Estimating Axial Point Data

Maksuda Lillah and Jeff Boisvert

Orientations of a geological object of interest can be obtained from direct angle measurements, from studying outcrops, exposed underground working or measuring formation dip with a dipmeter or similar equipment. Often the orientations of properties of interest such as grade and rock types follow these main deposit scale orientations. Incorporating this information in geostatistical modeling in the form of locally varying anisotropy (LVA) has been a subject of past research. Complex orientations can be incorporated into modeling if exhaustive LVA fields can be determined from these point measurements. A method to generate an exhaustive field of orientations from point measurements is presented. This is difficult as orientation data in geostatistics is axial and cannot be considered as traditional directional data.

Introduction

Axial or non-polar data requires processing before point samples can be used in estimation methodologies such as inverse distance or kriging. The statistics in handling 2D axial data to generate exhaustive LVA fields by kriging is detailed in Boisvert (2010). However, these measurements are often made in 3D and the typical 'double' the angle technique is not appropriate. Prior to estimating global continuity, the nature of 3D axial data must be addressed.

Data representing axes cannot be directly modeled for estimation as the angles are continuous, 0° is indistinguishable from 360°. Much theory of spherical statistics is similar to that of circular statistics as applied to axial data. Each input is defined by a strike (θ) and dip (Φ) angle and is decomposed into spherical polar coordinates. The direction X is then:

$$X(z,x,y) = (\cos\theta, \sin\theta\cos\Phi, \sin\theta\sin\Phi)$$

Using kriging to interpolate an exhaustive LVA field would give erroneous results as points with strike 45° and 225° are deemed opposite whereas they lie in the same axis and as far as considering a major direction of continuity in geostatistical algorithms, the angles are identical. Consider three sample points being used to estimate at an unknown location (Figure 1). The proposed technique is to determine which axial orientation would be consistent for all three locations. There are a total of 3 different situations (Figure 1). The variance of the angle data can be used to determine which of the three orientations should be averaged (Equation 2). As indicated on Figure 1, the combination where location u_3 is 'flipped' has the minimum variance (Table 1). It is important to distinguish this angle variance from the typical kriging variance used in estimation. This angle variance is only used to determine which orientation results in the most consistent estimate for a given location. For any number of conditioning data, the aim is to determine the combination of data that results in the minimum variance of the angles. Because estimation proceeds sequentially, from location to location the input data is not significantly different when reasonable search ranges are selected. This will allow for the quick calculation of appropriate sample orientations if the orientations used in the previously estimated location are known.

Consider data on S^3, $x_1, \dots x_n$ the sample mean is :

$$\bar{x} = \frac{1}{n}\sum x_i \qquad (1)$$

Where, r $\bar{x}_0$ *a unit vector and* $\bar{R} \geq 0$, $\bar{R} = \|\bar{x}\|$ $\quad \bar{x}_0 = \|\bar{x}\|^{-1}\bar{x}$

The mean resultant length has the following minimizing property, where *S(a)* is the arithmetic mean of the squared Euclidean distance between x_i and ***a***.

$$\begin{aligned} S(a) &= \frac{1}{n}\sum \|x_i - a\|^2 \\ &= 2(1 - \bar{x}^T a) \\ &= 2(1 - \bar{x}_0^{\,T} a) \qquad (2) \end{aligned}$$

S(a) is minimized subject to ($a^Ta=1$) ,

$$when\ a = \bar{x}_0$$

$$\min_a S(a) = 2(1-\bar{R}) = \sigma_{sp}{}^2 \qquad (3)$$

The spherical variance is computed for all combinations of orientations of the surrounding conditioning data found within the estimation search. As an illustration, assume we are using 3 nearby samples for estimation (Figure 1) denote the input axial orientation as 0 and the 'flipped' or opposite orientation 1. The combinations that need to be considered are:

Table 1: Angle variance for all combinations of input data.

u1	u2	u3	Variance
0	0	0	0.60
1	0	0	0.60
0	1	0	0.58
0	0	1	0.01

Cleary the sample at location u_3 should be rotated and point in a similar direction as the other data, this is reflected in the variance of the angles. Significantly more combinations are required for considering a larger number of conditioning data. The combination with the minimum variance is selected and traditional estimation of the components of x is implemented.

Conclusion

It is not possible to consider the estimation of axial point data in 3D without some constraint on the input data. When the data is 2D, doubling of the azimuth to consider wrapping at 360 degrees is a suitable alternative to converting axial data into directional data. There is no such option for doubling 3D axial data where orientation is measured with strike and dip angles. In 3D, the proposed methodology of dealing with directional data that minimizes the angle variance at each location provides an alternative to considering estimation of axial data.

References

Boisvert, J B. Geostatistics with Locally Varying Anisotropy, PhD (2010). University of Alberta, Canada.

Deutsch, C V, Journel, A G. Gslib: Geostatistical Software Library and User's Guide, Oxford University Press, New York, 2nd Edition, 1998.

Mardia,K , Jupp P. Directional Statistics. John Wiley and Sons, 2000.

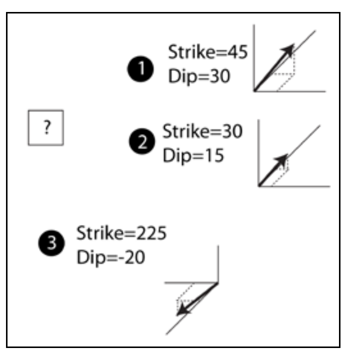

Figure 1: Estimating at an unknown (?) location with three conditioning data located at u_1, u_2 and u_3.