Paper 212, CCG Annual Report 14, 2012 (© 2012)

Calculating the Base of Net Bitumen for SAGD Reservoir Developement

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The most common geostatistical approach to well placement in steam-assisted gravity drainage (SAGD) is three dimensional (3-D) modeling of the reservoir. This study makes use of one dimensional well log data or columns from a geomodel to calculate the location of the production well. Achieving a high probability of good recovery and resource utilization are the main aims of the placement. Information provided by well log data is used to predict the lateral extent of the barriers in order to understand their influence on the drainage recovery process. This work utilizes Monte Carlo simulation to quantify the probability that a vertical thickness of a non-net is in fact a horizontal barrier. Most of the uncertainty in predicted oil-in-place results from the inherent uncertainty in the barrier size (non-net connectivity). Based on the understanding of the barrier extension, the well location is calculated to maximize recovery and resource utilization.

Background

Steam-assisted gravity drainage was originally proposed by Roger Butler in 1978 and soon after has been commercialized by the oil sands industry. The technology has resulted in a 40-fold increase in Alberta bitumen reserves estimate (McLennan et al., 2006). The reservoirs considered in this work for recovery assessment are the so-called ``good" reservoirs, containing 70-80% net-to-gross (NTG) ratios (e.g. McMurray formation which contains 140 billion cubic meters oil sands, which accounts for 20% of the oil reserves in Canada) (McLennan and Deutsch, 2003). In SAGD, the final recovery highly depends on the spatial distribution of the reservoir heterogeneities, as well as the vertical and horizontal well-placement. This necessitates detailed information of the reservoir which carries uncertainty due to the small scale of some heterogeneity. During the past decades, much work has been done on 3-D modeling of reservoirs, with the aim of achieving optimal well placement in SAGD.

Well-placement and planning is a crucial decision-making process; the most common practice is goestatistical 3-D modeling of the region before reservoir assessment is made. The 3-D modeling contributes by transferring the present (geological) uncertainty into the model. Methods have been developed to explain the geological distribution. However, most of these methods are computationally intensive as they consider numerous data with large amounts of variations (such as comprehensive flow simulations (Ballin et al., 1992)). In contrast, our approach in this work is to understand the uncertainty through one-dimensional data (e.g. based on a single set of well log data). Although the analysis is more computationally effective, it does not remove uncertainties due to limited knowledge. One strategy of 1-D analysis is to utilize correlations such as those between the thickness and the areal extension of the barriers to better assess the spatial distribution.

The ultimate goal in any reservoir study is to maximize economic recovery. The net-to-gross ratio in the reservoir is just one parameter for recovery determination; there are other characteristics that can significantly influence the final recovery. The continuity of the barriers, their positions, orientations, tortuosity, and vertical depths (from the surface of the reservoir) can strongly affect the flow rate, well placement and recovery. To consider each one of these properties in recovery analysis will quickly result in computational complexities that are unmanageable. With limited knowledge, parameters such as tortuosity would not be provided in the one dimensional well-data. Reservoir management and recovery quantification are still reasonable even though some parameters are simplified. In this paper, we first outline the 1-D methodology using a Monte Carlo technique. We will then apply this simplified approach to a case involving two barriers of known depths and thicknesses. The lateral extents (H_b) and positions (l_c) of these barriers are, however, allowed to have statistical variations. The uncertainties in H_b and l_c will be incorporated into the final estimation of oil recovery -- a quantity that we will call the *statistically-averaged* recovery. Later, the variations of recovery at different locations will be evaluated and decision-making under uncertainty will be discussed.

One-Dimensional Analysis

This analysis is for estimating the expected recovery from a reservoir based only on limited knowledge of the depths and the thicknesses of barriers at one specific location (e.g. data from a single borehole). In this approach, the recovery estimation is carried out by assuming a simplistic view that the barriers can have random lateral extent as shown in Figure 1. We will assume that barriers that exist at any depth will have a finite probability of being detected by the borehole. The probability of detection will depend on the lateral position l_c and lateral extent H_b of the barriers, both of which are treated here as random variables. The other two variables associated with the barriers, namely the depth z and the thickness t, are assumed known with no statistical variations.

To determine recovery, we will assume that a production well placed at a given depth is capable of recovering the oil above it, except for the volume that is situated directly above the barrier. We also assume that the oil is distributed evenly over the net region, and so the recovery could be represented by the volume of the net region that is accessible to the production well (i.e. region that is not blocked by barriers). We will denote recovery by the symbol R, and since R is represented by a volume, it will have units of m^3. According to these assumptions, a typical plot of the recovery R versus the well depth z will appear as in Figure 2 which also shows the corresponding 1-D representation of the model region.

Clearly, well placement should be made at a depth that corresponds to maximum recovery R. Note that as the production well depth reaches a level which coincides with the top of a barrier, there will be a sudden decrease in production as oil drainage will be blocked, thus leading to a ``step back" as shown. The magnitude of this ``step back" will be determined by the average size of the barrier which, in a statistical sense, is equivalent to the *probability* of the barrier (as will be explained later). Because of the ``step backs," the point of maximum R is not necessarily at the bottom of the model. Such case is depicted in Figure 2.

The R vs z plot can be understood as follows: Starting from the top of the reservoir (at $z = 0$), the recovery increases linearly with the well depth. This is because we have implicitly assumed a constant model cross sectional area that is given by H_m. As such, the volume accessible to the production well would just be proportional to the depth. This continues until the well reaches the top surface of a barrier, at which point the accessible volume decreases abruptly because the oil above the barrier can no longer be recovered by gravity drainage. The amount of this ``step back" to the left depends on the areal extension of the barrier. To keep the scheme simple, the amount of recovery stays unchanged from the top of the barrier to the bottom, assuming no production well would be placed over the extent of the barrier thickness. As the production well clears the bottom of the barrier, the recovery will again go up linearly, with the slope being inversely proportional to the corresponding cross sectional area. This process continues until the bottom of the model is reached.

We now need to evaluate the amount of ``step back," which reflects the amount of inaccessible oil due to the size of the barrier. As explained earlier, this barrier size is more properly interpreted as a *probability* of occurrence of a barrier. This probability is calculated as follows: As shown in Figure 3, a barrier is characterized by its thickness t, lateral extend H_b, and the lateral position of its centroid l_c (measured from the left).

The variables t and H_b are correlated as shown in Figure 4. As can be seen, the correlation is a linear line given by the relation $y_{fit} = 40x + 10$ which minimizes $\sum(y - y_{fit})^2$, where $\sum \Delta y = 0$. Every lateral extension H_b is assumed to have a normal distribution around its mean ($m = y_{fit}$). Note that any relationship can be used. We tried this relationship from some training image experienced. A relationship which can capture the non-linearity structure could be more effective in final evaluation of recovery. The standard deviation for every barrier (Δy_i) is

$$\sigma = \sqrt{(y_i^2 - \Delta y^2)/(N - 1)},$$

which is about 30 for this particular data. The quantity H_b is treated here as a Gaussian random variable with mean value m and standard deviation σ.

The lateral position l_c is another random variable that we assume here to be *uniformly distributed*, i.e. it is equally likely to take on any value between 0 and H_m. Next, we determine the probability of occurrence of a barrier. A barrier ``occurs" if it crosses the centreline of the model and is detected by the borehole. This (indicator = 1) will happen if

$$\begin{cases} l_c \leq H_m/2 \text{ and } l_c + H_b/2 \geq H_m/2 \text{ or,} \\ \\ l_c > H_m/2 \text{ and } l_c - H_b/2 < H_m/2. \end{cases}$$

The probability of occurrence is calculated using a Monte Carlo approach: A number of random trials (10,000 trials) were drawn for H_b, which followed a Gaussian distribution, and l_c, which was uniformly distributed. For

Table 1: Depth data for Figure 5.

Barrier Depth	Barrier Thickness
9	3
17.5	4.2
33.5	5

every trial, the above ``occurrence relations" were invoked to determine if the barrier was intercepted by the borehole. The final probability P_b was simply the ratio of the number of detections to the total number of trials, i.e.

$$P_b = \frac{\text{number of detections}}{10,000}.$$

The P_b evaluated as such can be interpreted as the *fractional occupancy* of the barrier within the model -- in a statistical sense, i.e.

$$P_b = \frac{< H_b >}{H_m},$$

where $< H_b >$ is the average lateral size of the barrier. Finally, the magnitudes of the step-backs in Figure 2 are given by

$$\text{step back} \equiv < H_b > \Delta z$$

where Δz is the vertical gap width between the present barrier and the one above it.

Examples on One-Dimensional Analysis

As a result of uncertainty in the lateral size and position of barriers, the evaluation of recovery is not unique. Based on the configuration of barriers and their thicknesses, there might be cases where the maximum recovery happens at different location so the production well would likely be placed at a depth that corresponds to maximum recovery R. However, due to different ``step backs," the optimal well placement would vary from case to case. This behavior is shown in Figure 5 that clearly demonstrates a scenario where the maximum recovery happens at the top of the third barrier in one case while it is maximum at the bottom of the model for the other case. In this scenario, the vertical extent of the model is $V_m = 40$ m, and the horizontal length is $H_m = 700$ m (*not realistic*). There are three barriers in the reservoir (i.e. n_b = 3), with their depth and thickness information listed in table 1. Figure 5 is generated using EXPECTEDRECOVRY program implemented for this purpose in GSLIB. Details on the calculation of expected recovery had been provided earlier. For the scenario that is plotted in red, the optimal well-placement is at the depth of 34 m while for the scenario in black, the optimal well-placement is at the depth of 40 m. Figure 6 demonstrates the calculated statistically-averaged recovery versus the depth which is give in table 4. As can be seen, the maximum recovery is at the top of the second barrier. That location is chosen as the optimal well-placement. However, this is not the case for all barriers' configuration as shown in Figure 5.

In this work, a one-dimensional Monte Carlo approach is used to evaluate the expected recovery from an oil sands deposit using the SAGD technique. The central parameter to this analysis is the probability of a barrier, which can also be viewed as the statistically-averaged occupancy $< H_b >$ of the model by a barrier. Such a parameter is needed to determine the ``step-backs" in Figure 6. The parameter $< H_b >$ in turn depends on two random variables, H_b and l_c, which are the lateral size and position of a barrier, respectively. As both these random variables are considered to be symmetrically distributed about their mean values, it should perhaps not be surprising that the statistically-averaged occupancy $< H_b >$ is just the mean of H_b, which is given by its linear regression with the barrier thickness t. This was indeed verified by comparing our value m (mean of the Gaussian distribution) to the Monte Carlo result based on 10,000 trials. It should also be noted that the parameter $< H_b >$ is completely independent of the standard deviation of the Gaussian distribution (as expected). It seems therefore pointless to take the Monte Carlo approach in this work. However, in cases where the random variables H_b and l_c are not symmetrically distributed about their means, the Monte Carlo

Table 2: Recovery statistics.

Barrier Depth	Expected Recovery	Standard Deviation for Recovery
4.80	358.913	1.086
12.00	605.339	90.717
15.00	551.532	202.218

analysis is important. Moreover, Monte Carlo analysis is required to transfer geologic uncertainty through to recovery uncertainty.

Quantifying Uncertainty on Recovery Estimation

In the last section, Monte Carlo simulation was applied to evaluate the recovery, accounting for all possibilities of the barriers' lateral extension and position. The amount of oil recovery has been associated to the recovery well placements -- the possible recovery positions are the ones above every detected barrier. Based on this process, one could get reasonable understanding of the recovery performance at different locations in the reservoir model. However, what is missing in the reported results is how much the recovery estimate could fluctuate about its mean. Note that the estimated recovery associated with a location for well placement is the expected value of the estimate, which is not a deterministic recovery that could be achieved at any condition. Our knowledge of what actually exists in the reservoir is quite limited. A more correct way of representing the estimated results is to report them with their corresponding variations.

Confidence intervals of the estimated values reflect the variation of the estimation about its mean. Representing every estimate with its corresponding variation is more effective and reliable decision making to consider the risk associated with an estimate. In an applied case, it is more appropriate to make decision based on possibilities rather than a single mean value; see Figure 7. Note that the uncertainty on the estimate increases as one goes deeper in the reservoir. The target locations for recovery in this work are above the barriers since the only data available is the barrier positions. The one-dimensional analysis discussed above transfers uncertainty in barrier distribution to uncertainty in recovery assessment. Basically, the uncertainty grows as the number of barriers increases in the area. Also, it is noteworthy to recall that the confidence interval is especially important for cases with less data available, as in our one-dimensional problem. Here, the variation of estimation represents one standard deviation around the mean.

Representing the variation around the estimated recovery helps one to get an overall picture of the reservoir before making decision, on well-placement. A single value for available resources at a particular location might suggest position **A** for well-placement, while the confidence interval for location **B** might offer larger recovery values at some other points. The challenge is to optimize the resource utilization and economic recovery simultaneously. However, profit typically is defined differently for different usages. One would allow for more risk on resource utilization for example and go for 90% variation about the mean to maximize the economic recovery. Different well placements at different locations account for different amount of variation in the estimated original oil-in-place. In general, the entire recovery process is a trade off: (1) placing the well too high result in loss of resources while it improves the probability of connections and (2) placing the well too low recovers more while it has to deal with more uncertainty regarding the net connectivity above the production well. The present study on recovery assessment and well placement is however, independent of details of profit calculations. Examples are provided in the following section to evaluate the cost resulting from overestimation and underestimation. In this study, a simple, general form of loss function (asymmetric absolute loss function) shown in Figure 8 is considered. The loss function determines the cost related to different decisions for well-placement. The objective is to minimize the cost which is equivalent to the optimal decision given the uncertainty space. The asymmetric property accounts for different cost for two cases of overestimation and underestimation. For example, in recovery, overestimation corresponds to the case where the realized recovery is less than expected, while underestimation corresponds to the case where the recovery is greater than the expected value. This would possibly mean that the optimal decision could have advantaged from more accelerating production or correctly sizing the required facilities.

$$\mathrm{L}(\widehat{\mathrm{OIP}}, \mathrm{OIP}) = |\widehat{\mathrm{OIP}} \text{ - } \mathrm{OIP}|$$
$$\widehat{\mathrm{OIP}}_{\mathrm{optimal}} = \arg\min\ \mathrm{L}(\widehat{\mathrm{OIP}}, \mathrm{OIP})$$

Note that in this study, we have imperfect knowledge on barrier distributions; in the recovery process and the overall original oil in place (OOIP) is not accessible. However, since we cannot evaluate the loss due to the well design and flow properties, recovery in our case is a fraction of OOIP which is the same for all locations.

Several programs have been developed throughout this study. The one considering the variations in recovery (OOIP) evaluation is called RecoveryVariance (see appendix). Its parameter file asks for the angles of slops corresponding to the overestimation and underestimation. A. G. Journel in (App) has proved that in case of asymmetric loss function, the minimum cost occurs at the p-quantile $\lambda_{\mathrm{small}}/(\lambda_{\mathrm{small}} + \lambda_{\mathrm{big}})$ of the production distribution. Having said that, after the user identifies the loss function, program will sort the recovery values and approximately look for the recovery corresponding to the quantile which minimizes the loss function. The user would decide on the well-placement based on the occurring maximum recovery at the optimal quantile (Deutsch, 1998) (see appendix).

Examples on Recovery with Variations

We know that the uncertainty increases as the reservoir gets deeper. The estimated recovery at the location above the deepest barrier in the model has the largest uncertainty; the most variation in estimation corresponds to the deepest location which inherits the uncertainty from all barriers above it. Similarly, the largest recovery value most likely occurs at the point furthest to the right of the lowest recovery location; see Figure 7. However, the optimal recovery is not guaranteed since this location might end up giving the low recovery. The decisions made are dependent on the company's policy on risk-taking which will make a difference in recovery and well placement. The judgment of a specialist is required.

In one experiment, we assumed having two barriers at depths of 4.8 and 12 m with thickness of 1.7 and 1.3 m, respectively. Note that as before, the model is 15 m thick. After applying Monte Carlo for each case individually (10,000 in total) and evaluating the expected mean of recovery with the associating standard deviation gives the result in Table 2. As can be seen from the table, the expected recovery at top of the second barrier is larger than the expected recovery at the bottom. However, as anticipated, the uncertainty at the lowest recovery position is larger, meaning that there is also a possibility that locating recovery wells at this position will result in a recovery of about $551 + 202 \simeq 753$ m^3, while the recovery at the top of the second barrier might lead to a recovery of $605\text{-}90 \simeq 515$ m^3. Therefore, the evaluation of confidence intervals enables us to reasonably consider different possibilities. However, there would be also cases which decision making is much simpler. All evaluations may turn out to be in favor of well placement at a specific location. In such as case, decision making will of course be easier, even if the estimates are not always accurate -- as long as the estimates do not represent overlap, they would be less problematic.

Now, consider a case where barriers are distributed in a way that (2.5 m thick located at 4.8 m depth, and 1.0 m thick located at 10.0 m depth) the reservoir assessment and recovery evaluation suggest a maximum expected recovery at the bottom of the model; see Table 3. Although we expect a larger uncertainty on this value compared to the higher recovery locations, the recovery at two locations considering their variances represent no overlap. In the best case scenario, maximum recovery is achieved at the top of the second barrier, $318+106 \simeq 424$ m^3. On the other hand, the worst possible recovery at the bottom of the barrier is $620\text{-}194 \simeq 426$ m^3. These two values are far enough apart to have no overlap. Everything else equal, it is more reasonable to choose the bottom of the model for well-placement in similar cases as this location has access to more resources.

Table 3: Recovery statistics.

Barrier Depth	Expected Recovery	Standard Deviation for Recovery
4.80	358.894	1.105
10.00	318.173	106.178
15.00	620.503	194.657

Resource Conservation versus Discounted Recovery

Let us look at the example in the previous section again. As was discussed, the recovery is determined based on quantile analysis (conservative strategy is assumed with tan$^{-1}(55)$ as the cost of overestimation (see Appendix 1) and tan$^{-1}(25)$ as the cost of underestimation) which result in a recovery of about 504 m^3 at the top of the second barrier and a recovery of about 405 m^3 at the bottom of the reservoir (quantile of $31.25\% = \lambda_{\text{underestimation}}/(\lambda_{\text{underestimation}} + \lambda_{\text{overestimation}})$. However, the expected recovery is larger if the recovery happens at the base of the model. These cases are common in this study. Our one-dimensional knowledge inherits considerable uncertainty which would normally lead to a more complex decision making process. A trade off between resource utilization and economic recovery is a concern in this analysis.

The decision in this context should be able to account for the relative importance of resource conservation versus discounted recovery. Resource utilization ensures that the reservoir has been recovered to its fullest potential; a decision on recovery at a lower quantile (i.e., < 33%) will facilitate resource conservation. On the other hand, there are situations where operators are more concerned with the amount of recovery which might be discounted. The optimization on the overestimation side should account for recovery discount. A higher quantile, say > 67%, evaluates the recovery aggressively (see Figure 9). Finally, in cases where both are of equal importance, the decision based on the expected recovery is the safest route. For example, in the earlier case also where the recovery at different locations introduced overlap, a conservative decision would suggest recovery at the top of the second barrier with a recovery of 562 m^3 resulted from quantile analysis, and a recovery of 457 m^3 at the bottom of the reservoir. Here, the quantile analysis (quantile of $68.75\% = 1$ - $[\lambda_{\text{underestimation}}/(\lambda_{\text{underestimation}} + \lambda_{\text{overestimation}})]$) for a risk-taking company suggests an almost equal recovery of 655 m^3 at both locations. For the risk-taking company, the quantile analysis (quantile of 68.75%) again confirms the well placement at the bottom of the model (724 m^3 against 370 m^3).

Comments on Application

The uncertainty in the reservoir study comes from a lack of knowledge of its geometry, property distribution, fluid flow, response to external stimuli (in well-placement), and economy-related variables such as uncertainty in prices. We consider the uncertainly in geologic properties resulting from sparse well data. As mentioned earlier, the recovery in our case is due to the barrier's lateral extension. Our knowledge of the thickness and depth of the barriers helps us build our understanding of the reservoir by estimating the size of the barriers as well as their positions. Linear regression analysis on a few hundred barriers from the training image library has been used to quantify the barriers' uncertainty in length. The resulting parametric relationship between the thickness and length of the barrier has been used to predict the mean of the barrier length.

One-dimensional study of the borehole data results in quick identification of the non-net thickness in the reservoir. This knowledge can be utilized to map the thickness base elevation. Similarly, this can be used to map local trends and gradients. As the well pairs in SAGD should be placed parallel to the base contours, the quick iso-base contours resulting from the non-net thickness and distribution of the barriers can help with well placement. One-dimensional well data analysis can also be applied to reconcile nearby wells. One could estimate the distribution of barriers using lateral extensions of the non-net intervals for every well, and then compare to the nearby wells. This may identify problematic data and help reconcile different elevations.

Concluding Remarks

The work done in this paper is aimed at simplifying the reservoir study using limited data from a single well at a time. The understanding of reservoir is based on non-net connectivity in the reservoir and how the barriers are distributed over the given space. The thicknesses and locations of barriers are combined with our lateral extension analysis to helps with the estimation and well-placement. Although this analysis cannot account for a reservoir's properties such as flow, it provides an initial understanding of reservoir specifications without going through many complexities.

This work has been limited to the uncertainty of the lateral extension of the barriers and their locations based on well data. Several other considerations could be applied to analyze the effect on the drainage process and optimal well-placement of SAGD. This could affect the recovery assessment and optimal well placement. For example, in our 1-D analysis, we rely on the connectivity of non-net to estimate the pore volume and possible presence of bitumen. Obviously, permeability in our case is an unseen parameter which in reality is an important factor for practical recovery. Apart from the geological limitations, recovery requires human expertise in the design and configuration of well placement; this makes recovery a function of additional variables as well as the geological parameters.

Later also, the 1-D analysis applied to decision making on well-placement for optimal recovery. Reservoir study is only one aspect of the work. One would subsequently require applying judgment on the results of the study. However, decisions are not easy to make as the study already includes considerable uncertainty. In our one-dimensional analysis, as has been discussed, variabilities in barrier lateral extension and its position impose uncertainty on reservoir understanding and recovery evaluation. Some examples given in this paper discussed the decision making process based on quantile analysis. This is however, mostly depends on the strategy on resource utilization versus the economic recovery.

Our future work considers the 1-D analysis which leads to interesting questions and discussions on 3-D processing. The interesting question is how and why the ranking process should be used and how to manage the results for recovery purposes. There would be upcoming questions in 3-D processing when intensive simulations are tried to be ignored. Questions such as what realizations are to be used? What would be the best ranking scheme? And if clustering could be an efficient replacement to the ranking scheme?

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Appendices

Parameter Files

For the general case of statistical recovery evaluation, a program called ``SIZPOSRECOVERY'' has been developed in GSLIB to draw samples for H_b and l_c based on the thickness of barriers. The program prompts for the parameter file. Properties such as the linear regression between thickness and lateral extension (t and H_b) correlation, standard deviation, number of barriers, size of the model and the input file which includes the thicknesses and depths of the barriers can be changed as required. An example of this parameter file is in appendix. This parameter file has been modified for the example of a two-barrier model with the input data file

```
                    Parameters for SIZPOSRECOVERY
                     ************************
START OF PARAMETERS:
depth.dat                  - input Depth data file
1   2                      - columns for depth scale, barrier depth
150   15                   - model size for hoRizontal length, vertical length
2                          - number of barriers
Hbproperty.out             - output file
recovery.out               - output file
40   10                    - slope, interception
30   30                    - Standard deviation for every regression fit-barrier
```

listed in table 4. With the following arrangement, there are cases where the maximum recovery occurs at the top of the second barrier, and some other cases where the bottom of the model is the place for optimal well-placement. The uncertainty regarding the optimal well-placement is the result of limited knowledge which prevents us from determining the length of barrier and its centroid with certainty. The parameter file of

Table 4: Depth data for case study.

Barrier Depth	Barrier Thickness
4.8	1.7
12.0	1.3

``RecoveryVariance'' asks for the angles considered in overestimation and underestimation in addition to the other parameters which have been explained in ``SIZPOSRECOVERY'' parameter file.

```
                    Parameters for RECOVERYVARIANCE
                    ******************************
START OF PARAMETERS:
depth.dat                - input Depth data file
1    2                   - columns for depth scale, barrier depth
150    15                - model size for horizontal length, vertical length
2                        - number of barriers
recoveryStat.out         - output file
recovery.out             - output file
40    10                 - slope, interception
25    30                 - Standard deviation for every regression fit-barrier
55    25                 - Asymmetric Cost Function: overestimation,underestimation
```

Loss Function

Let's assume a loss function as below:

$$L(z - u) = \begin{cases} \lambda_1(u - z) & z \leq u \\ \lambda_2(z - u) & z \geq u \end{cases}$$

The objective is to find z corresponding to minimum loss:

$$\begin{aligned}
&E\{L(z - u)\} \\
&= \int_{-\infty}^{u} \lambda_1(u - z)f(z)dz + \int_{u}^{\infty} \lambda_2(z - u)f(z)dz \;\; \text{and} \;\; f(z) = \frac{dF(z)}{dz} \\
&= \int_{-\infty}^{u} \lambda_1 u dF(z) - \int_{u}^{\infty} \lambda_2 u dF(z)dz - \\
&\quad \int_{-\infty}^{u} \lambda_1 z dF(z) - \int_{u}^{\infty} \lambda_2 z dF(z)dz \\
&= \lambda_1 u\{F(u) - F(-\infty)\} - \lambda_2 u\{F(+\infty) - F(u)\} \\
&\quad \lambda_1 z\{F(u) - F(-\infty)\} - \lambda_2 z\{F(+\infty) - F(u)\} \\
&= \{\lambda_1 + \lambda_2\}F(u)(u - z) + \lambda_2(z - u) \;\; \text{and} \quad z = \int_{-\infty}^{+\infty} zf(u)du = m(u) \\
&= (\lambda_1 + \lambda_2)F(u)(u - m(u)) + \lambda_2(m(u) - u) \\
&= 0 \\
&\quad \Rightarrow \;\; F(u) = \frac{\lambda_2}{\lambda_1 + \lambda_2} \\
&\quad \Rightarrow \;\; u = F^{-1}\left(\frac{\lambda_2}{\lambda_1 + \lambda_2}\right)
\end{aligned}$$

Then equating $F(u)$ to p (quantile):

$$p = \frac{\lambda_2}{\lambda_1 + \lambda_2} \;\; \text{and} \;\; 1 - p = \frac{\lambda_1}{\lambda_1 + \lambda_2} \;\; ; \;\; p \in [0, 1] \tag{1}$$

Note that, when λ_1 > λ_2, p < $0.5 \Rightarrow$ best estimate is < F$^{-1}(0.5)$. In practice, higher cost of overestimation will result in call for a more conservative estimate. If λ_1 < λ_2, p > $0.5 \Rightarrow$ best estimate is > F$^{-1}(0.5)$. This proves that for asymmetric loss function, the decision p-quantile can be simply found by the ratio in (1).

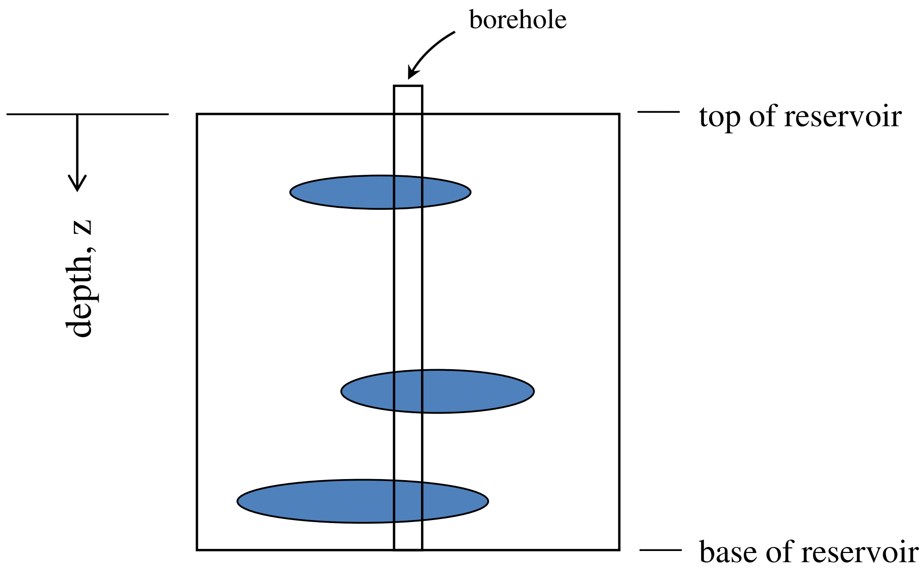

Figure 1: The possible positioning of barriers inside the model.

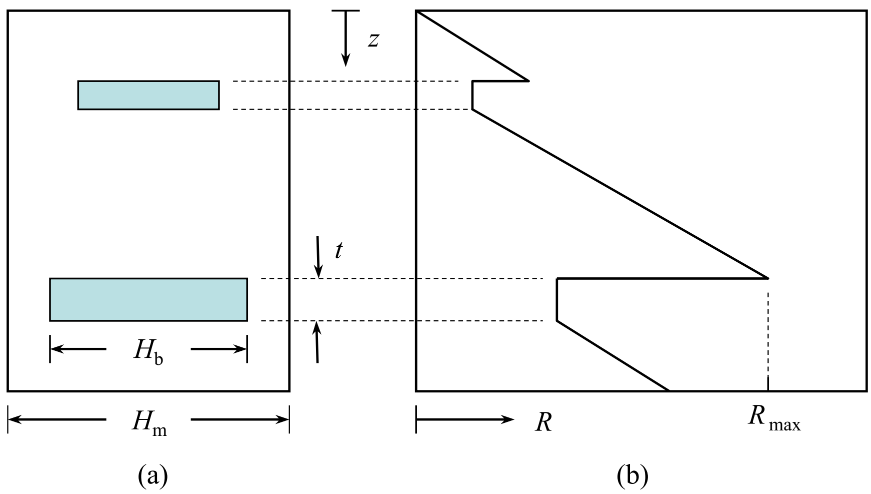

Figure 2: Model (a), expected recovery (b).

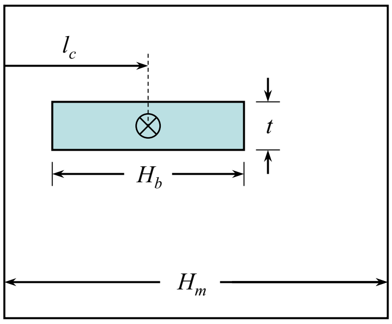

Figure 3: The parameters to define a barrier.

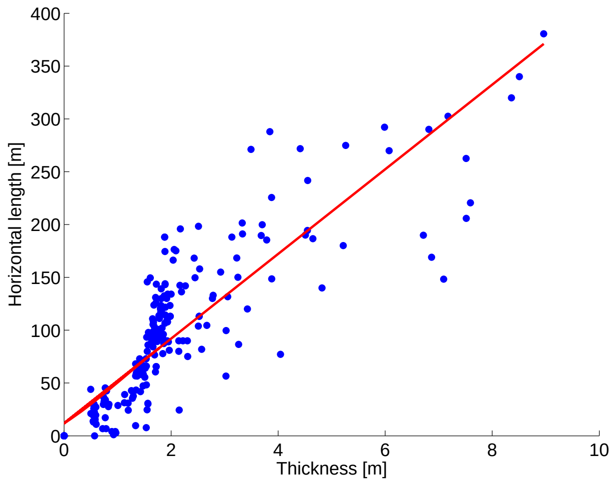

Figure 4: Illustrates the correlation between the lateral extension and thickness of the barriers. A few hundreds of barriers from image training library have been selected for this analysis. The details can be found in (Lajevardi et al., 2011),

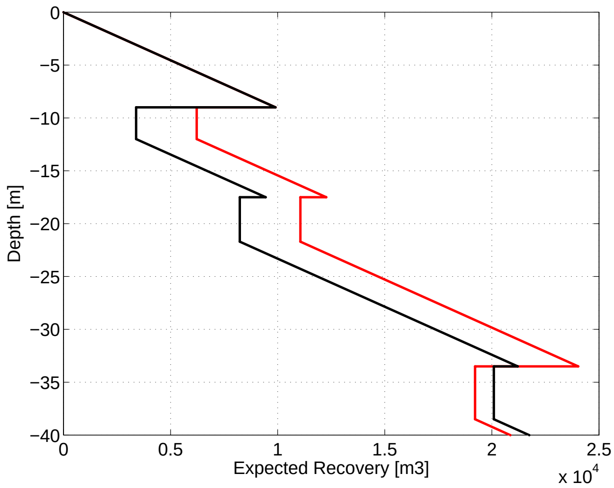

Figure 5: Recovery comparison for different lateral extension of the barriers.

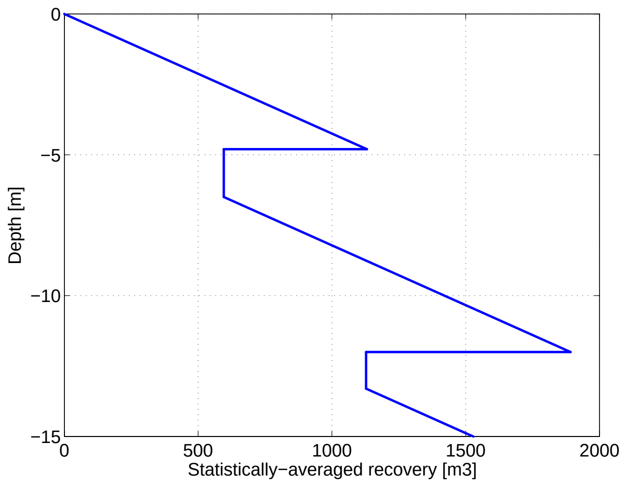

Figure 6: Illustrates the amount of recovery on average at every depth, based on many different distribution of barriers.

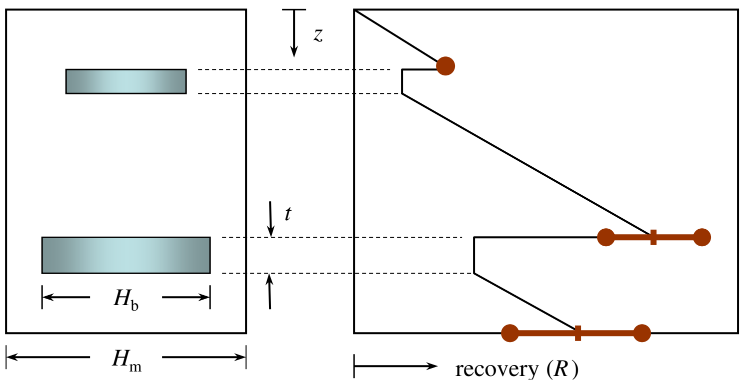

Figure 7: Recovery with confidence interval.

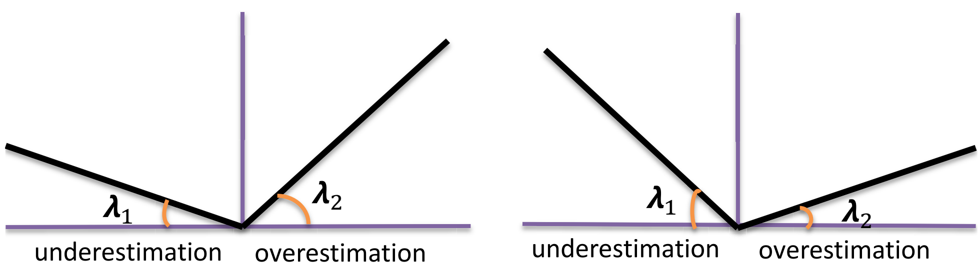

Figure 8: The loss function at left defines more cost in the case of overestimation and the one at right defines more cost regarding underestimation. The one at left is used when economic recovery is of more interest whereas the one at right is more concert on resource utilization.

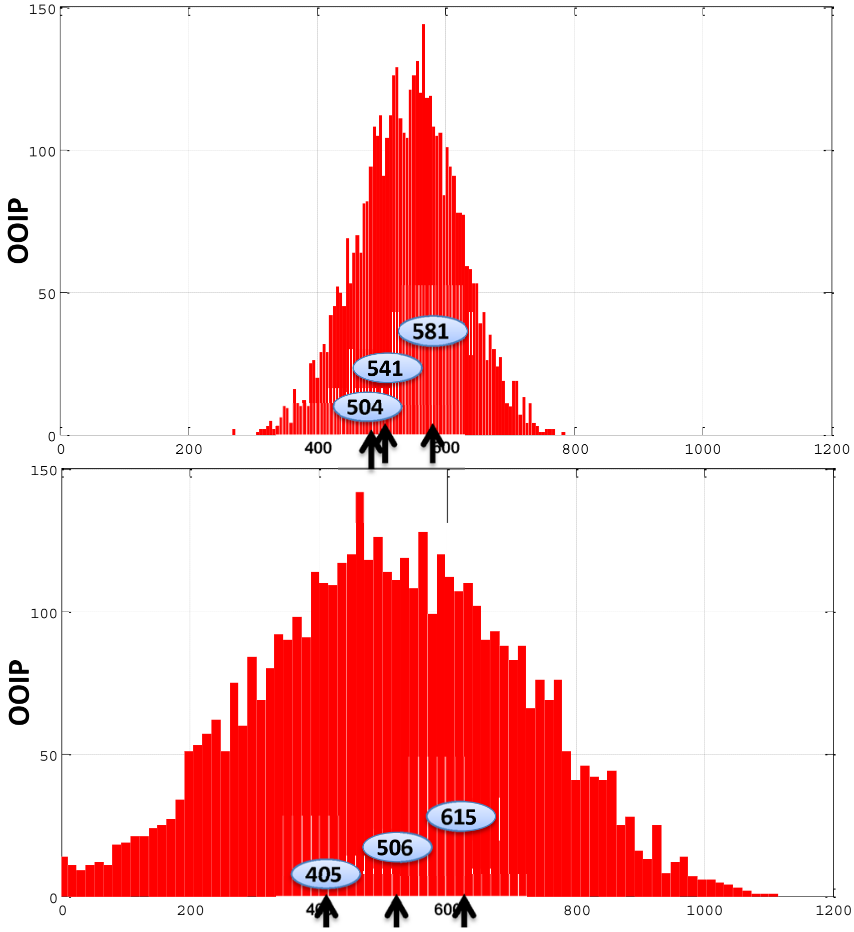

Figure 9: Quantile analysis for the recovery distribution of the example with no overlap.

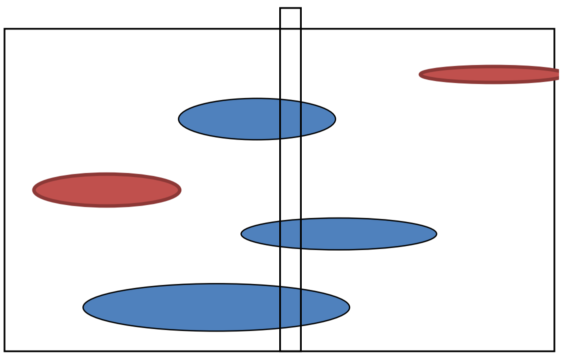

Figure 10: Illustrates the barriers in red can not be identified by the well.