y_l^i \le 0 \qquad \forall t = 1,2,\ldots,T \ , \ n = 1,2,\ldots,N \ , \ l = 1,2,\ldots,C\left(L\right) \tag{9}$$

$$\sum_{i=1}^{t} y_n^i - a_n^t \le 0 \qquad \forall t = 1,2,\ldots,T \ , \ n = 1,2,\ldots,N \tag{10}$$

$$a_n^t - a_n^{t+1} \le 0 \qquad \forall t = 1,2,\ldots,T-1 \ , \ n = 1,2,\ldots,N \tag{11}$$

Where Eq. (5) is grade blending constraints; these inequalities ensure that the head grade is within the desired range in each period. g_n is the estimated grade of block n and , gl^t and gu^t are allowable lower limit and upper limit of input grade at period t. There will be separate constrains for each element of interest and any contaminants in each period. There are two equations (upper bound and lower bound) per element per scheduling period in Eq.(5). Eq. (6) is the processing capacity constraints where pl^t and pu^t are the lower limit and upper limit (target production) for the designed processing plan; these inequalities ensure that the total ore processed in each period is within the acceptable range of the processing plant capacity. There are two equations (upper bound and lower) per period per ore type. Eq. (7) is the mining constraints where ml^t and mu^t are lower and upper limit for mining limits; these inequalities ensure that the total tonnage of material mined (ore, waste, overburden, and undefined waste) in each period is within the acceptable range of mining equipment capacity in that period. There are two equations (upper bound and lower bound) per period. Eq. (8) represents inequalities that ensure the amount of ore of any block which is processed in any given period is less than or equal to the amount of rock extracted in the considered time period.

Eqs. (9) to (11) control the relationship of block extraction precedence by binary integer variables a_n^t which is equal to one if extraction of block n has started by or in period t, otherwise it is zero, i is the index for set of the blocks, C(L), that are needed to be extracted prior to extraction of block n. This model only requires the set of immediate predecessors' blocks on top of each block to model the order of block extraction relationship. This is presented by set C(L) in Eq. (9).

The amount of ore processed and amount of material mined are controlled by two separate continues variable rather than binary integer variables. In this model, there is T (number of periods) multiply by N (number of blocks) integer variables.

The estimated block model may be used to maximize NPV, therefore NPV_{es} is from Eq. (12):

$$NPV_{es} = \sum_{t=1}^{T}\sum_{n=1}^{N}\left(v_n^t \times z_n^t - q_n^t \times y_n^t\right) \tag{12}$$

In this model, the number of decision variables equals two times of the number of blocks multiplied by the number of periods. Therefore, it would be time consuming process to solve this linear programming. Boland et.al. (2009) and Askar-Nasab and Awuah-Offeri (2009) tried to solve this problem with clustering the blocks to reduce the number of variables. Using some grade aggregation methodology and based on lithological information, similar blocks are summarized to a group and are dealt as one variable which will be extracted in the same period. Each group of blocks is called a mining cut. Grouping the blocks into mining-cuts is done without sacrificing the accuracy of the estimated (or simulated) values and to model a more realistic equipment movement strategy. The mining-cut clustering algorithm developed uses fuzzy logic clustering (Kaufman and Rousseeuw, 1990). Coordinates of each mining-cut has been represented by the centre of the cut and its spatial location.

The proposed linear programming was formulated in MATLAB environment (MathWorks Inc., 2007). TOMLAB/CPLEX (Holmström, 1989-2009) was used as the Linear programming Solver. TOMLAB/CPLEX efficiently integrates the solver package CPLEX (ILOG Inc, 2007) with MATLAB.

MILP formulation based on Grade uncertainty without Stockpile

A Mixed Integer Linear Programming (MILP) model for optimizing long term production scheduling in open pit mines is developed with an objective function that maximizes the total NPV of the project under a managed grade risk profile. Grade uncertainty causes shortfalls from target productions. Therefore to get an optimum solution, NPV must be maximized and deviation from target production must be minimized simultaneously among all simulation realizations:

$$\begin{cases} \text{Max. NPV} \\ \text{Min. Deviation from target production} \end{cases}$$

Two main assumptions are made to model this optimization problem:

1. There is no stockpile to store any possible overproduced ore. Most of the time not having a stockpile is very unlikely in real life. However the assumption made here is a hypothetical case.
2. Long term scheduling is a dynamic process. This means that it changes during mine life. There are many situations that may happen at the operational level that management needs to change the extraction schedule such as misclassification of ore and waste, so called as information effect, failures at equipment, price changes at final production etc. Also every period the generated schedule is updated with new information such as blasthole data and new exploration drill holes. Therefore no long term schedule would be followed from the first year until the end of the mine life. However the goal here is to find a robust long term schedule using all useful information. In the case of violation from target production in operational level, the optimization should run again with new information to find the new optimum schedule.

The proposed method here is tries to postpone the uncertain blocks to later years when there will be new information and there is less uncertainty. The main idea is to minimize the risk of not meeting the target production, because the uncertainty may impose costs to the project. The cost of uncertainty is proposed at paper 301. The generated schedule using proposed method here have less risk at early year of production.

The objective function has two components. The same as Eq.(4), only one block model is used to maximize NPV which is most of the time estimated block model such as Kriging. The generated schedule is such that at all periods except the last one, the plant is fully fed by this block model and there is no deviation from target production. The second objective is applied to realizations. There is a probability that any schedule may not meet the target production because of grade uncertainty. These probabilities can be calculated using simulation values. The method presented here tries to minimize these probabilities at early years of mine production. There are two new variables op_l^t and up_l^t which are amount of over production and under production for realization l at period t. Each of these variable are multiple by discounted cost for over and under production called $c_{op}{}^t$ and $c_{up}{}^t$. These two parameters are chosen by user. They are the discounted penalty dollar values per ton for probable over and under production and they are discounted based on defined discount rate.

Also one should note that most of the time there is not enough ore to feed the plant at final year of mine life. Therefore as it is discussed at CCG annual meeting paper number 301, the cost of underproduction for final year is consider to be equal to zero: $c_{up}{}^T = 0$.

Therefore the mathematical form of the optimal mining schedule in the presence of grade uncertainty is presented at Eq.(13)

$$Max \sum_{t=1}^{T} \left\{ \underbrace{\sum_{n=1}^{N} \left(v_n^t \times z_n^t - q_n^t \times y_n^t \right)}_{first\ part:MAX\ NPV} - \underbrace{\frac{1}{L} \sum_{l=1}^{L} \left(c_{op}{}^t \times op_l^t + c_{up}{}^t \times up_l^t \right)}_{second\ part:MIN\ Risk.} \right\} \tag{13}$$

The first part of this model is maximizing the NPV using one block model usually estimate values, and the second part tries to minimize risk by deferring high uncertain blocks to the later years.

There are two more constraints for each period and each realization. These constrains control two new variables op_l^t and up_l^t. The other constraints of this model are the same as Eq. (5) to (11) which are defended for Eq. (4).

Two constraints are defined by Eq.(14) and (15):

$$\sum_{n=1}^{N} \left(o_{n,l} \times z_n^t - op_l^t \right) \le P_u^t \qquad \forall\, t = 1,2,\ldots,T \ ,\ l = 1,2,\ldots,L \tag{14}$$

$$\sum_{n=1}^{N} \left(-o_{n,l} \times z_n^t - up_l^t \right) \le -P_l^t \qquad \forall\, t = 1,2,\ldots,T \ ,\ l = 1,2,\ldots,L \tag{15}$$

Where $o_{n,l}$ is the tonnage of ore at block n in realization l. The number of decision variables are: $2 \times N \times T + 2 \times L \times T$. Numbers of binary variable are the same as Eq. (4). This model was implemented in MATLAB (MathWorks Inc., 2007) and solved by Cplex TOMLAB (Holmström, 1989-2009) library. Because the size of the problem is too large to handle with current hardware and software, fuzzy clustering technique (Askari-Nasab and Awuah-Offeri, 2009) was used to aggregate similar block to same group called mining cuts.

MILP formulation based on grade uncertainty with Stockpile

Not having a stockpile is a very severe assumption. Most of the time there is a stockpile to put over produced ore when there is enough material to feed the plant. These surplus ores are used when there is some problem with feeding the plant such as failure at extraction and hauling system or there is a grade blending problem with input material to the mill. Therefore any possible over produced ore would be processed at later years and the penalty value defined in Eq.(13) for over production would be less than in the presence of stockpile. This means that any plausible over production based on a realization will be kept in a stockpile and will be used in the next period of extraction.

The costs of over production are:

- The cost of re-handling materials from a stockpile
- The loss of discounted value of transferred ore to the next period

In the Eq.(13) the cost of over production for each period is deducted by cost of over production of next period which means that for each period there is only the loss of discounted value of ore that is transferred to the next period. Meanwhile any re-handling cost should be added to the cost of over production in each period. Therefore new optimization model for long term mine planning in presence of grade uncertainty and stockpile is presented at Eq. (16):

$$Max \sum_{t=1}^{T} \left\{ \underbrace{\sum_{n=1}^{N} \left(v_n^t \times z_n^t - q_n^t \times y_n^t \right)}_{first\ part: MAX\ NPV} - \underbrace{\frac{1}{L} \sum_{l=1}^{L} \left[\left(c_{op}{}^t - c_{op}{}^{t+1} \right) \times op_l^t + c_{up}{}^t \times up_l^t \right]}_{second\ part: MIN\ Risk.} \right\} \tag{16}$$

The only difference between Eq. (13) and (16) is in cost of over production. Having a stockpile reduces the cost of possible over production.

It is very unlikely that in the last period of mine life, one realization create over production, because most of the time there is not enough ore to feed the plant in the final year of mine life. But in a hypothetical case if there is any realization that generates over production in the final year, the ore will not be processed and the cost of over production is the same as the previous model and it means that there is no deduction in over production cost for the final year. This can be imposed to mathematical form as: $c_{op}{}^{T+1} = 0$. There are some modification to be made in the constraints the control the two variables op_l^t and up_l^t. Because any possible over produced ore are going to be used at next period, this should be considered by two constraints as shown in Eqs. (14) and (15). Modified versions of these two constrains are shown in Eqs. (17) and (18):

$$\sum_{n=1}^{N} \left[o_{n,l} \times z_n^t - \left(op_l^{t-1} + op_l^t \right) \right] \leq P_u^t \qquad \forall\, t = 1,2,\ldots,T \;,\; l = 1,2,\ldots,L \tag{17}$$

$$\sum_{n=1}^{N} \left[-o_{n,l} \times z_n^t - \left(op_l^{t-1} + up_l^t \right) \right] \leq -P_l^t \qquad \forall\, t = 1,2,\ldots,T \;,\; l = 1,2,\ldots,L \tag{18}$$

Note that there is no overproduction in period 0: $op_l^0 = 0$. The number of decision variables and binary variables are the same as in the previous model. Matlab and Tomlab are used to solve this optimization problem. Clustering method is used as before to reduce number of variables.

Discussion

In the early stages of production, the cost of uncertainty is higher than later years because of the having new information and less uncertainty at later years. It is a reasonable decision to postpone the extraction of very high uncertain block to later years which cause to reduce the probability of getting deviations from target production. Higher penalty values means that for early years of production lower uncertain blocks are preferred. The optimizer generates a schedule that maximizes NPV from the Kriging block model and minimizes the average penalty value calculated from L realizations. There is trade off at choosing high grade and low uncertain blocks at early years of production. On the other hand higher $c_{op}{}^t$ and $c_{up}{}^t$ cause less NPV generated from first part of the Eq. (13) and (16). Therefore it is very important to find the optimum values for these parameters. At this part two techniques are proposed.

a) Numerical method: in this method different $c_{op}{}^t$ and $c_{up}{}^t$ are used. Also both of over production and under production costs are considered to be equal and is called c . The optimization is run with different cost values. For each number NPV_{es}^c is calculated, which is the NPV coming from first part of Eq. (13) and (16). Cost of uncertainty is calculated (based on notation in paper 301) and the differences:

$$Delta = NPV_{es}^c - Unc.Cost \tag{19}$$

Figure 1 shows the expected results for this method. As it is shown at this graph, NPV_{es}^c decreases slowly with higher c factors, where Cost of uncertainty decreases rapidly. The optimum c value is chosen where the delta value exceeding to its maximum value.

b) The second method is based on Cost of uncertainty that is presented in paper 301. With equal $c_{op}{}^{t}$ and $c_{up}{}^{t}$ values, the second part of the Eq. (13) is changed to the cost of uncertainty that presented in paper 301.

$$\begin{aligned}&\frac{1}{L}\sum_{l=1}^{L}\left[c^{t}\times op_{l}^{t}+c^{t}\times up_{l}^{t}\right]=\frac{1}{L}\sum_{l=1}^{L}\left[c^{t}\times\left(op_{l}^{t}+up_{l}^{t}\right)\right]\\&=\frac{1}{L}\sum_{l=1}^{L}\left[c^{t}\times\left(op_{l}^{t}+up_{l}^{t}\right)\right]=\frac{1}{L}\sum_{l=1}^{L}\left[c^{t}\times\left|P_{l}^{t}-\text{Target}_{l}^{t}\right|\right]\end{aligned} \tag{20}$$

On the other hand the cost of uncertainty is:

$$CoU=\frac{1}{L}\sum_{l=1}^{L}\left[\left(\bar{g}^{t}\times P_{r}^{t}\times\text{Price}^{t}\text{ - }P_{c}^{t}\right)\times\left|P_{l}^{t}-\text{Target}_{l}^{t}\right|\right] \tag{21}$$

By comparing Eq. (20) and (21), $\bar{C}^{t}$ the average penalty cost for over and under production can be calculated from eq. (22):

$$\Rightarrow\bar{C}^{t}=\frac{c^{t}}{L}=\frac{\bar{g}^{t}\times P_{r}^{t}\times\text{Price}^{t}\text{ - }P_{c}^{t}}{L} \tag{22}$$

Where P^{t} is the input ore to the mill in period t, Target^{t} is the target production for period t, $\bar{g}^{t}$ is the average grade of input ore at period t, P_{r}^{t} is the processing recovery at period t, Price^{t} is the selling price of final product at each period, P_{c}^{t} is the processing cost and L is the number of realizations.

In the Eq.(13) there is no stockpile. Therefore any possible over produced ore is not processed and will be considered as waste. On the other hand modeling the stockpile is a very difficult problem, because it is needed to have one extra variable to control the average grade of stockpile and this extra variable is multiple by the decision variables that control the portion of extraction and processing and this causes the model to be a nonlinear optimization problem. One way to solve this problem is to have average grade of stockpile as an input. The Eq. (16) is used the average grade of stockpile as an input parameter. The penalty function, that is applied for both of Eqs. (13) and (16), is deferent at over production situation. Figure 2 shows the penalty values at different periods that are discounted for not having any stockpile at left and with stockpile at right. At each graph the vertical axis is the penalty dollar value per tonne of over and under production. Horizontal axis shows the under (left side) and over (right side) tonnage of material that are sent to the mill to be processed. As it is shown, the slop of the lines is reduced by time. It means that the penalty value at period one for over and under production is less than period 2. Also having stockpile case to reduce the penalty value for over production of ore. This cost is related to re-handling of material at stockpile and the revenue loss of ore materials that are transferred to latter years.

Case Study

The same oil sand deposit that is used in paper 301 is used in this section. GSLIB (Deutsch and Journel, 1998a) programs were used to generate ordinary Kriging block model and 50 conditional realizations. The Kriging block model was used in the first part of both models to maximize NPV. The realizations were used at second part to minimize deviation from target production. In this case study a 0.5$ per tonne penalty value was consider for both of over production and under production $c_{op}{}^{t}$ and $c_{up}{}^{t}$. Using average grade 9.5 mass percent bitumen in all periods, processing recovery of 95 percent, selling price of 2.8125$ per tonne, and processing cost of 0.5025 $ per tonne and Eq. (22) the c factor is calculated with 50 realizations as $\bar{C}^{t}=0.5$. The gap of 1% was used at Cplex optimizer. The final gaps for LP without stockpile and with stockpile respectively are 0.98% and 0.68%.

Figure 3 shows the schedules generated by two methods. In both methods, the optimizer did quite good too feed the plant over 7 years of production and there only is shortfall in the last period which is because of less ore remains. Figure 4 shows the cumulative cash flow over periods for Kriging, Etype and simulation realizations for both methods. Note that the generated schedule was followed for each of realizations. Average input grade to the mill in each period was presented in Figure 5. Having stockpile relaxes the optimizer to be able to extract and stock high grades ore at period three and this increases the average grade at period four. Figure 6 illustrates input tonnage to the mill with Kriging, Etype and realization block models for both methods. It is clear that having stockpile reduced the probability deviation from target production at early years of productions (period 3 and 4). This fact is clear in Figure 7, this figure shows the box plot of realizations and deviation from target production. Deviations from target production in period 3 and 4 the first method (without stockpile) are 2.5 and 5.28 percent. These values for method with stockpile are 0.39 and 3.32. This shows that a stockpile reduce the risk of not meeting the target production.

Table **1** summarizes the result of the two methods when each realization follows the two schedules. The expected values of NPV for first and second methods are 2319.18 and 2322.60. Table 2 shows the statistics of realizations for cumulative cash flow over the periods. The first two periods have a negative cash flow because of pre-stripping. The summery results of the three methods are shown in Table 3. First method is Kriging without uncertainty and stockpile. These results are form paper 301 CCG 2011 annual meeting. Second and third methods are the optimization of NPV with Kriging and minimization for deviation from target production using simulation realization with and without stockpile. The NPV_{es}^{c}, which is calculated based on Kriging block model, is maximum in the first method because there are fewer constraints and more flexibility for optimization: 2461, 2449.3 and 2453.8. On the other hand cost of uncertainty is higher in this method comparing to the next two methods: 178.9, 151.6 and 141.9. Also the stockpile reduced the cost of uncertainty. The Delta value which is defined in Eq. (19) is calculated in final column. It is clear that the Delta value for method with stockpile is higher than two other methods. Also using realizations creates a higher delta value than the schedule creates with only Kriging block model.

Conclusion

In this paper two methods were presented to generate long term production schedules using Linear programming technique. The Net present value is maximized based on the estimate block model, usually created by Kriging methods. The second objective is to minimize the deviation from target production or minimize the cost of uncertainty. In both methods, high uncertain block are going to be extracted in latter years when more information is provided by new drill holes. There is a trade off with extracting high grades and less uncertain blocks. This is controlled by two factors called cost of over and under production in each period. Having high values for each of these parameters means that less uncertain blocks are preferred in early years of production. Also high values for these parameters reduces the NPV that is calculated with Kriging. Two methods were presented to calculate the optimum values for these parameters.

There is not any over or under production for the Kriging block model. Probability of deviations from target production at each period is calculated using simulation realizations. The generated schedules are more robust because the probability of not meeting the target production is less in early years of production.

There are two types of variables in the proposed optimization models: binary variables that control the precedence of block extraction and decisions variables that are the portion of the block is going to be extracted and sent to the mill in each period. The number of variables for both of these methods is too much be handled by current commercial software for a real case study. For example there are 800,000 decision variables for a project with 20,000 blocks and 20 years of mine life. To solve this problem, clustering technique is used. Blocks in the same level with similar grades are aggregated into mining cuts and the number of variables is reduced. Recently there are some studies are going on to find better techniques and more complex criteria.

The future work for this study is to use pushbacks to reduce the size of problem. Also push backs can be used to define the large scale strategy of mine development and is used widely in industry. The methodology is to first use known techniques for push back design such as Lerch-Grossman algorithm

(1965) then find the block at each pushbacks and add constrains such that for example blocks at push back one to be extracted prior to the blocks at push back two.

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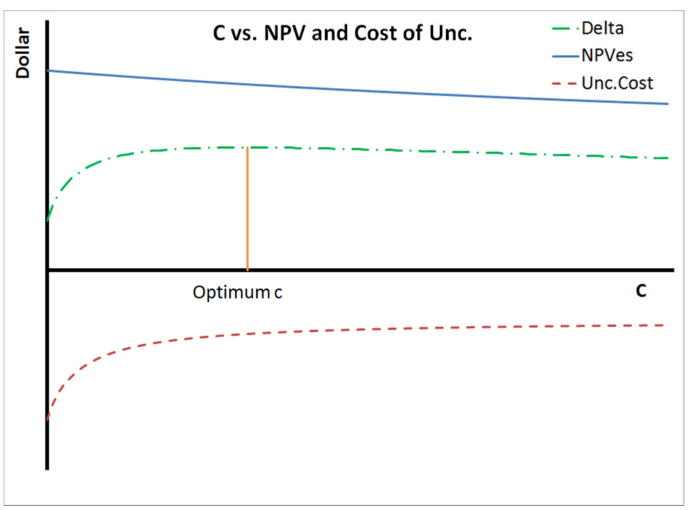

Figure 1. Expected graph for optimum C factor based on numerical method.

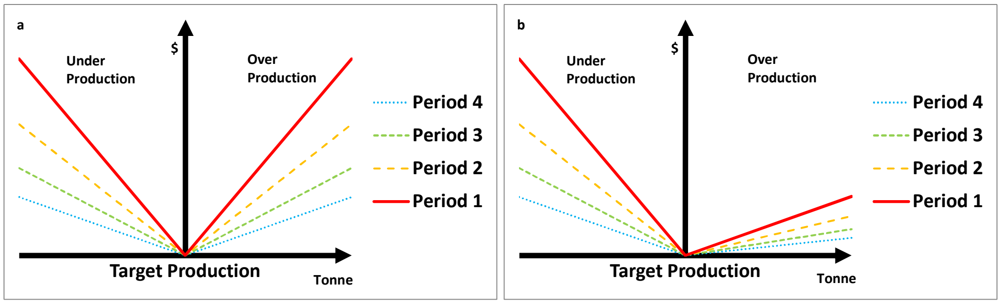

Figure 2. Penalty function for over and under production at different periods based on a discounting factor, a: no stockpile and b: with stockpile

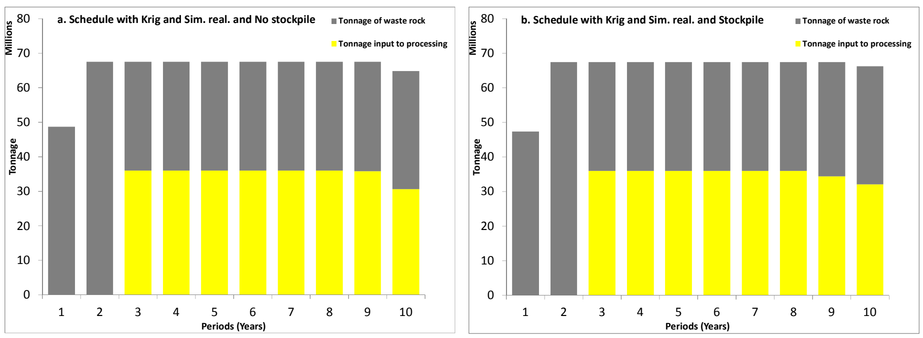

Figure 3. Schedules generated using krig model and simulation realizations a: without stockpile and b: with stockpile.

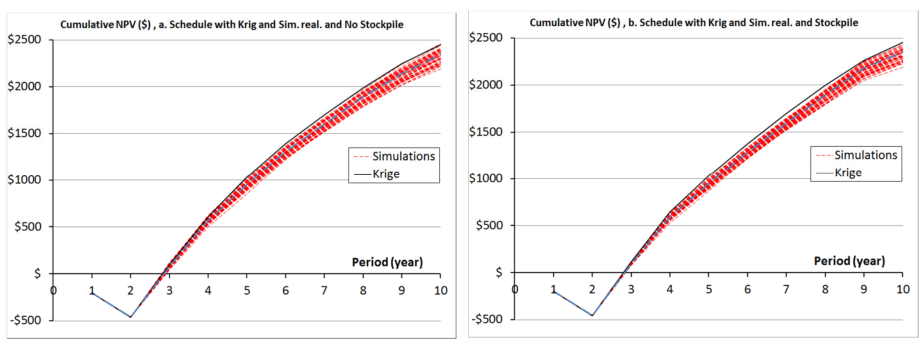

Figure 4. Comulative NPV over periods for kriging (back line), etype (dashed blue line) and simulations (dash red line), a: without stockpile and b: with stockpile.

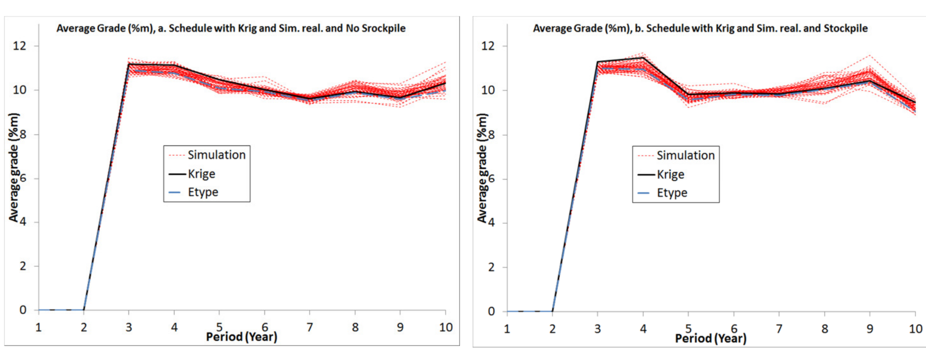

Figure 5. Input head grade to the plant over periods for kriging (back line), etype (dashed blue line) and simulations (dash red line), a: without stockpile and b: with stockpile.

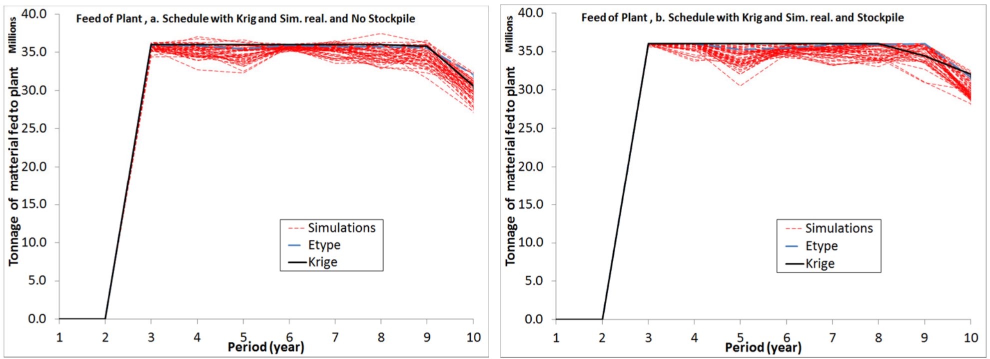

Figure 6. Feed of the plant over periods for kriging (back line), etype (dashed blue line) and simulations (dash red line), a: without stockpile and b: with stockpile.

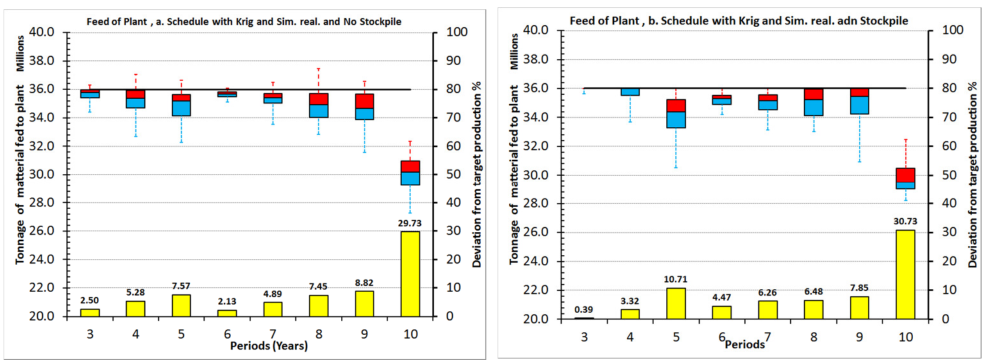

Figure 7. Boxplot and deviation from target production (yellow bars), calculated using simualtion values, a: without stockpile and b: with stockpile.

Table 1. Summary statistic of realizations when generated schedule with Kriging is followed, at above without stockpile and bottom with stockpile.

a: LP With Krig & Sim. Realizations Without Stockpile	**Ore Millions Tonnes**	**STRO**	**Input Bitumen Millions Tonnes**	**Average %**	**NPV Millions Dollars**
Mean	276.14	1.37	28.27	10.24	2319.18
Std. dev	3.60	0.03	0.44	0.09	60.74
Min	269.28	1.29	27.28	10.02	2189.42
Quartile 1	273.31	1.34	27.89	10.19	2269.00
Median	276.50	1.36	28.25	10.24	2316.67
Quartile 2	278.83	1.39	28.61	10.29	2367.12
Max	284.06	1.43	29.12	10.52	2428.65
Krig	282.44	1.31	29.11	10.31	2449.44
Etype	282.22	1.32	28.49	10.10	2346.25

b: LP With Krig & Sim. Realizations With Stockpile	**Ore Millions Tonnes**	**STRO**	**Input Bitumen Millions Tonnes**	**Average %**	**NPV Millions Dollars**

Mean	275.92	1.37	28.24	10.24	2322.60
Std. dev	3.61	0.03	0.44	0.09	59.42
Min	269.29	1.29	27.22	10.02	2191.25
Quartile 1	272.97	1.35	27.84	10.19	2269.99
Median	276.37	1.36	28.23	10.24	2318.70
Quartile 2	278.39	1.39	28.56	10.29	2372.09
Max	284.47	1.43	29.10	10.52	2430.31
Krig	282.44	1.31	29.11	10.31	2453.85
Etype	282.07	1.32	28.48	10.10	2349.99

Table 2. Summery statistics of cumulative cash flow at each period, at above without stockpile and bottom with stockpile.

Period	1	2	3	4	5	6	7	8	9	10
Mean	-203.8	-460.4	77.7	555.5	943.5	1,294.5	1,590.8	1,875.0	2,119.8	2,319.2
Std. dev	0.0	0.0	16.9	26.1	35.2	37.4	41.0	47.7	53.7	60.7
Min	-203.8	-460.4	42.8	501.9	851.8	1,219.6	1,516.5	1,784.7	2,017.3	2,189.4
Quartile 1	-203.8	-460.4	66.3	537.5	918.5	1,273.6	1,560.1	1,837.7	2,075.1	2,269.0
Median	-203.8	-460.4	76.6	549.7	943.9	1,289.8	1,588.3	1,883.2	2,125.4	2,316.7
Quartile 2	-203.8	-460.4	90.9	578.2	971.6	1,328.1	1,627.5	1,911.1	2,163.3	2,367.1
Max	-203.8	-460.4	120.5	620.4	1,015.7	1,360.7	1,669.2	1,954.8	2,202.1	2,428.6
Krig	-203.8	-460.4	101.6	609.3	1,032.0	1,391.6	1,698.3	1,991.2	2,244.6	2,449.4
Etype	-203.8	-460.4	80.0	561.5	951.4	1,304.4	1,602.7	1,890.4	2,138.0	2,346.3

Period	1	2	3	4	5	6	7	8	9	10
Mean	-198.1	-454.8	93.2	590.7	939.0	1,277.4	1,585.1	1,878.4	2,162.2	2,322.6
Std. dev	0.0	0.0	9.2	25.6	31.3	32.9	37.7	46.2	54.1	59.4
Min	-198.1	-454.8	76.9	525.7	870.9	1,214.9	1,520.8	1,792.2	2,049.0	2,191.2
Quartile 1	-198.1	-454.8	86.9	573.0	916.6	1,254.4	1,557.1	1,841.2	2,119.9	2,270.0
Median	-198.1	-454.8	93.3	594.0	939.9	1,272.8	1,579.4	1,886.8	2,165.2	2,318.7
Quartile 2	-198.1	-454.8	100.6	607.2	962.5	1,305.2	1,616.9	1,915.1	2,202.6	2,372.1
Max	-198.1	-454.8	114.9	649.2	1,008.3	1,337.5	1,654.8	1,957.1	2,255.8	2,430.3
Krig	-198.1	-454.8	115.1	644.7	1,028.1	1,379.8	1,697.3	1,997.4	2,264.9	2,453.8
Etype	-198.1	-454.8	94.6	591.0	946.4	1,285.8	1,598.3	1,895.0	2,179.9	2,350.0

Table 3. Summary of NPV and Cost of uncertainty at different methods.

Method	NPV_{es}^{c}	Cost of Unc.	elta
Kriging Without Unc. No Stockpile	2461.0	178.9	2282.1
Kriging With Sim. No Stockpile	2449.3	151.6	2297.7
Kriging With Sim. and Stockpile	2453.8	141.9	2311.9