Short Note on the Application of the Coefficient of Variance in Mine Planning

Behrang Koushavand, Hooman Askari Nasab and Clayton V. Deutsch

The goal of mine planning is to determine which block should be extracted at which period to maximize the Net Present Value(NPV) of the project. Askari-Nasab and Awuah-Offei (2009) have presented the mathematical programming formulization to maximize NPV. The advantage of this model was less integer variables and much faster converge time. The disadvantage of this model was they did not consider uncertainty on the grade. There for only one block model can be used at their model.

Geostatistical methods are widely used to model geological uncertainty. Kriging (Deutsch and Journel, 1998a; Goovaerts, 1997) is the most common estimation method used in industry. Geostatistical simulation algorithms are widely used to quantify and assess uncertainty. The generated realizations are equally probable and represent plausible geological outcomes (Deutsch and Journel, 1998b; Goovaerts, 1997; Journel and Huijbregts, 1981).

On the other hand Recently some authors are tried to used uncertain models at mine planning(Dimitrakopoulos, 1998; Dimitrakopoulos et al., 2001; Dimitrakopoulos and Ramazan, 2004, 2008; Godoy and Dimitrakopoulos, 2003; Leite and Dimitrakopoulos, 2007; Ramazan and Dimitrakopoulos, 2004). Paper 301 and 307 at CCG 2010 also provide some new techniques to use conditional simulation realizations at mine planning. The main problems with these methods are either the presented linear programming methods are very slow or they are using heuristic method to solve the optimization where optimality of the solution is not guaranteed. Boland et.al.(2009) and Askar-Nasab and Awuah-Offeri (Askari-Nasab and Awuah-Offeri) tried to solve this problem with clustering the blocks to reduce the number of variables. Using some grade aggregation methodology and based on lithological information, similar blocks are summarized to a group and are dealt as one variable which will be extracted at the same period. At this paper a new technique is presented that is much faster than new methods that are using simulation realization at mine planning.

Methodology

At this model instead of minimizing deviation from target production, the variance of production is going to be minimized. The model has two parts: maximizing NPV and minimizing the variance of the production.

$$\begin{cases} \text{Max. NPV} \\ \text{Min. Variance of production} \end{cases}$$

The variance of production can be calculate as eq.(1) :

$$\text{Variance of production} = \operatorname{var}\left(g_n \times o_n \times r \times z_n^t\right) = o_n \times z_n^t \times r \times \operatorname{var}\left(g_n\right) \tag{1}$$

$\operatorname{var}\left(g_n\right)$ is the variance of grade at block n and is a numerical measure of uncertainty. The dimension of NPV is dollar value; therefore it is needed to make variance of production to have the same dimension of NPV. Therefore a dimensionless measure of uncertainty is required; the "coefficient of variance" is used at this model.

A new term called "cost of variance" is defined as eq.(2):

$$\text{Cost of Variance} = Cv_n^t = o_n \times r \times \frac{\sigma_n}{\bar{g}_n} \times Cu^t \tag{2}$$

Where o_n is the tonnage of ore at block n, r is the processing recovery factor, $\frac{\sigma_n}{\bar{g}_n}$ is the coefficient of variance and is defined as the ratio of the standard deviation of block n to the mean of that block and it is a dimension less value. Either Kriging value or Etype mean can be used as mean of the block. Etype variance can be used as for variance of the block n. Ordinary Kriging (OK) variance is not the true variance and one should know that Kriging variance is not represent the variance of the blocks at original unit. The other method to calculate variance of the block is using simple Kriging at Gaussian framework. A quantile to quantile back transformation is performed to transfer sufficient number of values from Gaussian unit to original units. The Program called `PostMG` is used for this purpose. The correct distribution of local uncertainty can be get and variance of the block can be calculating form. Cu^t is the discounted pseudo-cost of variance and is not a real cost. The discount factor may be different from original discount factor that is used to calculate revenue. One may penalize the uncertainty more at early years by choosing high discount rate for Cu^t. The discounting rate for this cost may be different from original discounted rate. The new model is defined as eq.(3):

$$Max \quad \sum_{t=1}^{T}\sum_{n=1}^{N}\left[\left(v_n^t - Cv_n^t\right)\times z_n^t - q_n^t \times y_n^t\right] \tag{3}$$

All the parameter at this model is the same as Eq (4) at paper 307 CCG 2011. This model is superior to model (13) at paper 307 because there is no extra variable and the speed of algorithm is exactly the same as traditional mine planning model (4) at paper 307.

Case Study

The same case study as paper 307 is used at this paper too and $Cu^0 = 0.5\ \$/Tonne$. Figure 1 shows the schedule created with this method. Figure 2 shows the effect of grade uncertainty on the feed of the plant and this is because of at this case there is no constraint on the lower limit of the production. Therefore the optimizer tried to minimize the variance of input ore to the plant with less input ore at early yrar. As is can be seen from Figure 2, there is very small deviation from kriging (black line) at first year of production (period 3). But the input ore is less at first period. To solve this problem, a lower limit of the production target is set to the target limit which is 36MT per year. Figure 3 shows the schedule with new constraints on lower limit of production schedule. As the same procedure at paper 301, any probable over produced ore has been cleaned. From Figure 4 to Figure 7 there are two graphs for each figure. At left the surplus ore are not removed and it is the row version. And at right the post processed version where the over probable produced ore is removed from simulation realizations. Table 1 shows some summery statistics of input ore, strip ratio, input bitumen, average grade and NPV. The NPV of kriging is higher than any other realizations because the kriging block model is used at optimization. Also the average mean of NPV calculated from realizations is higher when surplus ore is not removed (M$2290.97 vs. M$2287.18). Table 2 shows the statistics of cumulative cash flow over the periods.

Conclusion

At this paper A new method is presented based on coefficient variance to minimize the variance of production at generated schedule. It is an optimization method that maximizes NPV using input block model and minimize the variance of the input ore by penalizing the Coefficient of variance. The number of variables is exactly the same as traditional method which done not consider grade uncertainty. Therefore the speed of algorithm is much more than new proposed method based on grade uncertainty. The short come of this method is to set lower limit of production. The optimizer tries to minimize the variance and this cause to shortfall at production at early years. The other changing question is to find the correct value of Cu^t. There is a proposed method at paper 307 CCG 2012. The method is running the optimization with different Cost parameter to find the optimum value.

References

Askari-Nasab, H., Awuah-Offeri, K., 2009, Mixed integer programming formulations for open pit production scheduling. MOL Report one 1, 1-31.

Boland, N., Dumitrescu, I., Froyland, G., Gleixner, A.M., 2009, LP-based disaggregation approaches to solving the open pit mining production scheduling problem with block processing selectivity. Comput.Oper.Res. 36, 1064-1089.

Deutsch, C.V., Journel, A.G. 1998a. GSLIB : geostatistical software library and user's guide. In Applied geostatistics series (New York, Oxford University Press), p. 369.

Deutsch, C.V., Journel, A.G., 1998b, GSLIB : geostatistical software library and user's guide, 2nd Edition. Oxford University Press, New York, 369 p.

Dimitrakopoulos, R., 1998, Conditional simulation algorithms for modelling orebody uncertainty in open pit optimisation. International Journal of Mining, Reclamation and Environment 12, 173-179.
Dimitrakopoulos, R., Farrelly, C.T., Godoy, M., 2001, Moving forward from traditional optimization: Grade uncertainty and risk effects in open-pit design. Transactions of the Institution of Mining and Metallurgy, Section A: Mining Industry 111, 82-88.
Dimitrakopoulos, R., Ramazan, S., 2004, Uncertainty based production scheduling in open pit mining. 106-112.
Dimitrakopoulos, R., Ramazan, S., 2008, Stochastic integer programming for optimising long term production schedules of open pit mines: methods, application and value of stochastic solutions. Mining Technology : IMM Transactions section A 117, 155-160.
Godoy, M., Dimitrakopoulos, R., 2003, Managing risk and waste mining in long-term production scheduling of open pit mine. SME Annual Meeting & Exhibition 316, 43-50.
Goovaerts, P., 1997, Geostatistics for natural resources evaluation. Oxford University Press, New York, 483 p.
Journel, A.G., Huijbregts, C.J., 1981, Mining geostatistics. Academic Press, London, 600 p.
Leite, A., Dimitrakopoulos, R., 2007, Stochastic optimisation model for open pit mine planning: Application and risk analysis at copper deposit. Transactions of the Institutions of Mining and Metallurgy, Section A: Mining Technology 116, 109-118.
Ramazan, S., Dimitrakopoulos, R., 2004, Traditional and New MIP Models for Production Scheduling With In-Situ Grade Variability. 18, 85-98.

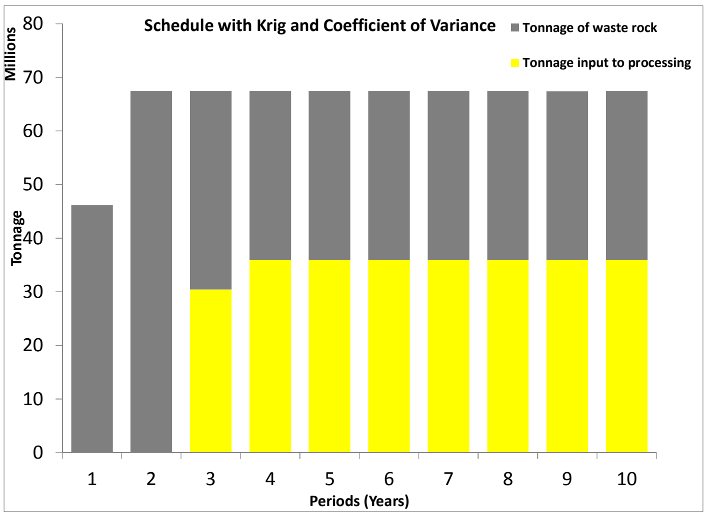

Figure 1. Schedules generated using krig model and simulation realizations

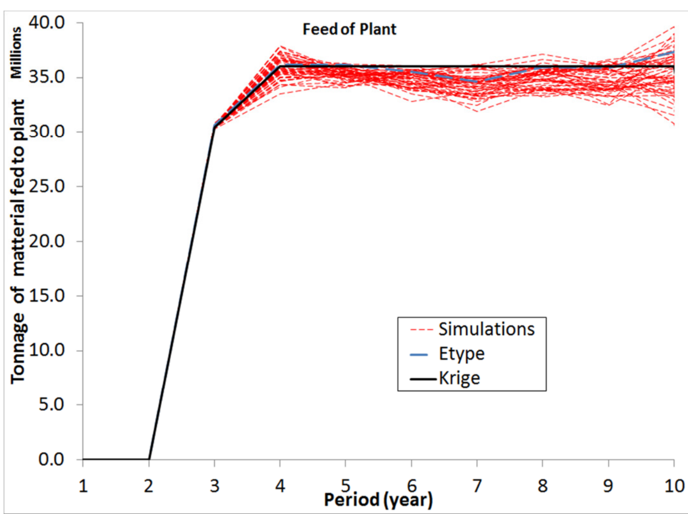

Figure 2. Feed of the plant over periods for kriging (back line), etype (dashed blue line) and simulations (dash red line)

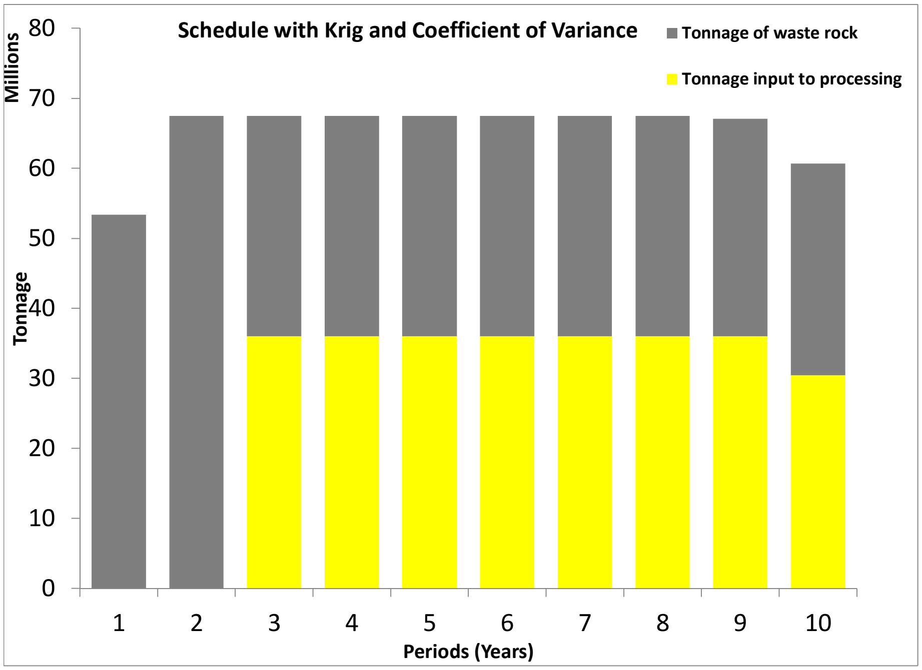

Figure 3. Schedules generated using krig model and simulation realizations with constraints on lower limit of production set to 36MT

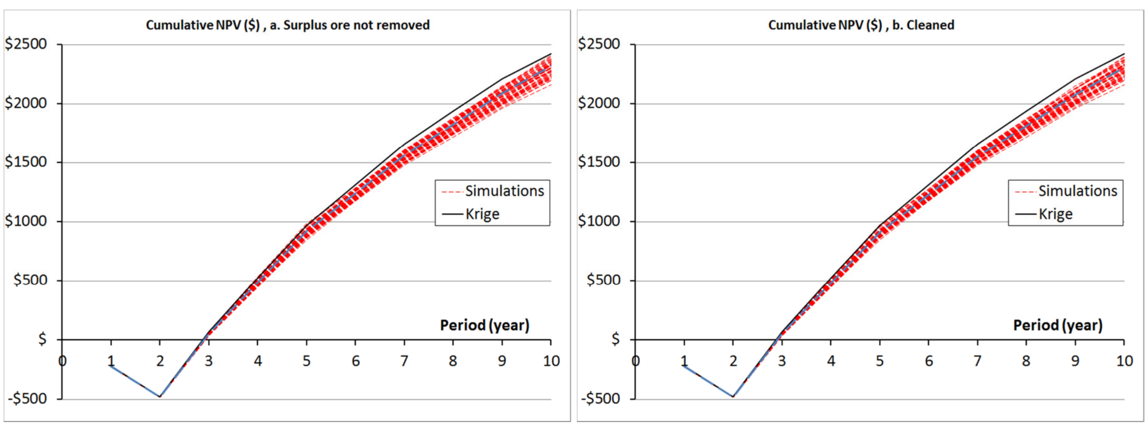

Figure 4. Cumulative NPV over periods for kriging (back line), etype (dashed blue line) and simulations (dash red line), surplus ore not removed at left and cleaned version at right

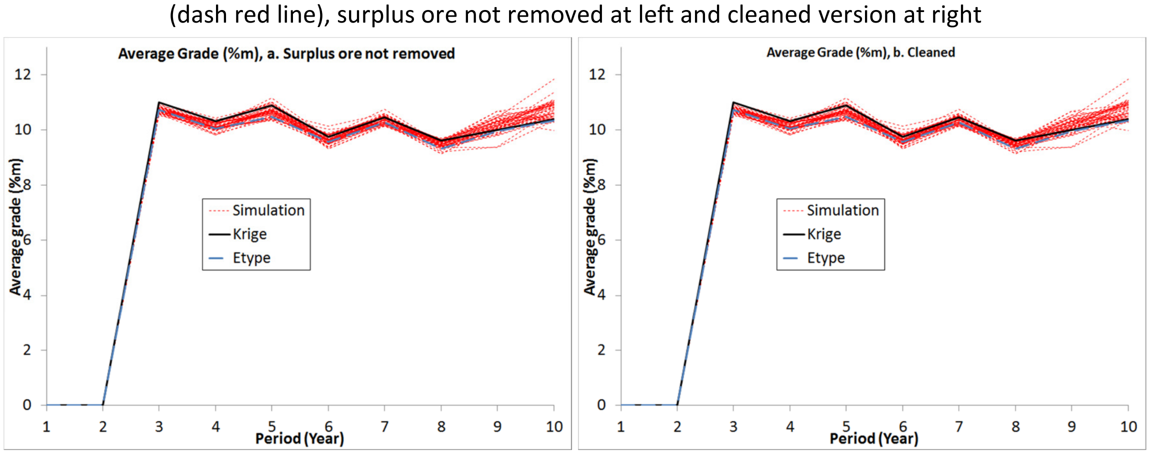

Figure 5. Input head grade to the plant over periods for kriging (back line), etype (dashed blue line) and simulations (dash red line), surplus ore not removed at left and cleaned version at right

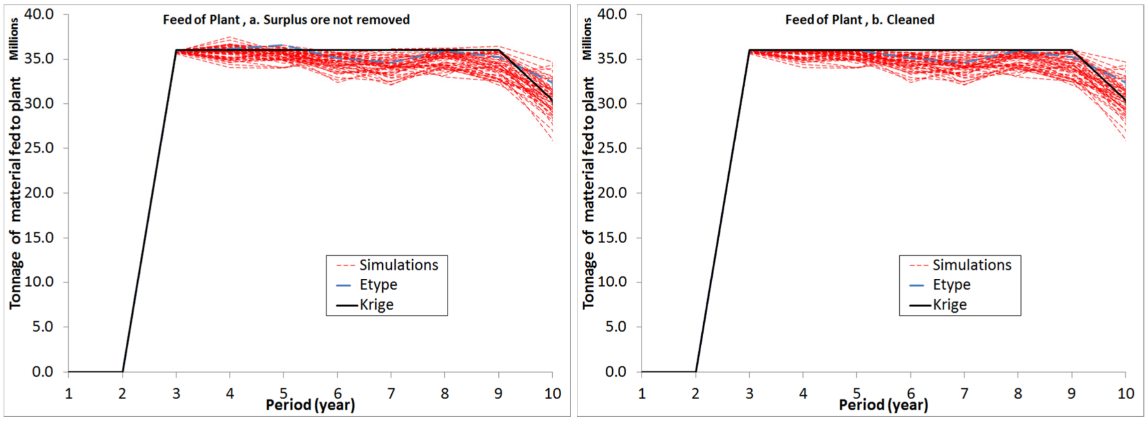

Figure 6. Feed of the plant over periods for kriging (back line), etype (dashed blue line) and simulations (dash red line), surplus ore not removed at left and cleaned version at right

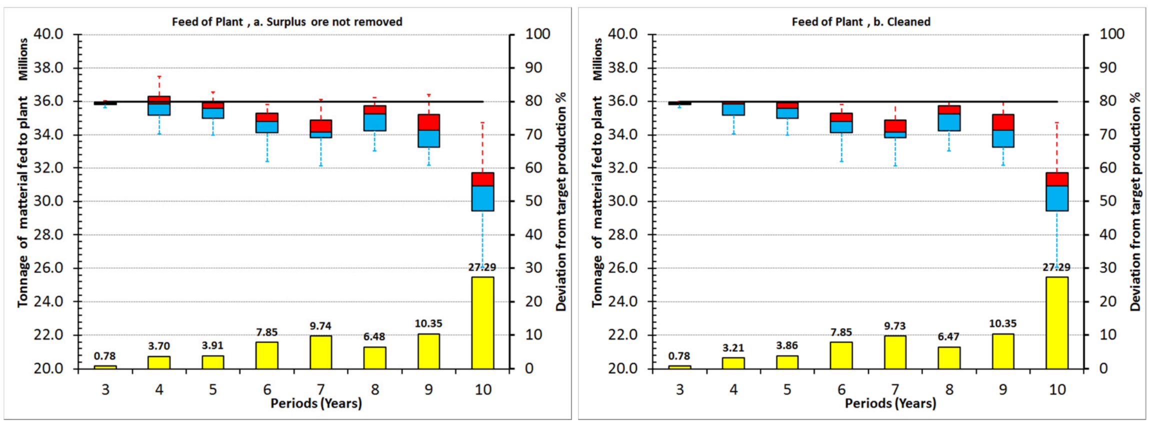

Figure 7. Boxplot and deviation from target production (yellow bars), calculated using simualtion values, surplus ore not removed at left and cleaned version at right

Table 1. Summary statistic of realizations when generated schedule with Kriging is followed, at above without stockpile and bottom with stockpile

a: LP With Krig & Sim. Realizations Without Stockpile	Ore Millions Tonnes	STRO	Input Bitumen Millions Tonnes	Average %	NPV Millions Dollars
Mean	276.03	1.37	28.27	10.24	2290.97
Std. dev	3.85	0.03	0.46	0.09	59.06
Min	268.97	1.28	27.26	10.03	2159.37
Quartile 1	273.14	1.35	27.87	10.20	2244.66
Median	276.14	1.37	28.25	10.25	2290.71
Quartile 2	278.38	1.39	28.63	10.29	2332.71
Max	286.83	1.43	29.36	10.52	2409.59
Krig	282.44	1.31	29.11	10.31	2421.27
Etype	282.03	1.32	28.48	10.10	2317.94

b: LP With Krig & Sim. Realizations With Stockpile	Ore Millions Tonnes	STRO	Input Bitumen Millions Tonnes	Average %	NPV Millions Dollars
Mean	275.77	1.37	28.24	10.24	2287.18
Std. dev	3.65	0.03	0.45	0.09	56.87
Min	268.97	1.28	27.26	10.03	2159.36
Quartile 1	273.13	1.35	27.85	10.20	2235.65
Median	276.05	1.37	28.23	10.25	2288.95
Quartile 2	278.03	1.39	28.57	10.29	2328.98
Max	285.49	1.43	29.23	10.52	2395.43
Krig	282.44	1.31	29.11	10.31	2421.27
Etype	281.32	1.32	28.41	10.10	2307.70

Table 2. Summery statistics of cumulative cash flow at each period, at above without stockpile and bottom with stockpile

Period	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Mean	-223.09	-479.70	47.00	484.13	908.59	1,228.27	1,547.82	1,807.94	2,064.99	2,290.97
Std. dev	0.00	0.00	7.86	19.46	30.87	31.38	37.10	42.20	50.28	59.06
Min	-223.09	-479.70	32.97	447.68	844.56	1,174.60	1,476.27	1,721.02	1,959.43	2,159.37
Quartile 1	-223.09	-479.70	41.65	469.02	884.18	1,202.83	1,521.84	1,770.04	2,023.89	2,244.66
Median	-223.09	-479.70	45.55	482.41	910.26	1,225.01	1,542.47	1,807.17	2,070.61	2,290.71
Quartile 2	-223.09	-479.70	52.90	498.75	929.12	1,259.19	1,583.94	1,848.07	2,100.73	2,332.71
Max	-223.09	-479.70	62.98	536.97	980.64	1,287.32	1,607.66	1,882.96	2,152.65	2,409.59
Krig	-223.09	-479.70	68.18	521.89	967.83	1,312.67	1,660.90	1,939.09	2,208.48	2,421.27
Etype	-223.09	-479.70	47.84	486.54	919.12	1,240.98	1,561.53	1,826.20	2,086.05	2,317.94

Period	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Mean	-223.1	-479.7	47.0	481.3	905.1	1,224.8	1,544.3	1,804.2	2,061.2	2,287.2
Std. dev	0.0	0.0	7.9	17.4	28.7	28.9	34.9	39.8	48.0	56.9
Min	-223.1	-479.7	33.0	447.7	844.5	1,174.6	1,476.3	1,721.0	1,959.4	2,159.4
Quartile 1	-223.1	-479.7	41.6	467.7	882.9	1,201.0	1,520.9	1,768.9	2,021.9	2,235.6
Median	-223.1	-479.7	45.5	479.6	908.1	1,224.2	1,540.1	1,804.3	2,070.6	2,288.9
Quartile 2	-223.1	-479.7	52.9	494.9	924.0	1,252.5	1,575.1	1,838.2	2,096.3	2,329.0
Max	-223.1	-479.7	63.0	526.2	969.9	1,276.6	1,602.4	1,876.0	2,150.6	2,395.4
Krig	-223.1	-479.7	68.2	521.9	967.9	1,312.7	1,660.9	1,939.1	2,208.5	2,421.3
Etype	-223.1	-479.7	47.9	484.5	908.9	1,230.8	1,551.3	1,816.0	2,075.8	2,307.7