Local Upscaling of Elastic Properties Honoring Shear Strain Response

Mehdi Khajeh, Jeff Boisvert and Rick Chalaturnyk

Rock mechanical properties are important parameters in geomechanical simulation. Considering stochastic heterogeneous models rather than conventional homogeneous layer cake models leads to more realistic simulation results and uncertainty assessment. These heterogeneous models are usually fine scale models generated by geostatistical techniques. Although considering high resolution property models is desirable, geomechanical simulation with a fine scale model is not computationally feasible for full–field, coupled geomechanical-flow simulation. Upscaling of property models is necessary to move from fine scale/high resolution property models to coarser scale/low resolution models appropriate for simulation. Local numerical upscaling was proposed by Khajeh et al. (2012) to describe the macroscopic elastic behavior of complex heterogeneous media and is expanded upon in this work. This numerical technique is not restricted to specific geologies as typical analytical techniques are. Moreover, transversely isotropic deformation is considered rather than the usual assumption of isotropic deformation. To compare the accuracy of the upscaling methodology, shear strain is considered as the geomechanical response of a synthetic facies model and the response of the coarse scale model is compared to other conventional analytical averaging techniques. It is found that the numerical upscaling technique better approximates the geomechanical response of the fine scale model.

Introduction

It is important to consider geomechanical responses for hydrocarbon reservoirs in stress-sensitive or stress-dependent reservoirs. Geomechanical effects impact reservoirs in a number of ways including subsidence, well casing deformation, cap rock failure and solid production. One advantage of geomechanical deformation of a reservoir is the failure of barrier facies in the reservoir such as shales in the McMurray formation of Alberta; this results in an increase in permeability for nearly impermeable layers and typically results in increased oil production.

Uncertainty in rock mechanical properties of reservoirs and the surrounding strata should be considered for stress dependent reservoirs. It is not possible to deterministically model rock mechanical properties because they are very dependent on facies; properties vary significantly between sand and shale. Geological uncertainty is unavoidable and should be considered in geomechanical simulations. Using geostatistical techniques and building multiple equi-probable geological realizations is standard and will be used to assess uncertainty in geomechanical response. Geostatistical realizations are built at a fine 'point' scale and simulated with a geomechanical simulator. However, the fine scale realizations cannot be used for geomechanical simulation because 1) considerable computational time required by geomechanical simulators to solve geomechanical equations and 2) current commercial geomechanical software cannot handle a large number of cells. There is a need to move from fine scale MEM realizations to coarser realizations for simulation.

Generating property models at a coarse scale that honor the geomechanical response of the fine scale realizations is the goal of upscaling. Many upscaling techniques could be considered for upscaling elastic properties, the following assumptions can be made (1) consider a simplified (spherically surrounded or stratified) configuration; (2) considering a limited number of layers, or; (3) considering isotropic deformation. These three assumptions limit the effectiveness of existing upscaling methodologies. A brief review of existing techniques for upscaling of elastic properties is provided.

Numerical upscaling has been proposed by Khajeh et al. (2012). Detailed description of this technique is provided in paper 118 in this report. Upscaling considering volumetric strain as the geometric response was implemented. In this work, shear strain is considered as the geomechanical response and the same process as in paper 118 is repeated. Power law averaging (Deutsch, 1989) is compared to the proposed numerical upscaling technique. The material properties and number of grid cells in each zone of the model (Figure 1-b) are summarized in Table 1 and Table 2 respectively.

The error resulting from upscaling is assessed by comparing shear strain of the fine scale model (Figure 3) to the response of the coarse upscaled models using power law averaging and numerical upscaling calculated as:

$$[\%]e = \frac{\sum_r \left|1 - \frac{\overline{e_{s_r}}}{e_{s_r}}\right|}{n_r} * 100 \quad (1)$$

$$\overline{e_{s_r}} = \frac{\sum_i e_{s_i}}{n_i} * 100 \quad (2)$$

where,

$\overline{e_{s_r}}$ is the average shear strain of the fine scale cells in the rth upscaled block.

e_{s_r} is the shear strain in the rth upscaled block.

n_r is the number of blocks in the upscaled model.

e_{s_i} is the shear strain in the ith fine scale cell within the upscaled block.

n_i is the number of fine scale cells in each upscaled block.

15 upscaling ratios (number of fine scale cells in each upscaled block) are considered, using horizontal ratios of 1:1, 5:1, 15:1, 30:1, 60:1, 100:1 and vertical ratios of 1:1, 4:1 and 8:1.

Review of conventional upscaling (averaging) techniques used for elastic properties

Mackenzie (1950) developed an analytical upscaling technique based on the concept of the self-consistent method applied to a spherical representative volume element (RVE). The bulk modulus and the shear modulus are determined by applying a hydrostatic pressure and a simple homogeneous shear stress to a large sphere. Mackenzie assumed that deformation is controlled by the isotropic elastic constitutive law. Hashin (1958) proposed a similar homogenization technique for spherical materials based on the concept of elastic energies. Hill (1965) calculated the effective elastic moduli of a two-phase composite. In contrast to previous works, Hill considered that the phases do not surrounded one another. Budiansky (1965) extended the model proposed by Hill (1965) to multiphase materials and stratified periodic materials. Salamon (1968) derived five elastic coefficients of a homogeneous, transversely isotropic medium equivalent to perfect, horizontally layered rock based on strain energy. These simplifications are not appropriate for the typical sand/shale geometries seen in the McMurray formation.

Arithmetic, harmonic and geometric averaging are three types of analytical up-scaling. A generalization of these averaging techniques is Power Law averaging, developed by Deutsch (1989):

$$A_w = \left(\frac{1}{n}\sum_{i=1}^{n} A_i^w\right)^{\frac{1}{w}} \quad (3)$$

Limited work has been undertaken related to numerical upscaling for geomechanical properties. Elkateb (2003) proposed a mathematical expression to determine the equivalent Young's modulus of a simplified layer cake model under isotropic deformation. This approach was not extended to heterogeneous media or anisotropic deformation.

Results

In Figures 2 and 3 upscaled Young's modulus maps for selected horizontal upscaling ratios (30:1 and 100:1) and for vertical upscaling ratios of 1:1 (Figure 2) and 8:1 (Figure 3) for different upscaling techniques are shown. It can be seen that averaging in the vertical direction has a large effect on upscaling because of the shorter variogram range in this direction. Considering a 30:1 horizontal variogram range has minimal impact on the upscaled maps whereas an 8:1 in the vertical direction is very significant.

The results of geomechanical response (i.e., shear strain maps) obtained from upscaled maps are shown in Figures 4 and 5. As with the Young's modulus, there is no significant difference between different upscaling techniques used for vertical upscaling ratio of 1:1. However changes in shear strains for vertical upscaling of 8:1 and horizontal upscaling of 100:1 are clear (Figure 5). There is more similarity between the numerical upscaling results and results obtained using the harmonic averaging.

Using the error definition in Equation 1, differences in the shear strain maps of the fine scale model response and the various upscaled maps are shown in (Figure 6). These results show that the proposed numerical averaging technique results in a lower error for all vertical and horizontal ratios considered.

Discussion and Conclusion

The effect of upscaling on shear strain was investigated. The main befits of the proposed technique is the consideration of anisotropic deformation and the lack of a restriction on the input facies configuration which is common for analytical techniques.

It was shown that the upscaling technique results in less upscaling error in comparison to conventional techniques. Finding an appropriate coarse block scale is important and should be selected through defining an acceptable level of error. In addition to producing less error, the advantage of numerical upscaling is that it is not limited to a specific facies configuration or geological formation. The numerical upscaling technique implemented in this work is appropriate for domains for which elastic deformation can be characterized by transversely isotropic behavior.

References

Budiansky, B., 1965. On the elastic moduli of some heterogeneous materials. Journal of the Mechanics and Physics of Solids. 13:223–227.

Deutsch, C.V. 1989. Calculating Effective Absolute Permeability in Sandstone/Shale Sequences. SPE Formation Evaluation. 4: 3,343–3,348.

Deutsch, C.V., and Journel, A. G., 1998. GSLIB: Geostatistical Software Library and User's Guide. Second edition, Oxford, New York.

Elkateb, T. M., 2003. Quantification of Soil Heterogeneity. PhD dissertation, University of Alberta, Edmonton, Canada.

Hashin, Z., 1958. The moduli of an elastic solid, containing spherical particles of another elastic material. Proceedings IUTAM Symposium on Non-homogeneity in Elasticity and Plasticity, Warsaw, Poland. 17:463–478.

Hill, R. A., 1965. Self-consistent mechanics of composite materials. Journal of the Mechanics and Physics of Solids. 13:213–222.

Khajeh, M.M., Chalaturnyk, R.J., and Boisvert, J.B., 2012. A Numerical Local Upscaling Approach for Elastic Rock Mechanical Properties: Dealing with Heterogeneity. Prepared for presentation at the 46th US Rock Mechanics / Geomechanics Symposium held in Chicago, IL, USA, 24-27 June. ARMA 12-654.

Lekhnittskii, S.G., 1981. Theory of Elasticity of an Anisotropic Body. Moscow: Mir Publishers.

Mackenzie, J.K., 1950. The elastic constants of a solid containing spherical holes. Proceedings of the Royal Society London. B63(1): 2–11.

Salamon, M., 1968. Elastic moduli of a stratified rock mass. Journal of Rock Mechanics and Mining Sciences 5:519–527.

Tables

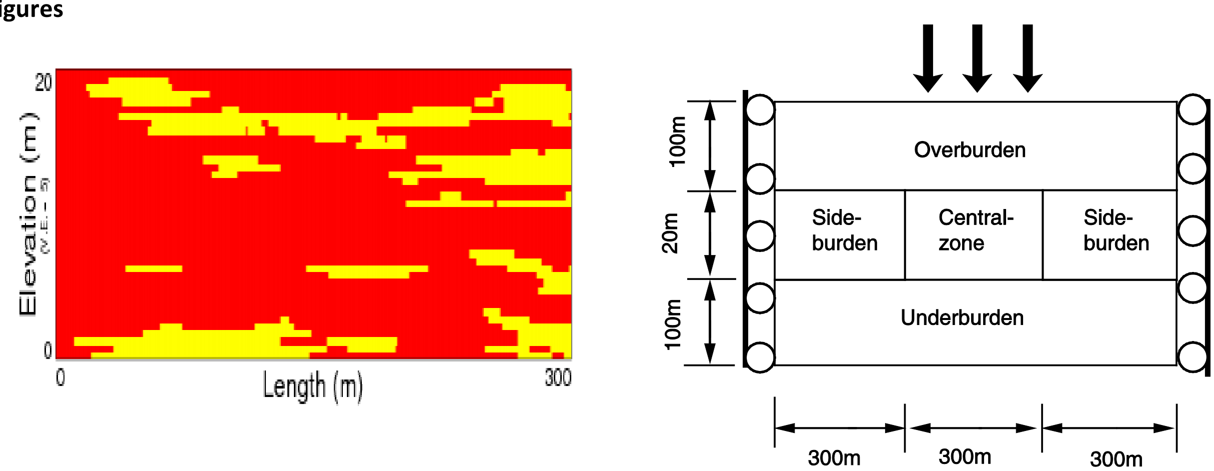
Table 1. Material properties in each section of Figure 1-b.

	Overburden	**Side Burden**	**Under burden**	**Central Section**
Deformation type	Isotropic	Isotropic	Isotropic	Transversely Isotropic
Young modulus (MPa)	250	480	5000	Shale : 300
				Sand : 600
Poison Ratio	0.3			Shale : 0.3
				Sand : 0.3

Table 2. Number of grid cells.

	Overburden	Side Burden	Under burden	Central Section
Horizontal	360	30	360	300
Vertical	10	40	10	40

Figures



(a) **(b)**

Figure 1. (a) SIS facies realization considered with 20% shale. Red (dark) is sand; yellow (light) is shale. Variogram parameters: one spherical structure with a vertical range of 4m, horizontal range of 120m and no nugget effect (b) Geometry of the area of interest used (from Khajeh et al., 2012).

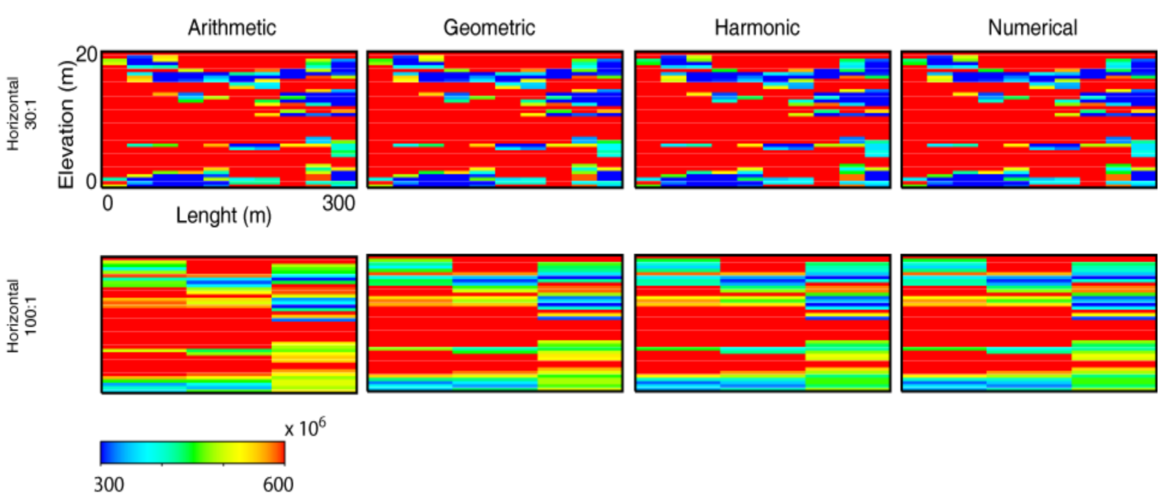


Figure 2. Young's modulus (MPa) for selected horizontal upscaling ratios, vertical upscaling ratio is constant at 1:1.

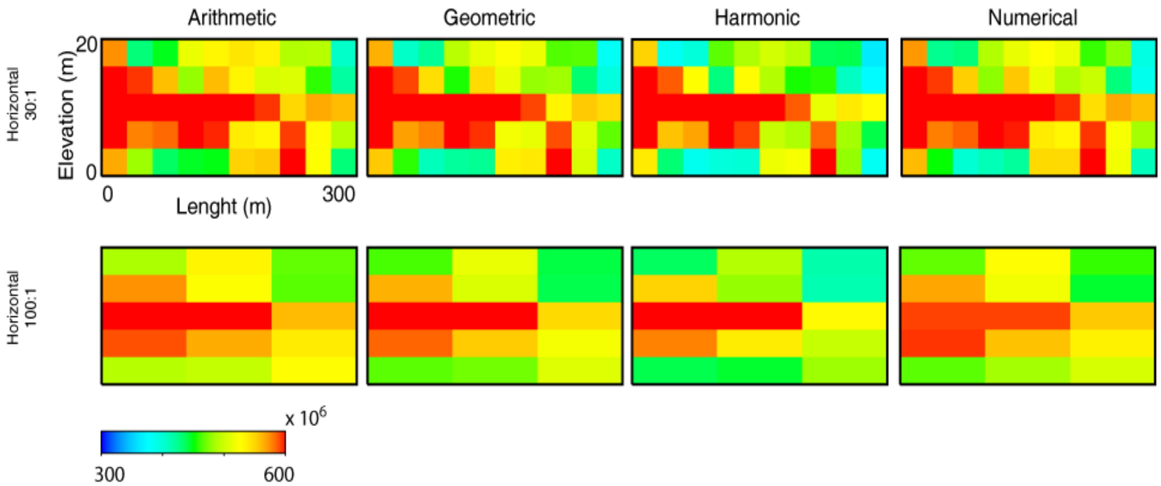


Figure 3. Young modulus for different horizontal upscaling ratios, vertical upscaling ratio is constant at 8:1.

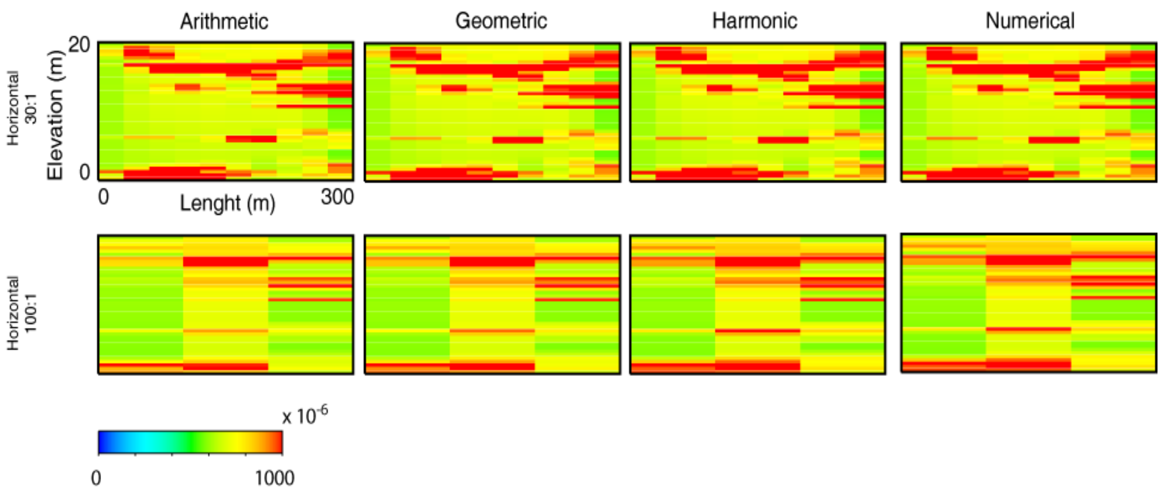


Figure 4. Shear strain for selected horizontal upscaling ratios, vertical upscaling ratio is constant at 1:1.

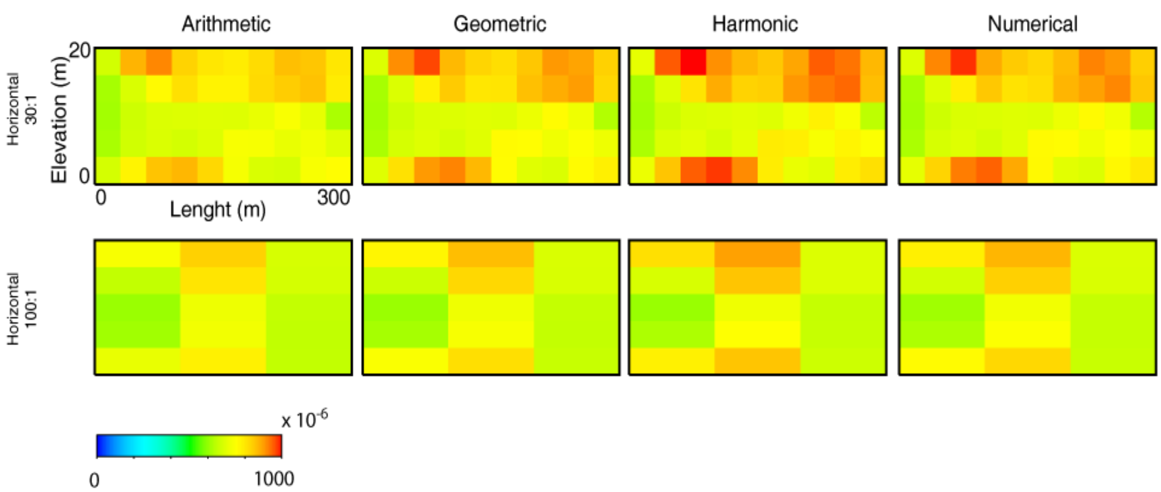


Figure 8. Shear strain for selected horizontal upscaling ratios, vertical upscaling ratio is constant at 8:1

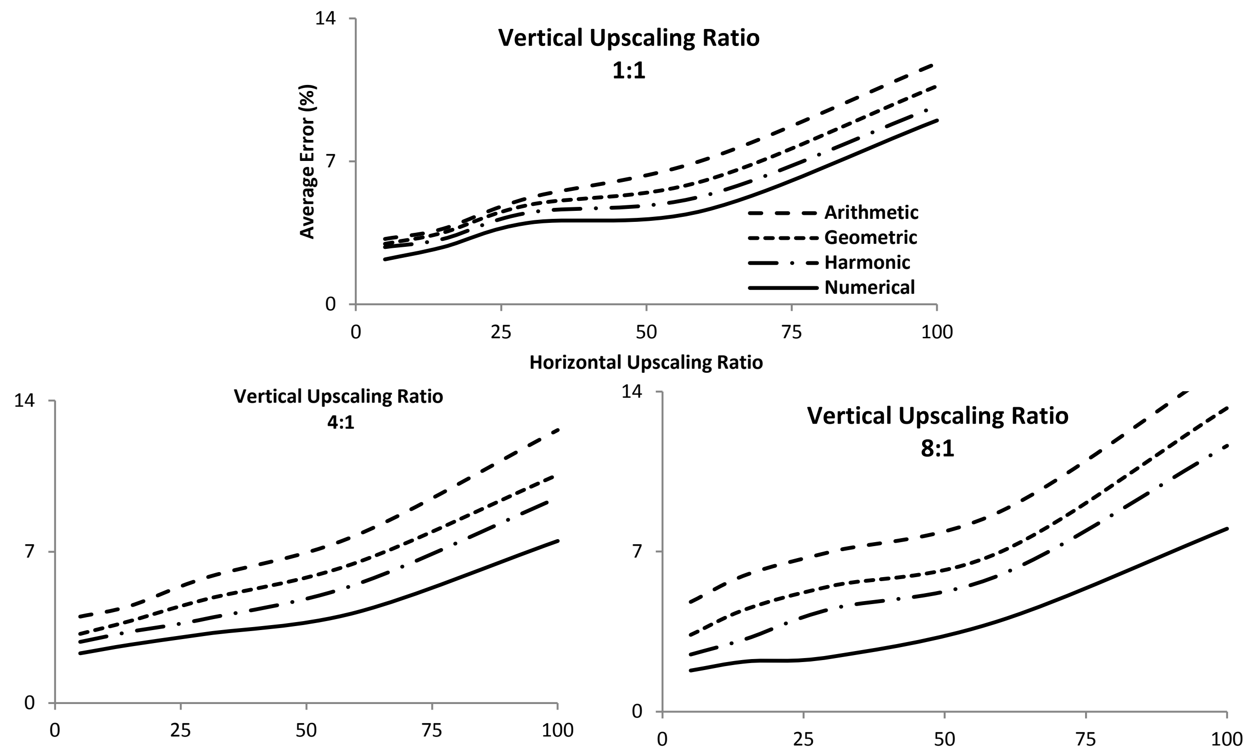


Figure 9. Average error for various horizontal and vertical upscaling ratios