Using the Perpendicular Distance to the Nearest Fracture as a Proxy for Conventional Fracture Spacing Measures

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Discrete fracture network simulation aims to reproduce distributions of fracture spacing, deviation from local fracture orientation, fracture intensity, the number of fracture intersections and fracture length. Ideally, this simulation would compare target histograms of true fracture spacing as measured by conventional means to histograms of fracture spacing in the models; however, for computational reasons, different measures may be preferred. The perpendicular distance between nearest neighbour fractures is one proxy for fracture spacing. This study uses simulation studies to explore the validity of using the perpendicular distance as a proxy for fracture spacing.

Introduction

Previous work has resulted in the creation of DFNSIM (Niven & Deutsch 2010a; Niven & Deutsch 2010b), which is a program for discrete fracture network (DFN) simulation. DFNSIM is able to generate DFNs that honour histograms of fracture spacing and relative orientation of fractures that are generated from outcrop or borehole data.

DFNSIM generates an optimized DFN by adding and removing fractures from the DFN at random and observing the effect of the change on an objective function. Among other factors, the objective function compares a histogram of a measure of fracture spacing for the DFN to a target histogram based on observed data.

Fracture spacing is the orthogonal distance between fracture planes and can be measured as the distance between fractures along a scan line, or down a borehole (Makel 2007; Twiss & Moores 1992). For the purposes of this article, fracture spacing measured in this way is referred to as the *true fracture spacing.* An early version of DFNSIM calculated relative histograms of true fracture spacing using the previously described scan line method. The methodology was as follows (See Figure 1):

1. Visit a random location within the modelling area of interest.
2. Propagate an imaginary scan line through the DFN at an orientation that is perpendicular to the average fracture orientation.
3. Measure the spacing between intersections of the fractures and the imaginary scan line.
4. Repeat *N* times (say *N=1000*), visiting a new location each time.

In this way, DFNSIM would calculate true fracture spacing similar to how a geologist might. However, experience has shown that optimizing a DFN requires anywhere from 10^3 to 10^7 iterations depending, on the required fracture intensity. Recalculating the histogram of fracture spacing (which requires hundreds or potentially thousands of imaginary sampling lines) each time the DFN is permuted becomes computationally expensive. Thus, an alternative measure of fracture spacing (or proxy) was required.

The methodology for DFN simulation, encoded in DFNSIM, proposes that the *average perpendicular distance between a fracture and its nearest neighbour* is used as an alternative measure of fracture spacing. Figure 2 shows the calculation of the average perpendicular distance between a fracture and its nearest neighbour. Since fracture spacing is normally measured perpendicular to the fractures, the distance between nearest neighbours is measured normal to the fracture planes. Note, that the perpendicular distance between a fracture and its nearest neighbour still meets the definition of fracture spacing from the literature (Makel 2007; Twiss & Moores 1992).The perpendicular distances (d_{p1} and d_{p2}) between each fracture's centroid and the other fracture are calculated. Note that $d_{p1} \neq d_{p2}$. Thus, the average of d_{p1} and d_{p2} is taken to be the perpendicular distance to the nearest fracture. If the imaginary ray from one centroid does not intersect the fracture, the intersection is taken where it would have been if the fracture was infinite in extents.

This purpose of this study is to establish the validity of using the average perpendicular distance to the nearest fracture as a proxy for the true fracture spacing measured using scan lines.

Perpendicular Distance as a Proxy for Fracture Spacing

A DFN with 1000 fractures was simulated in an area of interest that is 1000 x 1000 m. All fractures have the same orientation and are the same size (30 m in length). A relative histogram of true fracture spacing was calculated and is shown in Figure 3. The mean fracture spacing is 32 m.

For the same fracture network, the average perpendicular distance to the nearest fracture can be calculated as described in the previous section. An additional challenge is that the average perpendicular distance to the nearest fracture depends upon the bandwidth that is chosen (for more information on the bandwidth, please see (Niven & Deutsch 2010b)). There is an inverse relationship between perpendicular distance to the nearest neighbour and bandwidth. As the bandwidth increases, the average perpendicular distance decreases. The relationship between bandwidth and mean perpendicular distance to the nearest fracture is shown in Figure 4. The histogram of average perpendicular distance for the DFN with 1000 fractures, calculated using a bandwidth of 30 m, is shown in Figure 5. The mean perpendicular distance to the nearest fracture is 8.1 m.

One hundred DFN realizations were simulated by using the same settings and varying only the random number seed. For each realization the average true fracture spacing and average perpendicular distance to the nearest neighbour were calculated. Figure 6 shows a scatterplot of the average fracture spacing versus the average perpendicular distance for each of the 100 DFN realizations. The scatterplot shows zero correlation between the average true fracture spacing and the average perpendicular distance per realization. The effect of increasing and decreasing bandwidth was investigated but was not found to make a difference to the results.

Fracture Spacing Histogram Reproduction

Figure 7 shows a map of lineaments from an area of Northern Alberta (Pana et al. 2001). The figure shows two fracture sets, one trending southwest-northeast and the other trending southeast-northwest. The southwest-northeast fracture set was digitized for this example. In total, there are 425 fractures from the SW-NE set that were digitized. The right side of Figure 7 shows one of the fracture sets digitized.

The relative histogram of true fracture spacing was calculated using 1000 imaginary scan lines and is shown in Figure 8 (shown with blue diamonds). Meanwhile, Figure 9 shows the histogram of perpendicular distance to the nearest fracture for the digitized lineaments (also shown with blue diamonds).

Next, the digitized lineaments are taken as the truth and the lineaments were modelled using DFNSIM. Target histograms of perpendicular distance to the nearest fracture are built from the digitized lineaments.

The target, initial DFN and final optimized DFN histograms of perpendicular distance to the nearest fracture are shown in Figure 9. Note the excellent fit between the target and final optimized DFN histograms of perpendicular distance to the nearest fracture.

The true fracture spacing histograms for the initial and final DFNs are also calculated, which are shown in Figure 8 along with the target histograms. The true spacing of the final DFN is only a slight improvement over the initial "random" DFN. An attempt was made to calculate optimized DFNs using different bandwidths to calculate the perpendicular distances in hopes of improving the histogram match with no success.

Reducing the Problem to One Dimension

Figure 10 shows a depiction of fractures perpendicular to a one-dimensional line at coordinate locations x_i ($i = 1,...,n$). The intersections between the fractures and the line, which could be seen as a scan line, defines the true fracture spacings:

$$s_i = x_i - x_{i-1}, \quad i = 2,...,n \tag{1}$$

The perpendicular distances between fractures are also calculated as the minimum of: 1) the distance to the previous fracture, or 2) the distance to the next fracture. Or, mathematically:

$$p_i = \min(\, x_i - x_{i-1}\,, \quad x_{i+1} - x_i\,) \tag{2}$$

Note that the perpendicular distance p_i is always less than or equal to s_i. 10,000 fractures were simulated randomly along the one-dimensional line, and then were sorted by x-coordinate location. The sorted fracture centroid locations allow calculation of spacing between them as in Equation (1). The perpendicular distances between each fracture and its nearest neighbour are also calculated as in Equation (2).

Next, the average true spacing, along with the average perpendicular distance is calculated for this realization of fractures. The process is repeated, until 20 realizations of fractures are generated, calculating the mean true spacing and mean perpendicular distance to the nearest neighbour for each realization. Figure 11 shows the relationship between the mean values of true spacing and perpendicular distance for the 20 realizations. There is zero correlation between the average true fracture spacing per realization and the average perpendicular distance to the nearest neighbour.

Individual distances and spacings, rather than the average spacing and perpendicular distances, are examined next. Figure 12 shows the relationship between individual values of perpendicular distance to the nearest fracture and true spacing between fractures for one of the 20 realizations. This is a scatter plot of p_i vs. s_i where i=2,…,n. Note earlier, $p_i \le s_i$. The correlation between s_i and p_i is 0.5. While the correlation of 0.5 between the true spacings and the mean perpendicular distances is significant, this case only considers one dimension (fractures on a line). The relationship shown in Figure 12 has a lower correlation in two or three dimensions.

Perpendicular Distance to Nearest Fractures in Opposite Directions

It was thought that there might be an improvement in the correlation of true spacings and perpendicular distances by considering an alternative measure of perpendicular distance. For the next study, the average of two perpendicular distances in opposite directions was considered. Figure 13 shows the calculation of this modified perpendicular distance that considers opposite directions. If the fractures occur along a 1 dimensional line, this amounts to:

$$D_i = average(x_i - x_{i-1},\ \ x_{i+1} - x_i) = \frac{[x_i - x_{i-1}] + [x_{i+1} - x_i]}{2} = \frac{x_{i+1} - x_{i-1}}{2},\ \ i = 1,\ldots,1000 \tag{3}$$

A simulation study was conducted where 1000 fractures, f_i, are simulated along a 1D line ($f_i,\ i = 1,\ldots,1000$). Fracture locations and orientations are generated according the following rules:

- $x_1 = 0.5$
- $x_i = x_{i-1} + \text{norminv}(\text{rand}(), \mu{=}1, \sigma{=}0.5), \quad \text{i=2,...,1000}$
    - i.e. the previous value is added to a value randomly selected from a normal distribution with mean = 1 and standard deviation = 0.5.
- All fractures are oriented perpendicular to the line

The idea here is to simulate fractures on a one-dimensional line that have semi-regular spacing. The histogram of true fracture spacing is shown in Figure 14. Next, the perpendicular distances that consider both directions (Equation (3)) are calculated. Figure 15 shows the scatter plot of true spacings versus perpendicular spacings considering opposite directions. The correlation coefficient is 0.71, which is an improvement over the perpendicular distance in one direction. Then, the 1000 non-random fractures are used as the “Truth” and a modified version DFNSIM is used to model those fractures. The target histogram of average perpendicular distribution in opposite directions is built by the modified version of DFNSIM. 1000 fractures are simulated with the target histogram of average perpendicular distance in opposite directions. The target, initial DFN and final DFN relative histograms of average perpendicular distance in opposite directions are shown in Figure 5. Note the excellent match between the target and final DFNs.

Figure 17 shows the correlation between true spacing and average perpendicular distance for the initial DFN (r = 0.707). Figure 18 shows the same correlation for the final optimized DFN (r = 0.57). Note the drop in correlation between the initial and final DFNs.

For one final figure, consider Figure 19, which shows the fracture spacing for the "truth" fractures generated at the beginning, along with the true spacing of the initial DFN and final DFN. The final DFN is an improvement over the initial DFN in terms of true fracture spacing, but is not even close to matching the "truth".

Conclusions and Future Work

Early attempts at DFN simulation designed to reproduce histograms of true fracture spacing using the scan line methodology failed due to computational limitations. As a result, the perpendicular distance between a fracture and its nearest neighbour was introduced. Several simulation studies were conducted to assess the validity of using the perpendicular distance measure as a proxy to the true fracture spacing. There are differences between the two measures, but the proxy perpendicular distance measure is deemed reasonable for discrete fracture simulation..

Future work should include re-visiting DFN simulation using the histogram of true fracture spacing. While early attempts failed, there may be some efficiencies that can be implemented in the code to reduce the computational burden. For example, it may be possible to re-use the same scan lines with every perturbation in the DFN. As well, it may be possible to pre-calculate all the spacings once, which can then be saved in memory and re-used as necessary.

References

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Twiss, R.J. & Moores, E.M., 1992. *Structural Geology*, New York: Freeman and Company.

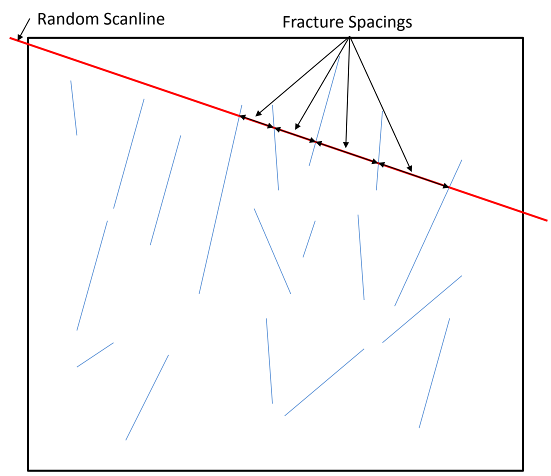


Figure 1: Fracture spacings are calculated along a random scan line, that is perpendicular to the average fracture orientation.

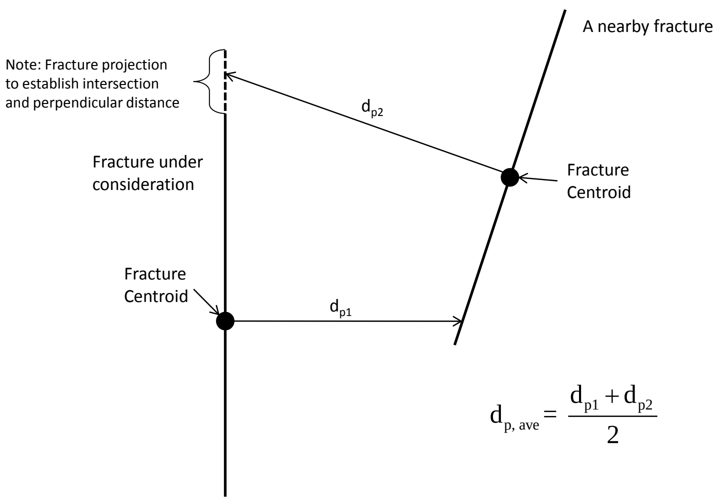


Figure 2: The procedure for calculating the perpendicular distance to the nearest fracture.

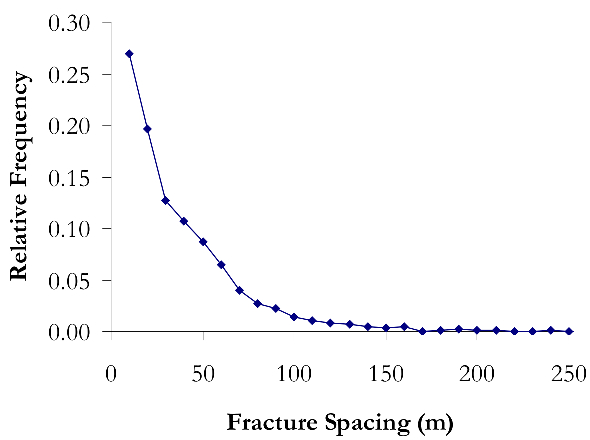


Figure 3: Relative histogram of fracture spacing for a DFN with 1000 fractures.

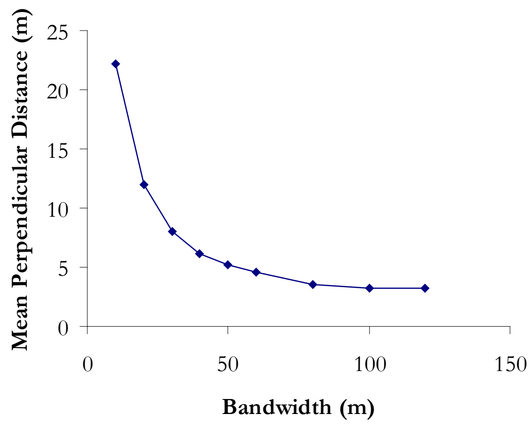


Figure 4: The inverse relationship between bandwidth and mean perpendicular distance to the nearest fracture for a DFN with 1000 fractures.

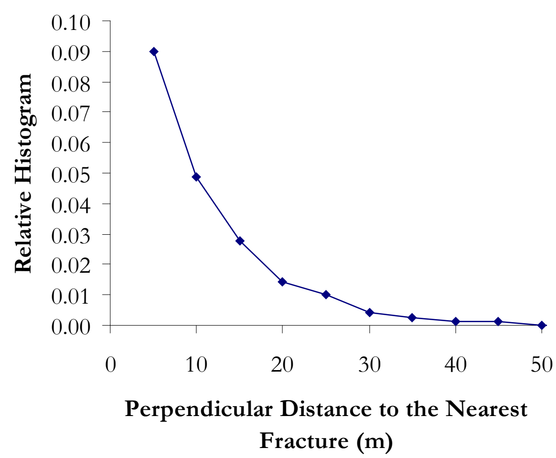


Figure 5: Relative histogram of perpendicular distance to the nearest fracture for a DFN with 1000 fractures, using a bandwidth of 30 m.

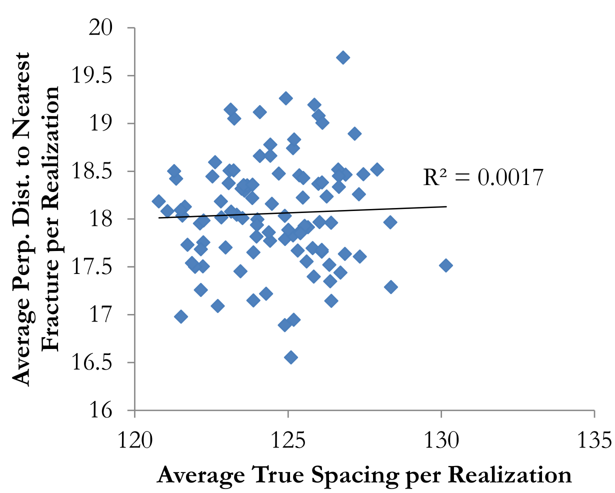


Figure 6: The mean fracture spacing versus the mean perpendicular distance for each of 100 DFN realizations.

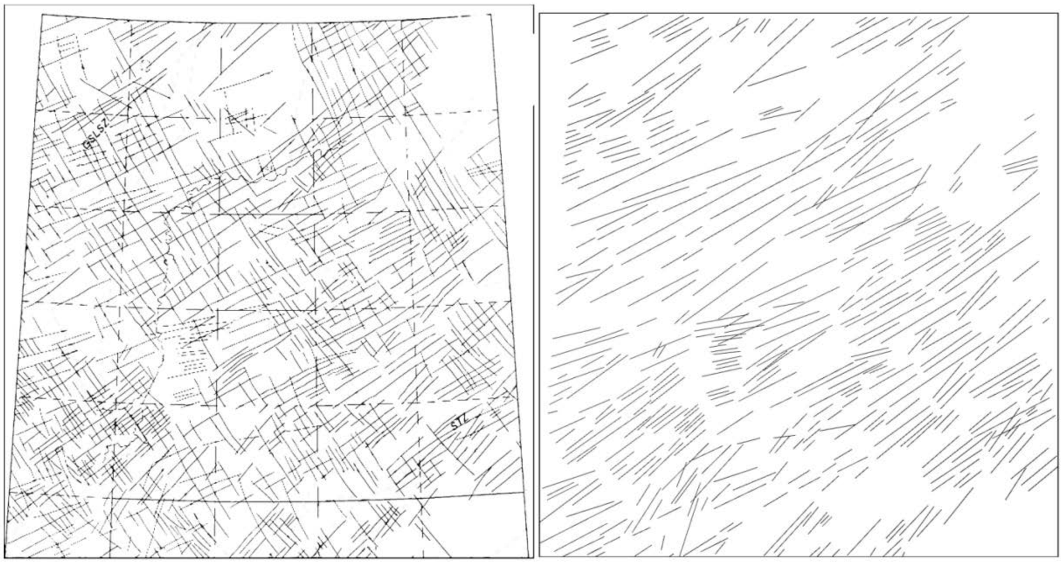


Figure 7: Lineaments measured from satellite imagery. Right: Digitized lineament set.

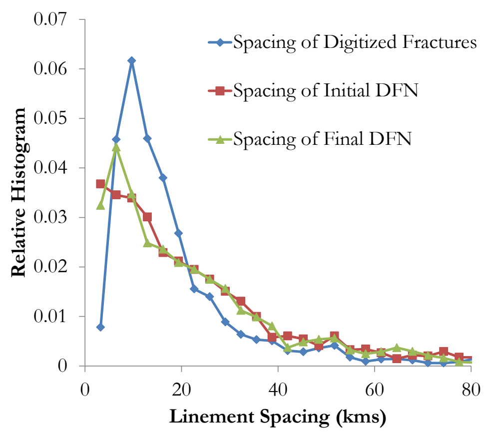


Figure 8: True fracture spacing histograms for the digitized lineaments, initial DFN and final (optimized) DFN.

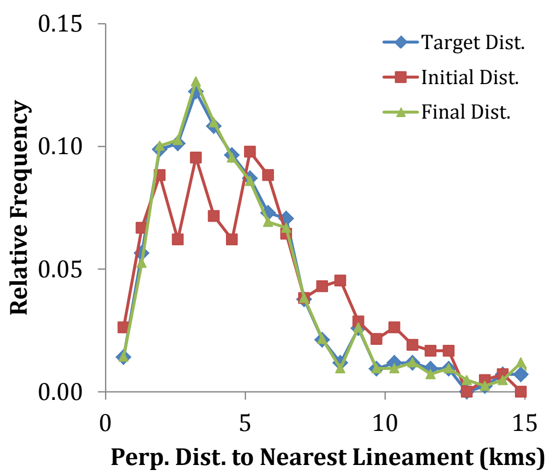


Figure 9: Target, initial DFN and final DFN histograms for perpendicular distance to the nearest fracture.

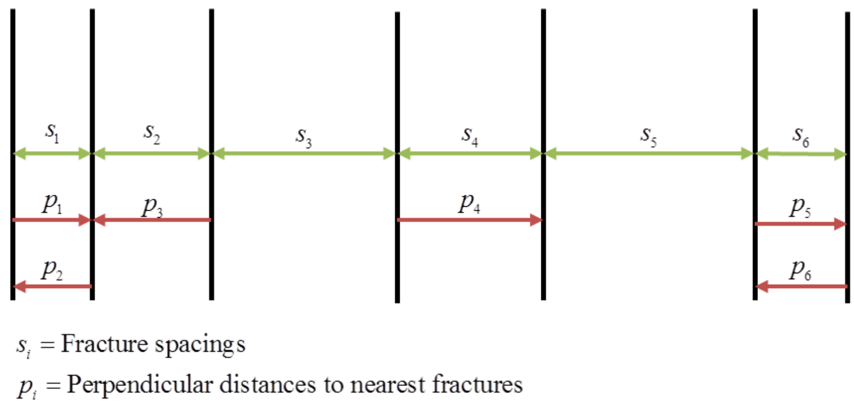


Figure 10: 11 one-dimensional fractures, simulated along a line.

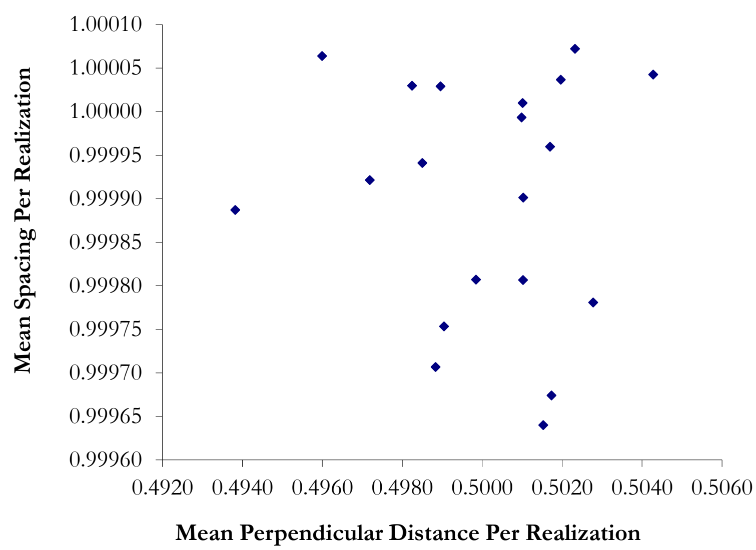


Figure 11: Relationship between mean spacing and mean perpendicular distance per realization.

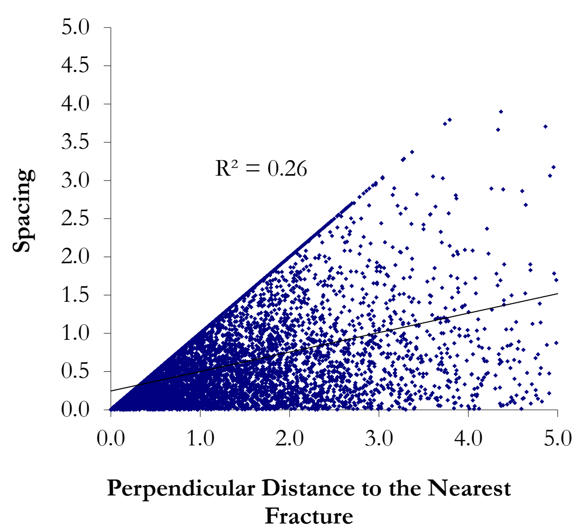
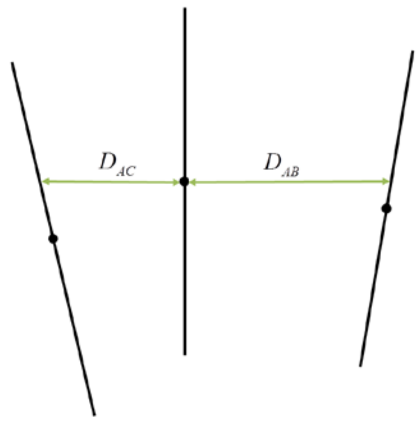


Figure 12: Relationship between individual values of perpendicular distance and spacing.



$$\text{Perp. Dist. from A to Nearest Neighbours} = D_{AB} = \frac{D_{AC} + D_{AB}}{2}$$

Figure 13: New method for calculating the perpendicular distance to the nearest neighbours. The perpendicular distance is the average of the two distances in opposite directions.

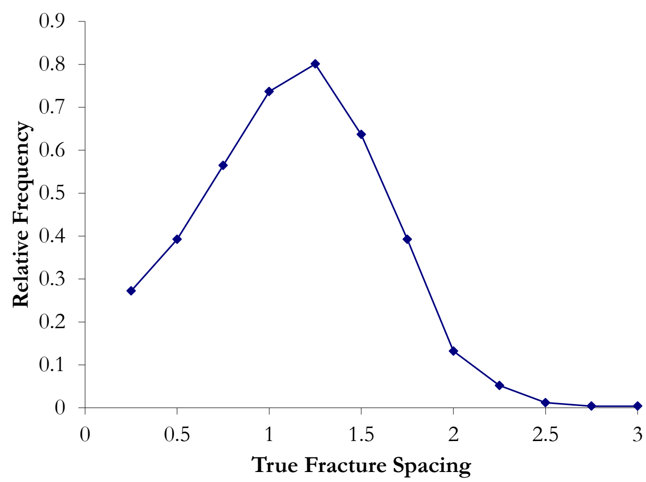


Figure 14: Relative histogram of true fracture spacing.

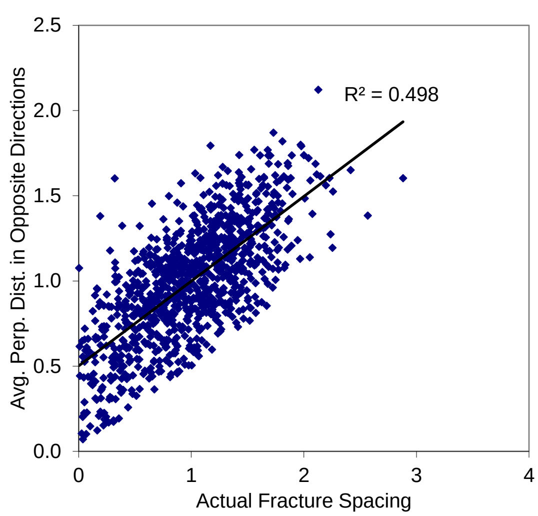


Figure 15: Correlation between true fracture spacing and average perpendicular distance in opposite directions.

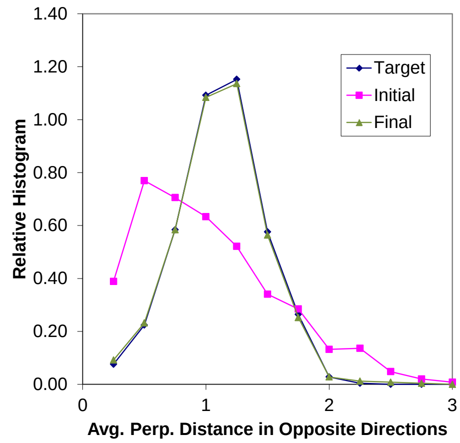
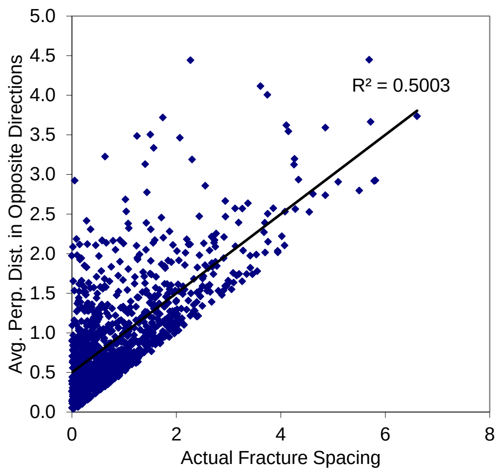


Figure 16: Target, initial DFN and final DFN histograms of average perpendicular distance in opposite directions.



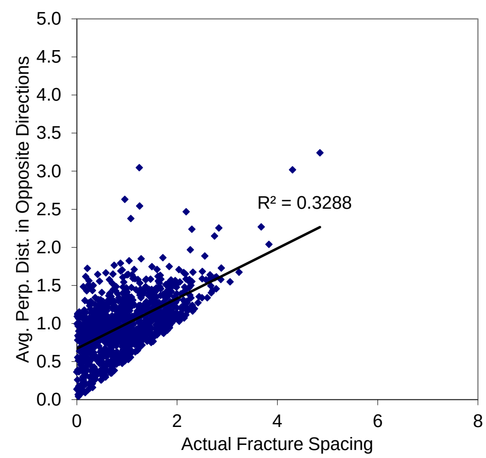


Figure 17 (left): True fracture spacing versus average perpendicular distance in opposite directions for the initial DFN. **Figure 18** (right): for the final DFN.

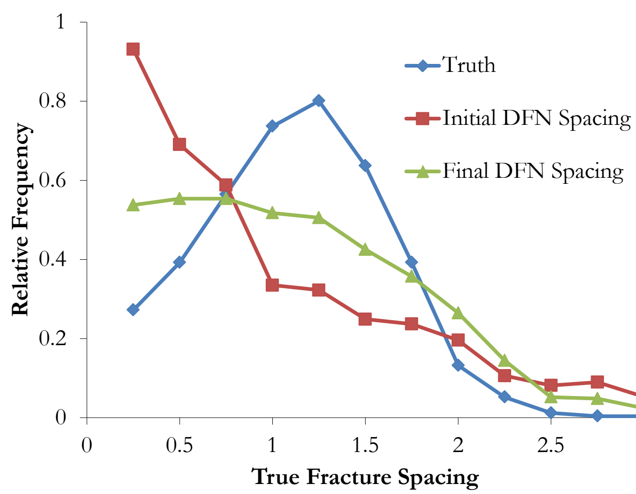


Figure 18: True fracture spacing for the original fractures (i.e. the truth), the initial DFN and the final DFN.