Tensor Based Approaches for LVA Field Inference

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The importance of locally varying anisotropy (LVA) in model construction can be significant; however, it is often ignored because of lack of sufficient data and methodologies to infer the exhaustive anisotropy field. Several sources of data are available for LVA field inference. If an area is sampled extensively, direct inference from data may be possible. Areas where data is sparse requires further geologic interpretation. Several data sources are explored in this work to infer LVA fields but the focus is on generating LVA fields from exhaustive data. Three methods are compared (1) a method based on the principal component decomposition of a local window (2) a gradient based method (3) a transformation to Fourier space. All methods perform well for characterizing local orientations, but the Fourier transformation is computationally demanding for large 3D models.

Discussing the Problem

Anisotropy is the concept that variables are more continuous in one direction than another. In a geological context, anisotropy describes the continuity of geologic variables such as facies type, porosity, permeability, mineral grade, concentration etc. Processes that occur in nature such as fracturing, folding etc can affect the overall spatial distribution of these properties.

Profitable extraction of resources requires effective modeling of deposit properties, such as grade per ton, volume of oil, contaminate concentrations etc. Modeling involves estimating the values of these properties at unsampled locations, predicting values at every point in domain and evaluating end processes such as mine design and flow simulation. Incorporating anisotropy in geologic modeling can increase accuracy as many deposits display complex, nonstationary behavior. However, existing techniques that aim at predicting values at unsampled locations either do not consider the geologic anisotropy of the formation or assume a single direction of continuity. Thus, complex nonlinear features that can improve model realism and performance are ignored. The current challenge is to build estimations of variables considering that the variables are continuous in locally varying directions (locally varying anisotropy- LVA). An important step is the inference of the LVA field which describes the direction and magnitude of the underlying anisotropy of the domain. This report will elaborate on the methodologies to build LVA fields from secondary geologic sources and also show extensions of the techniques from 2D to 3D models which should improve resource estimation and model predictions.

In 3D the orientation of the LVA field is defined by three angles strike (α), dip (β) and plunge (ϕ), where α is calculated from north (y-axis) by rotation about the z-axis, β is a dip rotation about the x-axis and ϕ the rotation about the y-axis. The magnitude of anisotropy is defined by the ratios: r_1 in the ratio of the range in the minor and major directions, and r_2 the ratio of vertical and major direction. In this work, the magnitude of anisotropy is assumed constant and known. The focus is on the orientation of anisotropy.

Drill hole Data

Drill hole data can be characterized as (1) primary or, (2) secondary. Primary data contains information on the variable of interest, i.e its underlying LVA features; secondary data is the measurement of a different variable that shows same LVA characteristics as the former. If secondary information is to be used, the continuity of the variable must show the same spatial features as the primary variable.

Remote Sensing

Data for LVA field inference can come from exhaustive geophysical remote sensing such as seismic, magnetic or gravitational surveys. These remote sensing surveys offer low resolution data for bulk petro physical properties. Usually the data are processed to define variables that are related to the variable of interest. For instance, in petroleum applications, seismic surveys are sensitive to changes in porosity. The seismic data can be used to infer acoustic impedance which is often calibrated to porosity (Deutsch, 2002). In such cases, these remote sensing surveys can be used to infer the LVA field.

Structural Models

In stratigraphic formations, the continuity of the deposit lies parallel to the direction of the deposited layer. The property of interest may follow the stratigraphy and the boundary of the layers would delineate the LVA of the property examined. The orientation of anisotropy can be obtained by locally averaging the slopes of the top and bottom contours of the defining layer (Boisvert, 2010) (Figure 1). In considering similar formations, it is suggested that each layer be identified and modeled separately (Deutsch, 2002).

Simulation techniques

Direct measurements of the LVA field can be obtained from down the hole cameras, formation dip from dipmeter etc. These data are axial in nature or undirected, which is to say that the continuity of the variables is the same in the direction θ and $\theta+180^o$. When data are 2D, the angles are restricted within $0 \leq \theta \leq \pi$, and then doubled and transformed into circular coordinates (Mardia and Jupp, 2000). Standard kriging techniques are applied to local values at unsampled locations to populate the modeling domain for all points in the spherical coordinates. The components are then recombined to create the final LVA field (Boisvert, 2010). To estimate axial point data in 3D refer to paper 123 in this report.

Geological interpretation

Accurate LVA maps are based on detailed understanding of the deposit geology. A straightforward technique is to draw the LVA field manually based on the information of the variable of interest. This method borrows heavily from the practitioner's experience and expertise on the geology.

Moment of Inertia

The drawback of manually generating the LVA field is that it can be tedious, subjective and often deterministic. An alternative technique for LVA field inference is to generate the LVA through automatic methodologies directly from the data available. The moment of inertia technique can be applied when exhaustive secondary data is available. A covariance map is generated by considering the data in a local window and calculating the local moment of inertia for the neighborhood. The covariance values are considered the mass. The moment of inertia tensor is a matrix where each component represents the moment of inertia about a different coordinate axis. The eigenvalue and eigenvector of the matrix in calculated and the eigenvector corresponding to the largest eigenvalue provides the major direction of continuity for the deposit (Hassanpour, 2007).

Evaluation of local orientation estimation techniques

The principles for calculating local orientation and changes in local orientations are largely borrowed from similar techniques from computer vision and image processing. Orientations in images are perceived when contours or visible boundaries are present and show alignment. Techniques that show local orientations through directional mathematical operations are of interest, such as Soille and Talbot (1998; 2001) or a similar examination of the data translated in Fourier domain and the effect of the energy of the power spectrum by orientation-selective filters (Kass and Witkin,1987). In a scalar space a vector describes local orientation. Starting with data in the spatial domain, for each point x in the *n*-dimensional space R^n a vector field $\boldsymbol{v}(x)$ is defined by the gradient of the space. A domain $W(x)$ – a square window in R^2 or a cube in R^3 – defines a local neighborhood centered at every point $\boldsymbol{x}$. Then, within this grid of points a cloud of points from is connects x_j belong to $W(x)$ to the origin by vectors $\boldsymbol{v}(x_j)$. A tensorial approach is adopted to study the average orientation inside $W(x)$ by the matrix of inertia of the cloud of points and obtain the principal axis of inertia by eigenvector decomposition.

Local orientation using Fourier Transform

It can be shown through a small subsection of a 2D outcrop image that the modulus of the Fourier transform of the area containing detectable linear patterns of contours would show a cloud of points with its main orientation resembling that of the pattern. The eigenvector of the point clusters gives the main axis of orientation.

For every point, *x*, in the image a square window is considered centered in *x*; the size of the window is always a power of 2- and the Fourier Transform is applied. The modulus of the image in the transformed space is:

$$Iv_M = |F[Iv]|^2$$

From the image Iv_m in coordinates (k_1,k_2) the inertia matrix, denoted sigma, and the two eigenvectors (λ_1, λ_2) are obtained from solving equation $\det(\Sigma - \lambda^* I) = 0$. Local orientations are given by the eigenvector corresponding to the largest eigenvalue.

Local orientations using gradient tensors

The outcrop image we are using show marked contours and patterns and can be considered piecewise constant. So at small block sizes the gradient vectors on average should be orthogonal to the dominant direction of the image pattern. Then, local orientation estimation is completed if a unit vector a is found that maximizes the average of angles between **a** and the gradient vectors in a defined window. The problem is to minimize:

$$\sum_{i=1}^{n}(\vec{a}^T\vec{g}_i)^2 = \vec{a}^T\sum_{i=1}^{n}(\vec{g}_i\vec{g}_i^T)\vec{a} = \vec{a}^T C\vec{a}$$

where the vector ***g*** is a gradient vector, and ***a*** is unit vector.

$$C = \begin{bmatrix} \sum_{i=1}^{n} g_x^{(i)} g_x^{(i)} & \sum_{i=1}^{n} g_x^{(i)} g_y^{(i)} \\ \sum_{i=1}^{n} g_x^{(i)} g_y^{(i)} & \sum_{i=1}^{n} g_y^{(i)} g_y^{(i)} \end{bmatrix}$$

Minimizing the term ***aCa*** is the eigenvector of *C* corresponding to the smallest eigenvalue.

Local Orientation using PCA

To implement a gradient tensor method using a PCA-based technique, we need to compute the gradient map for the image. In order to estimate orientations locally, the gradient field is divided into blocks or windows (either overlapping or non-overlapping), and for each block group the gradients into matrix G:

$$G = \begin{bmatrix} \nabla f(1)^T \\ \nabla f(2)^T \\ \vdots \\ \nabla f(N)^T \end{bmatrix}$$

N*2 matrix

Next we compute the singular value decomposition of the matrix G.

$G = USV^T$, here V is orthogonal and the first column (v_1) represents the dominant orientation of the gradient field. The local orientation of the image block is obtained by a rotation of v_1 by 90^o.

Adapting the methodologies for studying exhaustive geologic data

Consider a geological outcrop images as the underlying exhaustive data (Figure 2), which also serves well to display LVA. The obvious measure of the performance of the methodology is how well the LVA follows the contours of the outcrop image.

We are concerned with how the three methods compare in detecting the underlying LVA from outcrop images in 2D. Comparisons can be made visually by looking at the local orientations produced. Before using outcrop images, some test images with constant linear patterns (parallel lines running in various directions) are assessed to see if the methodology is producing the expected LVA (Figure 3).

Next, a measure of reliability is incorporated to select the dominant direction from the local cluster of vector fields. The quantitative measure of reliability (R) here is a ratio of the difference and the sum of eigenvalues of our tensor matrix. R = (difference of eigenvalue)/(sum of eigenvalues). In 2D it shows how much greater the continuity is in the major direction of continuity (Figure 4).

The outcrop image is sufficiently small (223×334) to allow for a range of windows to be tested; 7 different windows with a step size of 2 (input window size and 3 sizes up and 3 sizes down) are used and the window with the highest reliability is selected. This method is explained in paper 406 and shown here for comparison (Figure 5). The LVA field using the Fourier technique is shown in Figure 6. The PCA method is shown in Figure 7. A window size of 16×16 worked well for all methods implemented. All methods to do well reproducing the expected orientations of continuity with the maximum reliability obtained with the adaptive window method.

Extension to 3D

Only the gradient and PCA methods will be extended to 3D. The Fourier technique is very computationally demanding in 2D and would not be feasible in 3D. LVA modeling with our methodologies in 3D is done using a model of an ore body that has been created from a geologic training image (Figure 8). The gradient and the PCA techniques are shown in Figures 9 and 10 respectively. Results for both methods are reasonable but the gradient based method has a significantly higher reliability (Figure 11).

Conclusion

Image processing techniques were applied to infer local orientations of anisotropy. The gradient tensor technique gave satisfactory results in 2D and fairly robust results in 3D. PCA was also reasonable in both 2D and 3D. The Fourier technique was not feasible in 3D and will not be carried forward.

References

Armstrong M and Diamond P, 1984. Testing variograms for positive-definiteness. Mathematical Geology. 16(4):407-421.

Boisvert J, Manchuk J and Deutsch C, (2009). Kriging in the presence of locally varying anisotropy using non-Euclidean distances. Mathematical Geosciences

Boisvert J and Deutsch C, 2010. Programs for Kriging and Sequential Gaussian Simulation with Locally Varying Anisotropy Using Non-Euclidean Distances. Computers and Geosciences. In press, accepted March 2010.

Deutsch C and Journel A, 1998. GSLIB: Geostatistical Software Library and User's Guide. Oxford University Press

Deutsch C, 2002. Geostatistical Reservoir Modeling. Oxford University Press, New York, 376p.

Goovaerts P, 1997. Geostatistics for Natural Resources Evaluation. Oxford University Press. 496p.

Mardia K, Kent J and Bibby J, 1979. Multivariate Analysis. Academic Press. 521p.

Stroet C and Snepvangers J, 2005. Mapping curvilinear structures with local anisotropy kriging. Mathematical Geology. 37(6):635-649.

Figures

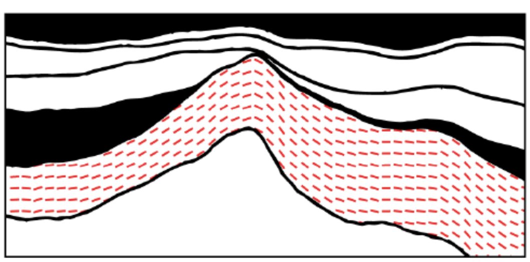

Figure 1 Cross-section through a reservoir with 6 different stratigraphic layer. LVA for one of the layer is shown (Boisvert, 2010).

Figure 2 Geologic outcrop image taken from
http://farm5.static.flickr.com/4093/4743195771_5520828258.jpg

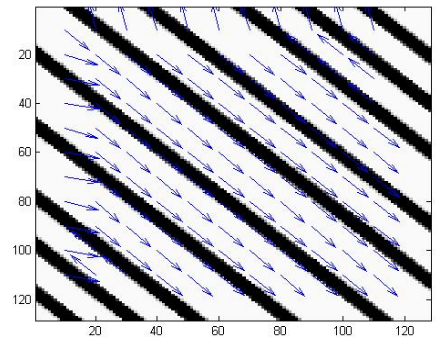

Figure 3: Arrows depict the single dominant direction from images.

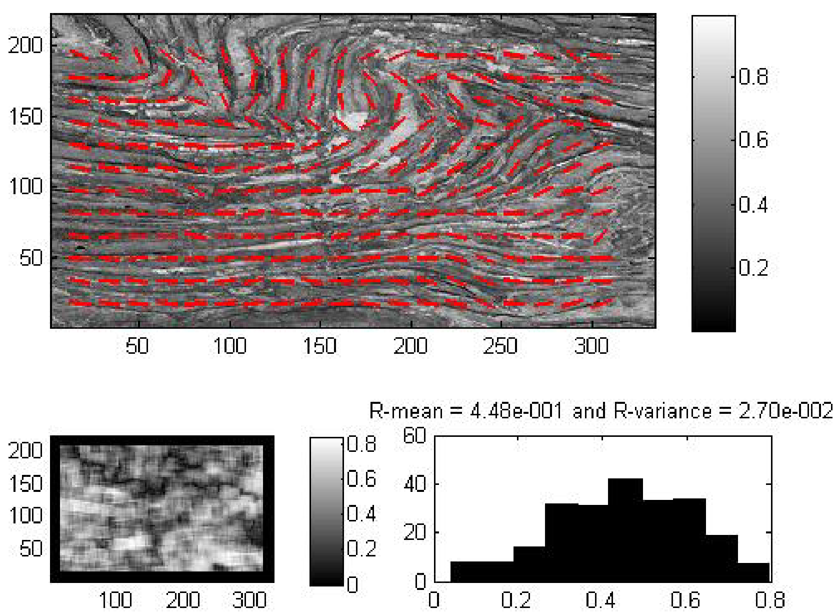

Figure 4 A basic implementation of gradient method, window of size 16.

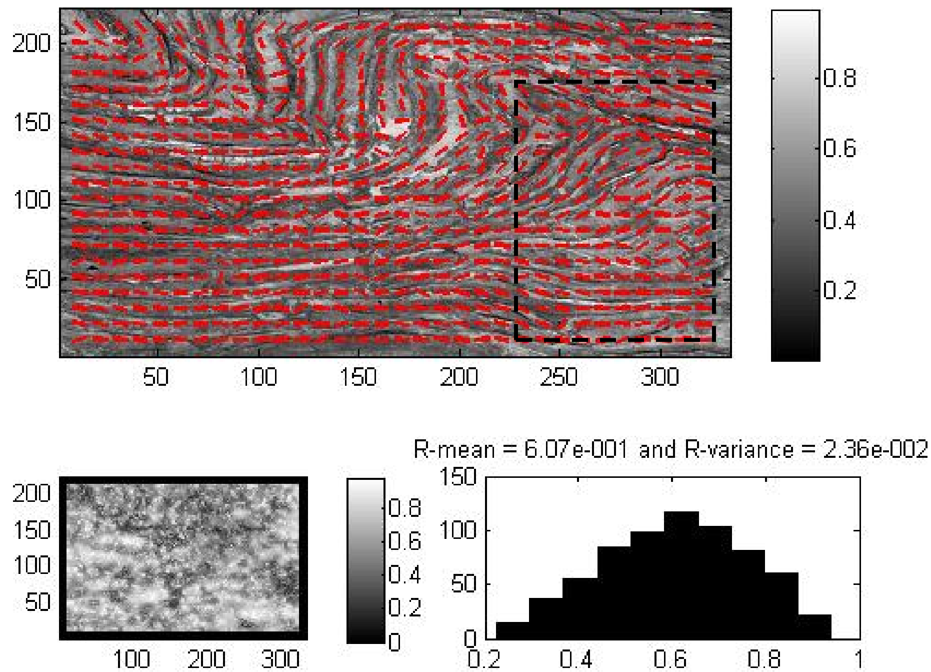

Figure 5 Inclusion of adaptive window with the input size of 10. The reliability map is significantly improved as is the LVA at small scale (top picture – near extreme right).

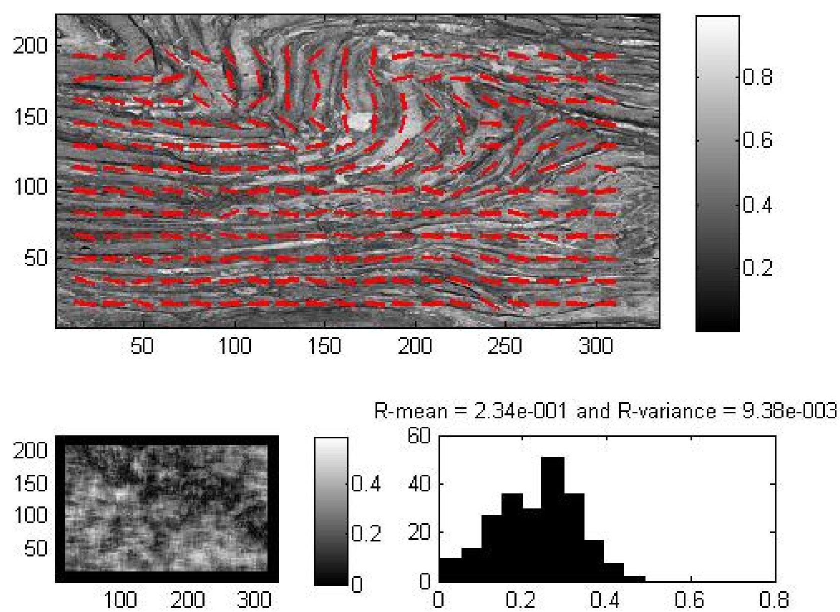

Figure 6: LVA field with the Fourier technique**.**

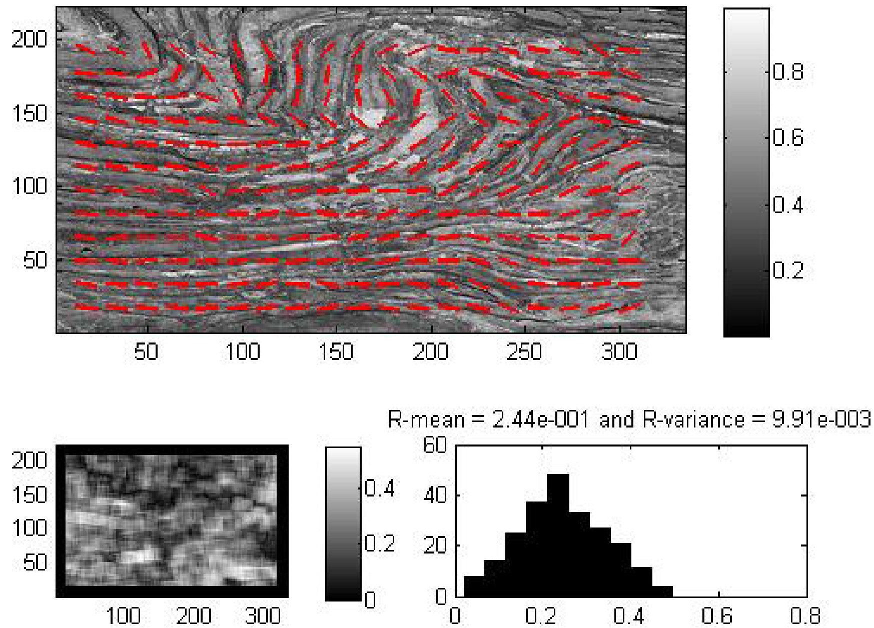

Figure 7 A simple PCA based methodology for generating LVA map, window size = 16.

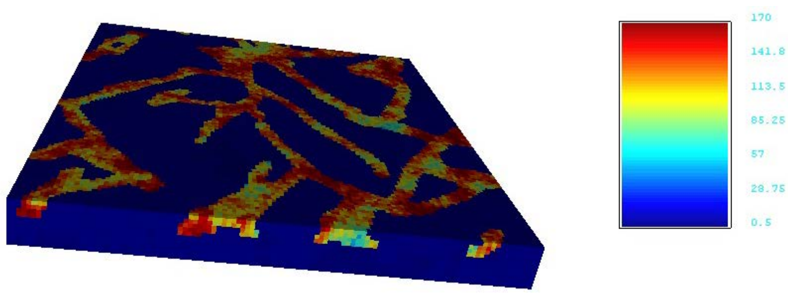

Figure 8 A 3D model depicting an ore body (created using training images).

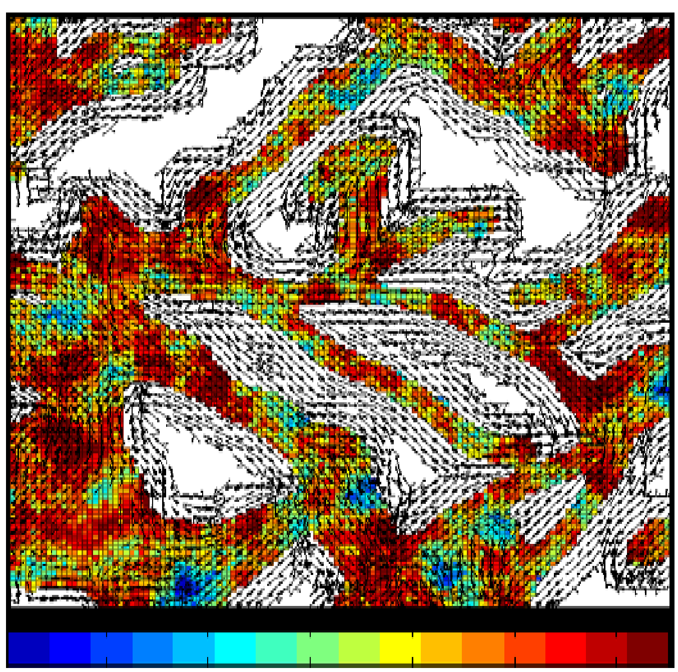

Figure 9 Gradient tensor based LVA generation**.**

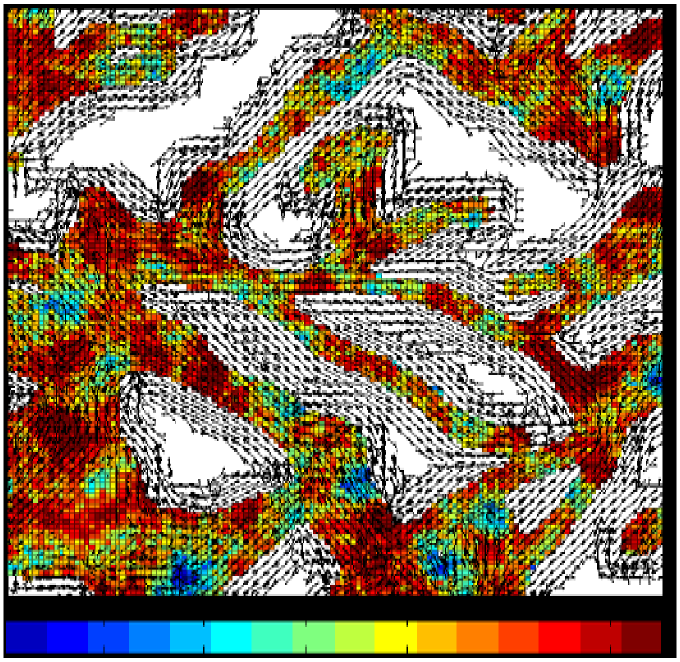

Figure 10 LVA from the pca based method.

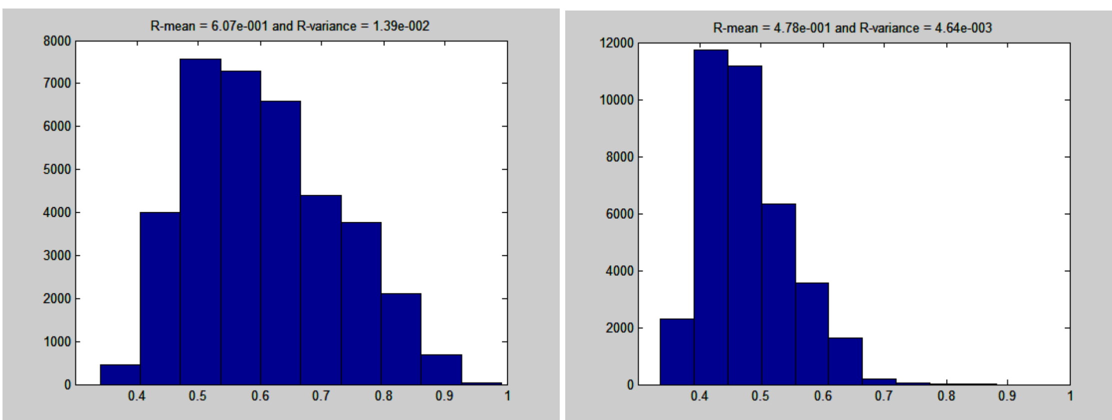

Figure 11 Comparison of reliabilities (gradient based method on the left, pca based method on the right).