Proxy Model Based on Butler's SAGD Theory

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Oil sand reservoirs play an important role in the economy of Canada. There are huge reserves in these reservoirs. Due to the large viscosity and low API of oil in these reservoirs, conventional methods cannot be used for producing oil from these reservoirs. For this reason, using Steam-assisted gravity drainage (SAGD) method is an efficient way of producing oil. SAGD is based on the drilling of two horizontal wells and injecting steam from one well and producing from another well. The bottom well is the producer which is close to the bottom of reservoir and the top well is the injector which is 5-10 meters above producer. For efficient SAGD operation, optimizing the trajectory of well is very important. It is desirable to keep the ratio of steam injection to oil production as low as possible. For optimizing well trajectory, reservoir simulation should be called several times. The CPU time of reservoir simulation is significant. For only one well pair the running time is about one day. By considering uncertainty and running different models, running time would be much larger too. For this reason, finding a proxy model that reasonably predict oil and water production is essential. In this paper, a real $3D$ *well pair has been considered. Top gas and top water are existed in this reservoir and made it a complicate example. This paper showed that there is a good match between proxy and simulator results.*

Introduction

Steam-assisted gravity drainage (SAGD) is an efficient method for producing heavy oil by steam-flooding. In this method, two horizontal wells should be drilled parallel to each other. Bottom well is production well and it should be drilled close to the bottom of reservoir. Top well is injection well and it should be drilled 5-10 meters above the production well. Length of each well is around 1 kilometer. Steam from top well should be injected to the reservoir to increase temperature of bitumen between wells and also above injector. This mechanism decrease bitumen viscosity and bitumen can be produced by gravity forces. Also process can be began by injecting steam into both of wells for 1-3 months in order to heated bitumen enough to start flowing to the lower wells, then injecting steam to the producers should be stopped and it should be injected only to the injector. During SAGD, cone shaped steam chamber moves to upwards during rising period and then will be moved horizontally during spreading period. Fig. 1 shows development of steam chamber by injecting steam.

In this method, amount of oil production depends highly on the efficiency of steam injection. Usually in these reservoirs there are shale barriers in different layers. If steam reaches to the shale layer, it cannot move upward any more. Also if there is a shale barrier between injector and producer, oil cannot be produced through producers. For this reason, location of producer and injector is very important in this process. Robust optimization should be done by considering uncertainty of static properties in the reservoir. Finding reservoir simulation model and trying different well trajectories for finding optimal value of oil production is not efficient. Due to the large computation time, it is almost impossible to find optimal location in a reasonable time. Even by considering uncertainty in the reservoir model, running time would be much more than using a single model. Finding a proxy model for fast modeling of SAGD process is a good option. In this paper, we used Butler theory for modeling SAGD process. Using this method, running time can be decreased significantly. Because running time of proxy is much less than flow simulation, considering uncertainty is possible and optimization can be done in a reasonable short time.

Butler theory

In this theory, only location of producer can be considered. As we talked before, location of injector is 5-10 meters above producer. Location of producer is close to the bottom of the reservoir and its trajectory can be optimized after finding a reliable proxy. In this theory, steam chambers grow to the top of the reservoir and then spread sideways. Using this theory (Butler 1987; Butler 2000), location of interface, rate of heat penetration and also oil production from different segments of interface can be found. For this reason, equations should be written for a small segment of the interface as shown in Fig. 2.

Using Darcy law, flow within an element with width of $d\xi$ can be written as:

$$dq_o = \frac{kg\sin\theta}{\nu_o}$$

In this equation q_o is rate of oil drainage ($\frac{m^3}{s}$), k is permeability (m^2), g is earth gravity (9.81 $\frac{m}{s^2}$), θ is angle of element with horizontal direction and ν_o is oil kinematic viscosity ($\frac{m}{s^2}$). For unheated reservoir with temperature T_r the corresponding equation is:

$$dq_r = \frac{kg\sin\theta}{\nu_r}$$

As a result, difference between dq_o and dq_r can be written by dq. After integration of dq the following equation obtained:

$$q = kg\sin\theta\int_0^\infty\left(\frac{1}{\nu}-\frac{1}{\nu_r}\right)d\xi.$$

Butler mentioned that for finding the following integral, viscosity should be defined as a function of distance from interface. He showed that the following relation exists between temperature and distance from interface:

$$\frac{T-T_r}{T_s-T_r} = e^{-U\xi/\alpha}$$

In this equation U is velocity ($\frac{m}{s}$) and α is thermal diffusivity ($\frac{m^2}{s}$).
As a result, for finding relation between viscosity and distance from interface, only relation between viscosity and temperature should be defined. He showed that the following relation exists between viscosity and temperature:

$$\frac{\nu_s}{\nu} = \left(\frac{T-T_r}{T_S-T_r}\right)^m$$

As he mentioned, parameter m has a constant value between 3 to 4. Also it can be found by solving the following equation (Butler 1987):

$$m = \left[\nu_s\int_{T_r}^{T_s}\left(\frac{1}{\nu}-\frac{1}{\nu_r}\right)\frac{dT}{T-T_r}\right]^{-1}$$

As a result:

$$\int_0^\infty\left(\frac{1}{\nu}-\frac{1}{\nu_r}\right)d\xi = \frac{\alpha}{U}\frac{1}{m\nu_s}$$

Using this relation, oil drainage flow at a point on the interface as a function of velocity U and the angle θ can be found from the following equation:

$$q = \frac{kg\sin\theta}{m\nu_s U}$$

He showed that by combining this equation and material balance equation the following equation for oil drainage flow can be found:

$$q = \sqrt{\frac{2\phi\Delta S_o kg\alpha h}{m\nu_s}}$$

In this equation ϕ is porosity, ΔS_o is recoverable oil saturation and h is height of reservoir. Also position of interface in horizontal and vertical directions can be found respectively from the following equations:

$$x = t\sqrt{\frac{kg\alpha}{2\phi\Delta S_o m v_s (h-y)}}$$

$$y = h - \frac{kg\alpha}{2\phi\Delta S_o m v_s}\left(\frac{t}{x}\right)^2$$

Also he showed that by writing differential heat equation and considering conduction heat and heat which is left behind the front, rate of heat accumulation ahead of front can be found easily. Then he showed that when temperature gradient varies linearly, rate of heat penetration can be found using the following formula:

$$\frac{d\gamma}{dt} = \frac{2}{\pi}\left(\frac{\alpha}{\gamma} - U\right)$$

In this equation, γ is degree of heat penetration.
Usually dimensionless variables have more applicability. For this reason, it is better to work with these types of variables instead of original units. For this reason, dimensionless variables can be defined as below:

$$x^* = \frac{x}{h}$$

$$y^* = \frac{y}{h}$$

$$\gamma^* = \frac{\gamma}{h}$$

$$Q^* = Q/\sqrt{\frac{kg\alpha\phi\Delta S_o h}{m v_s}}$$

Also Butler defined another dimensionless parameter for ease of using in some of equations. This parameter is B_3 and can be defined as:

$$B_3 = \sqrt{\frac{kgh}{\alpha\phi\Delta S_o m v_s}}$$

Using this parameter dimensionless oil flow rate would be:

$$Q^* = \frac{Q}{\alpha\phi\Delta S_o B_3} = \gamma^* B_3 \sin\theta$$

He found dimensionless time so that the product of dimensionless flow and dimensionless time gives the dimensionless area of reservoir which is $A^* = \frac{A}{h^2}$. As a result:

$$t^* = \frac{B_3 \alpha t}{h^2}$$

Also dimensionless rate of heat penetration can be defined as:

$$\frac{d\gamma^*}{dt^*} = \frac{2}{B_3\pi}\left(\frac{1}{\gamma^*} - B_3 U^*\right)$$

Also dimensionless velocity is can be obtained from the material balance equation:

$$U^* = -\left(\frac{\partial Q^*}{\partial L^*}\right)$$

Using above equations, rate of heat penetration, drainage oil flow and location of front by changing time can be found easily.

Although finding front location and calculating oil drainage using this method is very important, but for different applications like production optimization, finding amount of water production and as a result cumulative steam oil ration (CSOR) is very important. Different authors considered different methods for finding CSOR. Rose (1993) and Butler (2000) considered an efficient method for estimating steam oil ration.

Bulter (2000) mentioned that for predicting steam production, having this information is necessary:

1- Cumulative heat to the steam chamber and produced oil from T_r to T_s:

$$Q_c^* = \int_0^{t^*} Q^* dt^* = \frac{Q_c}{h^2 C_r \rho_r (T_s - T_r)}$$

2- Cumulative heat loss to over burden above steam chamber:

$$Q_o^* = \frac{(4/3) k_o (T_s - T_r) A \sqrt{t/\pi\alpha_o}}{h^2 C_r \rho_r (T_s - T_r)}$$

3- Cumulative heat to the reservoir:

$$Q_r^* = \int_0^{l^*} \gamma^* dl^* = \frac{Q_r}{h^2 C_r \rho_r (T_s - T_r)}$$

In above equations, k_o and α_o are thermal conductivity and thermal diffusivity of overburden. Also A is area of hot zone at time t.

Rose (1993) mentioned for estimating steam oil ration, the following equation can be used:

$$\text{CSOR} = \frac{\text{total heat transferred/unit volume of oil}}{\text{enthalpy of steam/unit volume of water}} = \frac{(T_s - T_r)(C_r \rho_r Q_c^* + C_r \rho_r Q_r^* + C_o \rho_o Q_o^*)}{Q_c^* \Delta H_w \phi \Delta S_o}$$

This formula is based on these three heats that butler mentioned.

This model assumes a vertical fracture running above and along the vertical wells and connecting the overburden to the wells. This assumption is not realistic. As another limitation, this model assumes producer is in the unconfined reservoir. Butler showed that when there is no flow boundary after interface reached to the boundary, its direction should be changed and moves downward. This changing in the direction can be shown using the following formula:

$$\left(\frac{\partial Q^*}{\partial x^*}\right) = \left(\frac{\partial y^*}{\partial t^*}\right)$$

But before reaching to this no-flow boundary, the following equation should be used for finding interface location:

$$\left(\frac{\partial Q^*}{\partial y^*}\right) = -\left(\frac{\partial x^*}{\partial t^*}\right)$$

Also when there are adjacent well, half of the spacing between them acts as a no- flow boundary and direction of interface should be changed and move along vertical downward direction.

For these reasons, Butler proposed considering two different periods.

1- Rising period
2- Spreading period which includes before reaching to no-flow boundary and after that

For eliminating vertical fracture above and along the vertical wells rising period should be considered. In this period, Butler assumed a chamber has the shape of a section of circle with the center of circle at the production well. Side of this chamber is a straight line with angle of 58° with horizontal. For this period, Rose showed that another series of equation should be used for calculating dimensionless flow rate and also dimensionless cumulative heat to the stem chamber and producing oil:

$$Q^* = 1.5{t^*}^{1/3}$$

$$Q_c^* = 1.125{t^*}^{1.333}$$

Also dimensionless height of steam chamber can be found from the following formula:

$$h^* = 2{t^*}^{2/3}$$

Next period is spreading period. All equations that we mentioned before are valid before interface reaches to the no-flow boundary. After that for calculating heat lost to the overburden the following equation should be used:

$$Q_o^* = \frac{(4/3)k_o A(T_s - T_r)[t^{1.5} - (t - t_l)^{1.5}]/\sqrt{\pi\alpha_o}}{h^2 C_r \rho_r (T_s - T_r)}$$

In this formula, t_l is the time that interface reaches to the no-flow boundary.
For finding the time of changing period from rising to spreading, Butler (2000) proposed the following solution. He mentioned that this time is a time that plot of oil production rate versus recovery for both of rising and spreading chambers intersect with each other. Fig. 3 shows this time.
This method assumes there is no relationship between different $2D$ sections along well length. For this reason oil and water rate for different sections should be summed together to find the final oil and water production. The following procedure can be used for finding front position and calculating oil drainage:

1- Assume a small non-zero value for γ^* and assuming $\theta = 90^{\circ}$.
2- Finding oil production rates for different time steps by assuming spreading period
   a) Finding $Q^* = \gamma^* B_3 \sin\theta$
   b) Finding interface velocity $U^* = -\left(\frac{\partial Q^*}{\partial L^*}\right)$
   c) Finding new γ^* from $\frac{d\gamma^*}{dt^*} = \frac{2}{B_3\pi}\left(\frac{1}{\gamma^*} - B_3 U^*\right)$
   d) Finding new Q^* and new position
3- Finding oil production rates for different time steps using rising period equations
4- Finding transition time between rising and spreading periods
5- Adjusting production rates
6- Calculating CSOR

Modifying proxy
In 2008, Jose Valter Vanegas added different options to this proxy for considering heterogeneity and also adjusting different parameters. The options that he added to the code are:
1- Considering heterogeneity in two ways:
   a) Calculating average of different parameters like porosity, permeability, saturation and diffusivity coefficient along the interface for different segments to have better approximation of parameter B_3. Using this method, different realizations for considering uncertainty can be considered.
   b) Defining effective volume factor for considering permeability heterogeneity in a better way. During steam rising, if permeability is very low, steam cannot pass through shale layers and oil production

from that segment could be stopped. This factor can be defined as ratio of vertically connected porous volume to the overall vertical section pore volume.

$$EVF = \frac{\sum_{i=1}^{nb_h} \sum_{j=1}^{nb_{vsi}} VP_{ij}}{Total\ VP}$$

2- Calculating average oil relative permeability for modifying permeability value. Because oil drainage should be calculated, oil relative permeability should be considered in parameter B_3. What he did was assuming a value for oil relative permeability in the first time step and multiplying it by average permeability of segment. For next time steps he calculated fraction of water and then read corresponding value of oil relative permeability from relative permeability table. If relative permeability table is not provided, correlation can be used for finding oil relative permeability.
3- Adjusting different parameters in the proxy. He showed that different parameters in the proxy should be adjusted in order to get a better match between proxy and reservoir simulator. Four parameters that he selected for adjusting proxy:
   a) Adjusting factor for oil production during rising period.
   b) Adjusting factor for oil production during spreading period.
   c) Adjusting factor for permeability.
   d) Adjusting factor for CSOR.

The first two factors can shift oil production during different period linearly. Adjusting parameter for permeability is for considering geomechanical effect. Due to heating the reservoir, permeability may be increased after some time. Also CSOR should be adjusted because steam temperature at production line is unknown and it can changes steam enthalpy and as a result CSOR. He used simulated annealing for adjusting this factor by minimizing misfit of oil and CSOR curves between simulator and proxy.

Adding new options to the proxy

In this work, different options have been added to the proxy for making it possible to predict oil and water productions of realistic models. These options are:

1- Considering wells with non-horizontal well trajectory.
2- Modifying effective volume factor coefficients. In this definition, if fraction of shale barrier in the horizontal direction is greater than the predefined fraction, proxy assumes that steam cannot pass through it and production from that section should be stopped. In SAGD, effect of laminated shale is significant.
3- Using different number of facies and considering one relative permeability table for each facies and also calculating relative permeability in a more efficient way. In this method, oil relative permeability for each cell along the interface can be computed and then multiplied by grid permeability for finding more realistic average of oil relative permeability. Because different facies can be used, also different relative permeability tables can be used and relative permeabilities are more realistic. Relative permeability tables can be copied from CMG data file to the proxy data file directly.
4- Using different PVT regions and as a result different PVT tables. Usually temperature and different PVT properties will be changed by changing depth. This has effect on finding oil viscosity and m in the B_3 parameter.
5- Using different rock thermal properties. Thermal rock types can be found from shale volume in each grid. Volume of shale can be found using different correlations from effective porosity values. Shale volume can be defined using the following formula. Then for different ranges, different thermal properties can be defines (CMG user manual).

$$V_{sh} = 0.8969 - 3.3681\phi_e + 2.7129\phi_e^2$$

6- Effect of pinchout, gas zone and water zone can be considered in this proxy. Pinchout has very low permeability and they are important in optimization. Also gas and water zones have significantly higher conductivity than oil zone and they cause increasing heat lost to the overburden. Also when steam chamber has contact with cold water zone, heat loss will increase significantly, and condensed steam produces through producing well. For this reason, in most of time steam, amount of injected steam and produced steam are equal. This effect should be considered; otherwise matching CSOR would be very difficult or even impossible. Fig. 4 shows a case that proxy oil production matched with simulator oil production very well, but proxy CSOR decreased at the late times significantly and this is effect of water and gas zone at the top of reservoir.

7- Adding another important optimization parameter is another option that has been added to the code. Adjusting steam chamber pressure is very important. If steam chamber pressure is high, interface move very fast and reaches to the boundary in early times. In this case, matching proxy results with simulator is very difficult. Also if thermal diffusivity is very small. Front moves slowly and cannot produce much of the oil in that place. User in the parameter file can select which parameters are better to be adjusted during optimization and leave other parameters unchanged.

8- For adjusting parameters, a robust optimization algorithm is critical. Without having a good optimization algorithm finding a good fitting is very difficult. Simulated annealing is a slow method and its efficiency is not very good. I added 3 different optimization algorithms to the code and user can select which one is better for adjusting. These algorithms are 1- Sequential quadratic programming (SQP) 2- Differential evolution 3- Particle swarm. SQP is a very robust constrained optimization algorithm. This method is the most efficient gradient-based method. SQP algorithm modified a little bit to prevent trapping in the local minima. Also differential evolution and particle swarm are very useful for optimizing multidimensional problems and because they are not based on the gradient computation, objective function can be non-differentiable, but in the SQP, objective function should be twice differentiable. In solving this optimization problem, objective function is square of mismatch between results of proxy and simulator. Also constraints should be defined for having acceptable value for these parameters.

Case Study

For case study, a real $3D$ SAGD model which is located in the Canada has been selected. This model has grid dimension of $49 \times 49 \times 83$. Grid size in y and z directions are $1\ m$ and in the x direction is $22\ m$. There is pinchout at the bottom of the reservoir. Also top water and top gas existed above the oil zone. 21 PVT regions existed in this reservoir. Also 5 different facies existed in this reservoir with 7 different thermal rock types. Each facies has a separate relative permeability curve. Fig. 5 shows horizontal permeability of the reservoir in two different sections. Also well trajectories can be seen in the right figure.

As can be seen in Fig. 5a, permeability in top part of reservoir is significantly less that the net part of reservoir. This part is related to the water zone and on the top of the reservoir there are about 5 layers related to the thin gas zone. Also well trajectory is not horizontal and there is a little deviation in different directions. Also Fig. 6 shows one slice of porosity and oil saturation in the XZ direction.

Obviously in the water zone you cannot see too much oil and porosity of this zone is much less than the porosity in the oil zone. In this example, wells are not at the middle of formation in y direction.

In this example, different sections have been considered separately. Then results of different sections added together for finding cumulative oil and cumulative steam oil ratio. Fig. 7 shows interface position for one of the slices at different time steps.

Also Fig. 8 shows comparison between results of proxy and CMG before and after adjusting parameters. For adjusting, five different parameters optimized 1- Adjusting factor for rising period 2- Adjusting factor for spreading period 3- Adjusting factor for permeability 4- Adjusting factor for CSOR 5- Adjusting factor for steam chamber pressure. SQP algorithm has been used for optimizing parameters. After finding approximate match, perfect match can be found by changing parameters manually.

Running time for finding proxy was about two hours and running time for optimized proxy is about 90 seconds which is much smaller than the simulator running time which is about than 1 day.

Conclusion

In this paper, a real $3D$ model has been considered. Top gas and top water existed in this reservoir and make it a complex example. Higher thermal conductivity of these zones and also excessive heat lost due to the contact between steam and cold water have been considered for better matching CSOR. Five adjusting factor has been considered. They were 1- Adjusting factor for rising period 2- Adjusting factor for spreading period 3- Adjusting factor for permeability 4- Adjusting factor for CSOR 5- Adjusting factor for steam chamber pressure. SQP algorithm has been used for optimizing these parameters. Finding optimal parameters took about two hours. Results showed a very good match between proxy and simulator results. Running time of optimized proxy is about 90 seconds which is much less than the running time of reservoir simulator which is about 1 day. This proxy can be used for predicting oil and water productions or optimization by changing model properties or well trajectories.

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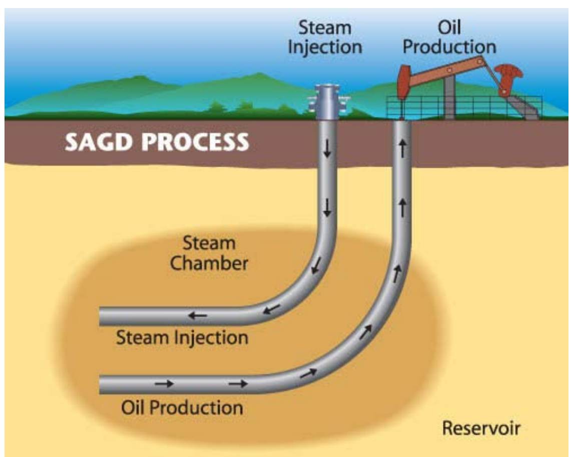


Figure 1: Development of steam chamber by injecting steam in SAGD process (source: www.centreforenergy.com)

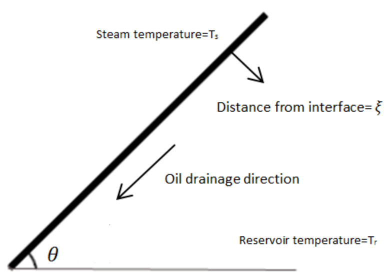


Figure 2: Small vertical section through interface

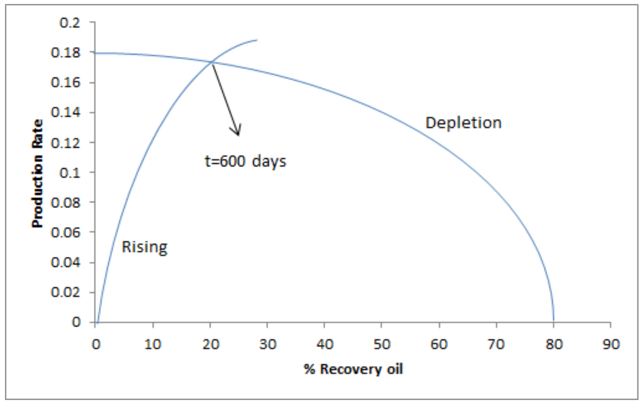


Figure 3: Finding the time for changing from rising to spreading period

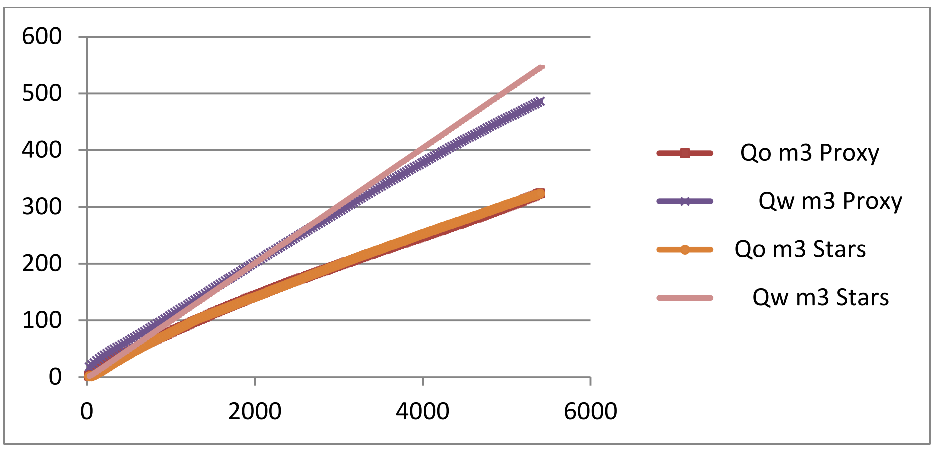
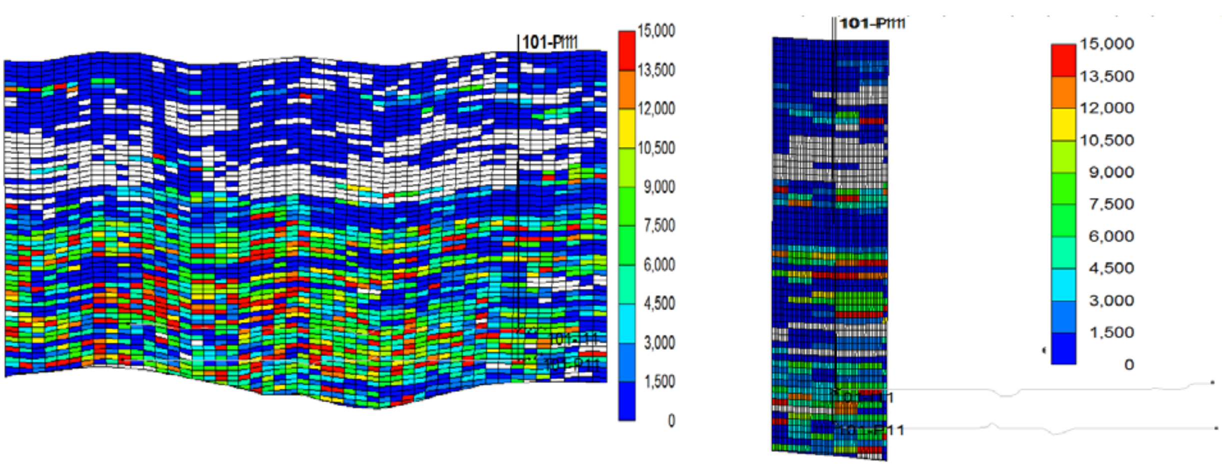
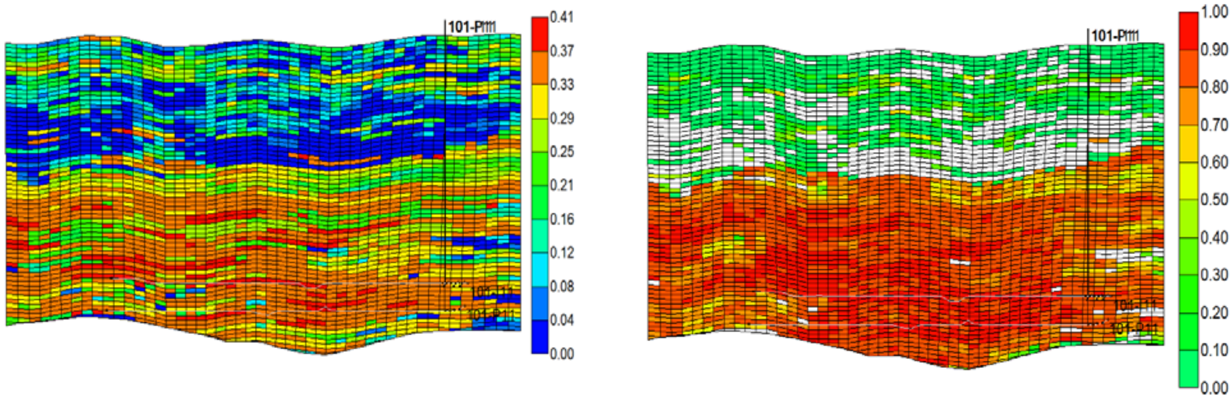


Figure 4: Decreasing proxy CSOR due to presence of water and gas zones



(a) XZ slice (b) YZ slice

Figure 5: Horizontal permeability (md)



(a) Porosity (fraction) (b) Oil saturation (fraction)

Figure 6: Porosity and oil saturation for one slice in XZ direction

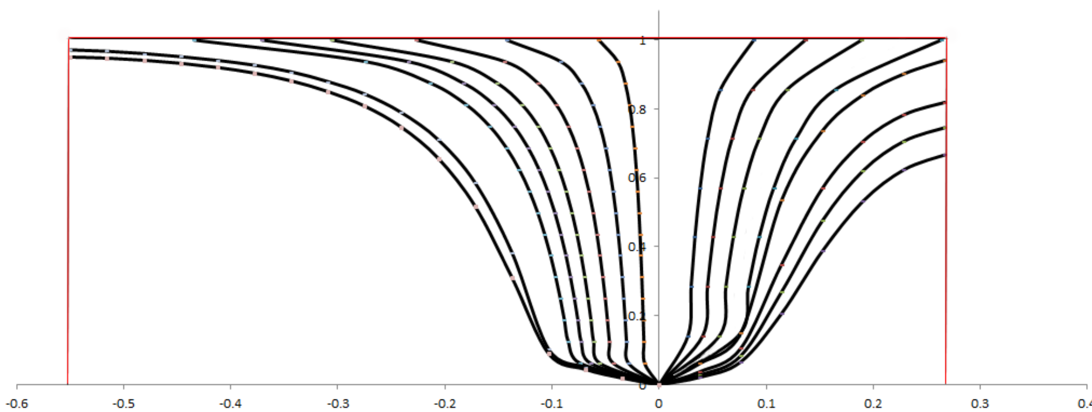
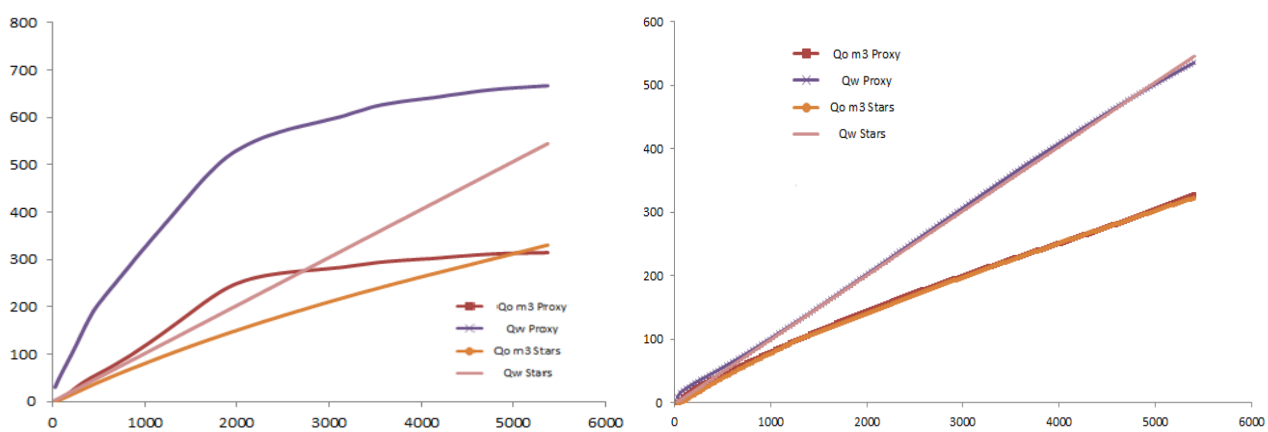


Figure 7: Interface position for one of the slices at different time steps



(a) Before (b) After

Figure 8: Comparison between results of proxy and CMG before and after adjusting parameters