Optimizing Thresholds in Truncated Pluri-Gaussian Simulation

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Truncated pluri-Gaussian simulation (TPGS) is an extension of truncated Gaussian simulation. This method is used to generate facies realizations and define complex contacts between facies. The data are recoded as Gaussian values, simulated and then transformed back into facies by rules which control truncations. The probability of each facies and the transition probability between facies are determined by the truncation rules. The proposed methodology is to start with initial thresholds and optimize them to find the optimal truncation rules. Optimality is defined by an objective function which assesses the transition probabilities between facies while preserving the desired proportions. This paper presents a method to find the optimum values for thresholds when a complex configuration between facies is required. The optimization procedure is illustrated on a synthetic example.

Introduction

Truncated pluri-Gaussian simulation has been used to create categorical models used in typical geostatistical workflows. The method is to simulate one or more continuous Gaussian realizations and truncate them in order to produce a categorical variable. The truncation rules control the resulting features in the realizations. The principle of this method was published in Galli et al. (1994). Statistical inference of the variograms underlying the Gaussian realizations is given in Galli et al. (1994).

The pluri-Gaussian method allows for additional flexibility when deciding on facies interactions. Properties of this method with a view in implementing the algorithm both for practical structural analysis and conditional simulation were studied by Le et al. (1996). Currently this technique is used in the petroleum (Remacre and Zapparolli 2003) and mining industries (Fontaine and Beucher 2006). The following parameters should be defined for implementing truncated pluri-Gaussian simulation (Le Loc'h and Galli, 1994):

1) The number of Gaussian random functions (each of them with zero mean and unit variance).
2) The matrices of covariance and cross-covariance, which fully define the spatial variability model of the Gaussian functions.
3) The threshold mask to transform the set of Gaussian realizations into a unique discrete facies realization.

In general, *M* Gaussian realizations can be used to define the relationship between facies. The relationships between neighboring facies can be more complex than is considered in the standard truncated Gaussian model (Galli et al., 1994). In this work, we propose to optimize the threshold mask in order to match desired statistics such as the transition probabilities between facies, proportions, etc. The inference of these statistics would like come from data, geological interpretations or training images.

Truncation and Thresholds in Pluri–Gaussian

When the number of Gaussian functions is two, truncation is characterized by the partition of the plane defined by the Gaussian realizations. Any number of combinations of facies can be considered. As the relationships between facies become increasingly more complex, the truncation rules have to be flexible to account for the relationships. Here we use M=2 independent Gaussian realizations z_1 and z_2. They have a spherical variogram model, and present North-South anisotropy. Both realizations have a 0 mean and a variance of 1 and three facies (Figures 1 and 2). This model is used as the truth and is sampled randomly; these samples are used to demonstrate the optimization algorithm presented below.

After generating the Gaussian realizations, the next step is to map the realizations to the desired facies. In Figure 2 the initial realizations for z_1, z_2 and the truth pluri-Gaussian realization is shown. The facies variable can take more than two values, more than one threshold is defined, n_f-1 thresholds for n_f possible facies.

Let a point in the simulation domain be defined by x and let 1_{Fi} be indicators of the i^{th} facies Fi. In general case, the following condition is satisfied:

$$1\ Gaussian\, realization \;\rightarrow \{x \in Fi \;\; \Leftrightarrow 1_{Fi} = 1 \Leftrightarrow \; t_{i-1} \le Z(x) < t_i\} \tag{1}$$

$$M\ \ Gaussian\, realization \;\rightarrow \{x \in Fi \;\; \Leftrightarrow 1_{Fi} = 1 \Leftrightarrow \; (Z_1(x), Z_2(x), \ldots, Z_M(x)) \in D_i\} \tag{2}$$

where t_i is the i^{th} threshold and M Gaussian realizations define a space with M dimensions. Let D_i be the subset of the Gaussian space. Here we consider two independent simulated Gaussian realizations and assume n_f=3 of facies and define 2 arbitrary thresholds (Figure 1).

Now we can determine the proportion of each facies at point x. Let G be the standard normal cumulative density function. The probability of having facies Fi at point x can write as:

$$1\ Gaussian\, realization \;\rightarrow p_{Fi} = P(t_{i-1} \le Z(x) < t_i) = G(t_i) - G(t_{i-1}) \tag{3}$$

$$\begin{aligned} M\ \ Gaussian\, realization \;\rightarrow p_{Fi} &= P\{(Z_1(x), Z_2(x), \ldots, Z_M(x)) \in D_i\} \\ &= \int_{D_i} g_{\sum}\ (z_1, \ldots, z_M)\, dz_1 \ldots dz_M \end{aligned} \tag{4}$$

where $g_{\sum}\ (z_1, \ldots, z_M)$ is the M-variate standard Gaussian function with mean 0 and variance 1, $\sum$ is the correlation matrix.

There is a one to one relation between proportions and thresholds. Accordingly, knowing the univariate distribution of the Gaussian realizations, it is straightforward to deduce the set of truncation thresholds that match the desired facies proportions. This is not the case if nonlinear truncation thresholds are considered as well.

Transition probability

Transition probability can be used instead of the indicator cross-variogram as the measure of spatial variability. Transition probability $t_{ij}(h)$ is defined by

$$t_{ij}(h) = \Pr\{j\ \ occurs\, at\, x + h \,|\, i\ \ occurs\, at\, x\} \tag{5}$$

where x is a spatial location, h is the lag (separation vector), and i, j denote mutually exclusive categories such as geologic units or facies ($x \xrightarrow{h} x + h$). The transition probability approach considers all transitions between facies and allows for the possibility of asymmetry, $t_{ij}(h) \ne t_{ij}(-h)$.

Conditional transition probability is probability of being in facies Fj at point x+h knowing that x is in a given facies:

$$\begin{aligned} P(x + h \in Fj \,|\, x \in Fi) = P(1_{Fj}(x+h) = 1 \,|\, 1_{Fi}(x) = 1) &= \frac{P(1_{Fj}(x+h) = 1 \,\&\, 1_{Fi}(x) = 1)}{P(1_{Fi}(x) = 1)} \\ &= \frac{E(1_{Fj}(x+h).1_{Fi}(x))}{E(1_{Fi}(x))} = \frac{C_{ij}(x, x+h)}{P_{Fi}(x)} \end{aligned} \tag{6}$$

The transition probability matrix can be defined as a bivariate probability. Markov chain methods prepare a more general framework and under stationary assumption, the “first-order” Markov chain is considered. In the “first-order” Markov, the chain current state is dependent only upon one previous state. (Krumbein, 1970- Li and Zhang, 2008).

Indicator cross-variogram with indicator variable $I_i(x) = \begin{cases} 1 & if\ category\, i\, occur\, at\, x \\ 0 & otherwise \end{cases}$ defined as:

$$\gamma_{ij}(h) = \frac{1}{2} E\{[I_i(x) - I_i(x+h)][I_j(x) - I_j(x+h)]\} \tag{7}$$

The transition probability in 80 different lags is considered and data are horizontally scanned. In the next step the objective function based on transition probabilities to optimize thresholds is built.

Case study

First, two independent Gaussian realizations (z_1 & z_2) are defined in 2 dimensions (Figure 2). These two realizations have been used as the exhaustive model. When considering two Gaussian distributions, truncation can be defined by the thresholds of the two Gaussian realizations. Figure 1 shows the thresholds considered.

Both realizations are 200x200 blocks of size 1x1. Table 1 summarizes the properties of two Gaussian realizations. These realizations are used to generate the facies realizations with the given thresholds, n_f=3, t1=-0.1 and t2=0.9. The proportion of each facies is given in Table 2.

Table1: Properties of two Gaussian realizations

Simulated	*Nugget Effect*	*Maximum continuity*	*Minimum continuity*
Realization 1	0.1	50	20
Realization 2	0.1	80	10

Table2: Facies proportion of data

Facies	*F1*	*F2*	*F3*
Proportion	0.125	0.207	0.668

Based on data extracted from the exhaustive model, two conditional Gaussian realizations have been generated with a similar variogram (Figure 4). The goal is to determine the thresholds that will best characterize this realization given the input sample data.

The objective function is defined by the sum of the square differences between transition probability of the data ($t_{ij}(h)^d$) and the simulated realizations ($t_{ij}(h)^r$) given a particular threshold. This objective function measures the mismatch:

$$O = \sum_{h}^{nTP} \sum_{j}^{nF} i \sum^{nF} (t_{ij}(h)^d - t_{ij}(h)^r)^2 \tag{8}$$

where nTP is the number of lags and nF is the number of facies. The thresholds values are adjusted to minimize the objective function while preserving the target proportions. The procedure is done for 53 different values in the range of [-3, 3] for both thresholds (Figure 3). The optimum value is obtained by t1=-0.2 and t2=1.1 which are close to the initial thresholds for the exhaustive realization.

For optimizing the threshold, the rock type rule that produces facies by intersecting lines as thresholds has been applied but the goal is to consider an arbitrary facies mask rather than being limited by linear thresholds. This will provide additional flexibility when characterizing the interactions between facies but is difficult to optimize. The first step in this procedure consists of finding the bivariate distributions between two Gaussian realizations, z_1 & z_2 (Figure 5).

The goal would be to consider the optimization of the mask rather than linear thresholds. To obtain the optimum thresholds, initial thresholds of t1=-0.15 and t2=1.1 are selected. Randomly, n bins in the boundaries between facies were selected and changed to the other facies (Figure 6) while honoring the input proportions. A random starting configuration could also be selected (Figure 6). Considering 500 changes per iteration resulted in the optimum objective function (Figure 7). By this method, we are able to find the optimum thresholds in the linear cases.

Conclusion

Truncated pluri-Gaussian simulation of facies has been used to set up the underlying geological model and prepare a good method to deal with complex relationships between facies. The boundaries between different facies on the mask are important. In this paper, a method for optimizing thresholds by changing facies specifications in the mask was proposed. Further testing on more realistic facies configurations is required. The main advantage of this method is its application in determining the facies mask for more complex configuration.

References

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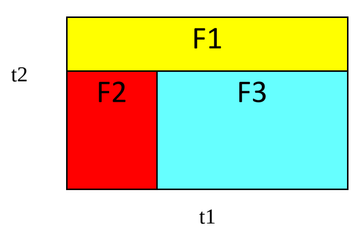

Figure 1: Example of thresholds mask for three facies.

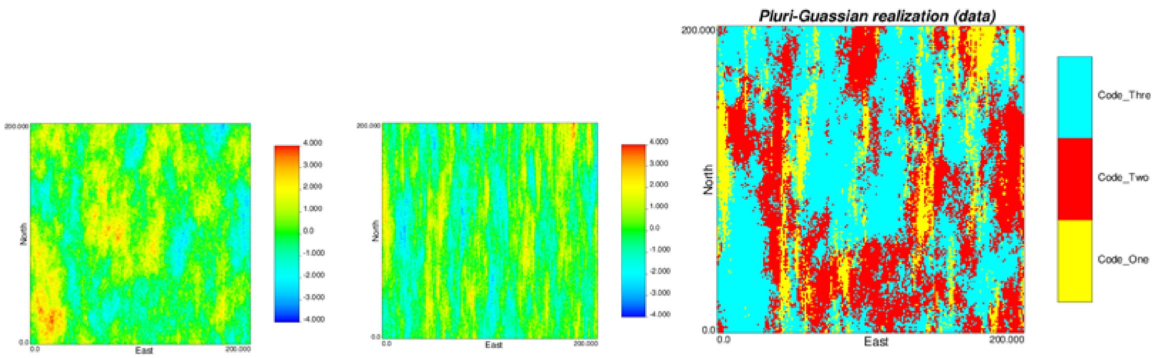

Figure 2: Initial realization z_1 and realization z_2 and pluri-Gaussian facies realization.

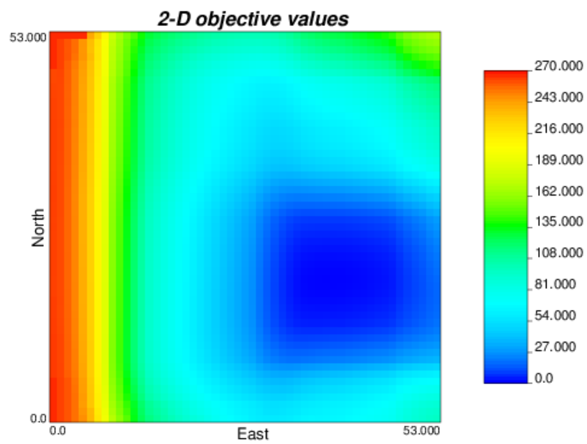

Figure 3: Pixel plot of objective values.

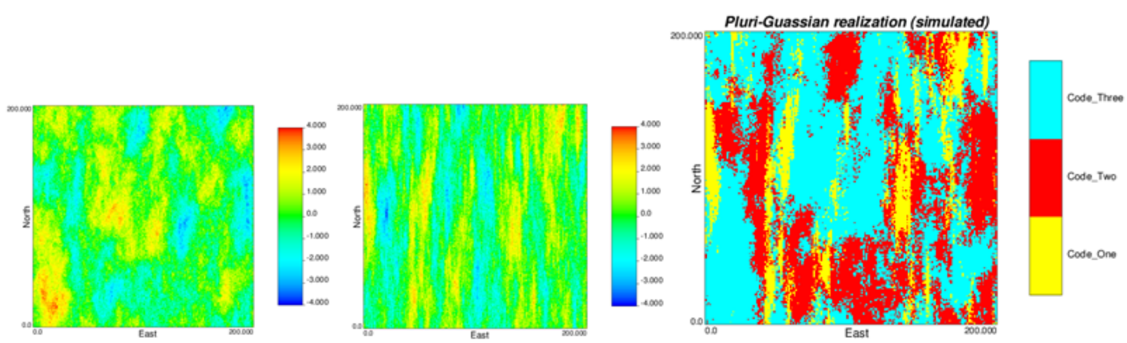

Figure 4: Conditional simulated realization z_1 and Conditional simulated realization z_2 and pluri-Gaussian facies.

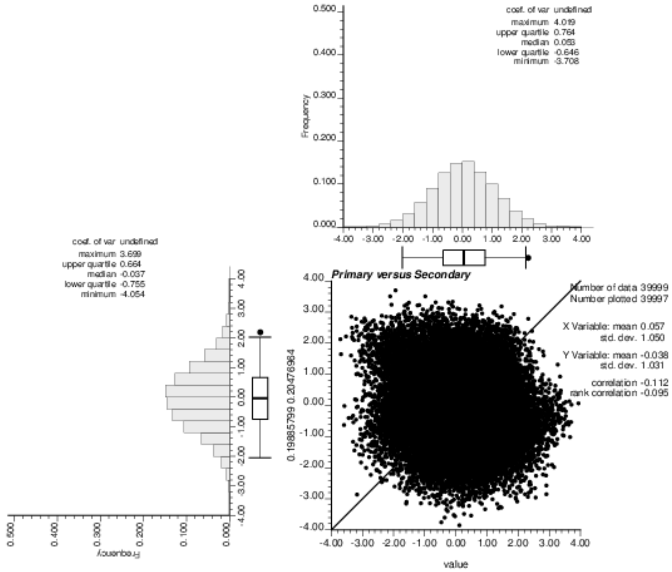

Figure 5: Scatter plot and bivariate distribution of two realizations.

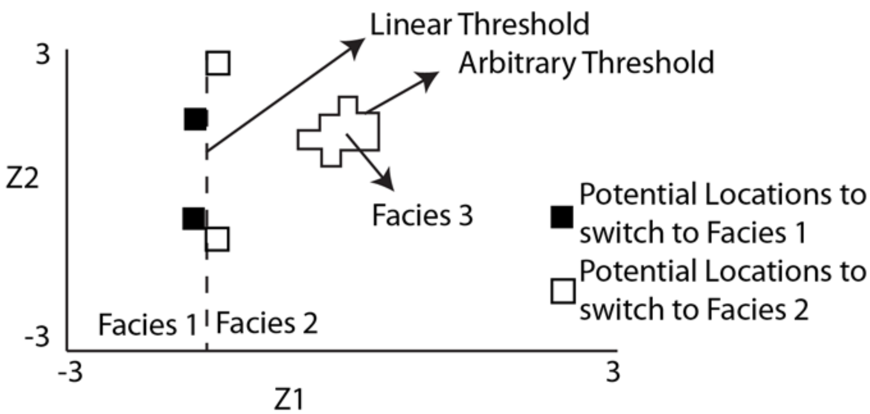

Figure 6: Complex facies mask showing linear and arbitrary thresholds. Potential locations for optimizing the mask are shown as solid and hollow cells.

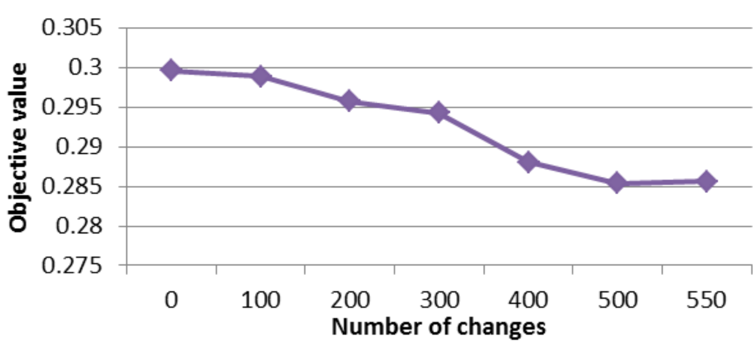

Figure 7: Objective function evaluation based on number of changing in the facies realization.