Prediction of Local Uncertainty for Resource Evaluation

by

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Abstract

Resource evaluation of a mineral deposit is an important and challenging task. Traditional estimation methods provide a best estimate at the block scale based on available data. The pitfalls of histogram smoothing and conditional bias associated with these methods are well known. Quantifying local uncertainty is an alternative to estimation that avoids these issues. Multiple methods have been established for quantifying local uncertainty and are presented here.

This thesis explores the local dependence of change of support methods as well as improves upon available post processing techniques associated with quantifying uncertainty for resource evaluation. Change of support parameters are examined through a simulation test. The advertised benefits and issues surrounding localization of uncertainty are evaluated and a flexible methodology for localization is demonstrated. Artifact reduction techniques are provided for improved localization when a single model is required. Lastly, a case study demonstrates practical implementation of geostatistical methods and post-processing for modeling block scale uncertainty and resource evaluation.

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List of Symbols

Symbol	Definition	First Use
$\mathbf{u}$	a vector of locations	11
$z(\mathbf{u})$	known data at a location	11
λ	a weight applied to a data value	11
$z^*(\mathbf{u})$	estimate at a location	11
$\mathbf{u}_\square$	a vector of unsampled locations	11
A	a domain	11
v	block scale location	13
Z	a random variable	13
ℓ	a realization	19
L	total number of realizations	19
f_{global}	global variance correction factor	25
$\bar{\gamma}$	the average variogram	25
γ	the variogram	25
σ^2	the variance	25
m	the mean	26
z'	a transformed value	26

CV	coefficient of variation	26
q	quantile value	27
q'	transformed quantile value	27
y	value in Gaussian units	28
ϕ	Hermite Polynomial Coefficient	28
$H(y(\mathbf{v}))$	Hermite Polynomial value	28
r	change of support coefficient	28
Y	a random Gaussian variable	28
p	index of Hermite Polynomials	28
r_V	panel change of support coefficient	28
r_v	SMU change of support coefficient	28
ρ	correlation coefficient	28
$D^2(\cdot, G)$	point scale variance in the domain	33
G	a domain	33
$D^2(v, G)$	block scale variance within the domain	33
ρ_{UC}	theoretical Uniform Conditioning correlation coefficient	39
$\bar{\gamma}_y$	average variogram in Gaussian units	40
ρ_y	correlation coefficient in Gaussian units	40
N	total number of SMUs	51
n	number of SMUs surrounding a location	51

List of Abbreviations

Abbrv.	Definition	First Use
SMU	Selective Mining Unit	1
MG	MultiGaussian	8
SGS	Sequential Guassian Simulation	8
UC	Uniform Conditioning	8
IK	Indicator Kriging	8
CDF	cumulative distribution function	17
DGM	Discrete Gaussian Model	23

Chapter 1

Introduction

1.1 Long Term Resource and Reserves Planning

Accurately assessing the value of a mineral deposit is an important and challenging task. The value of a mineral deposit is reported by two measures, the resource and the reserve. A resource is the quantified tons and grade of the mineral of interest. The reserve is defined as the amount of the calculated resource that can be economically extracted within a mine plan (Rossi and Deutsch, 2014). Each of these quantities is determined by creating a long range, life of mine, geostatistical model at an appropriate selective mining unit (SMU) scale. Many methodologies have been developed and studied for this purpose with the goal of achieving an accurate, precise and unbiased estimate, often complimented by a measure of uncertainty. The development and implementation of careful sampling and quality assurance procedures are required to collect reliable data to support the estimates. A reliable geologic interpretation and definition of estimation domains are also required. The focus of this thesis is neither on sampling nor geologic domaining; these tasks must be completed with an acceptable confidence. This thesis will focus on the effective development and utilization of block models of grades with uncertainty to support long term planning and decision making.

1.2 Data and Selective Mining Units

In a mining context, available data for resource and reserve modeling commonly includes a series of empirical measurements and geologic interpreta-

tions. In most cases empirical measurements consist of assay values from drill holes. An initial distribution of assay data is comprised of data collected at a small scale relative to the size of the deposit and most often the amount of data is relatively sparse. Based on the data within each geological domain, a geostatistical model must be constructed to quantify the value of the subsurface mineral of interest. To ensure the block model is suitable for technical and economic evaluation, geostatistical modeling is performed at a deemed relevant selective mining unit (SMU) scale. Modeling at this scale avoids the need to predict the small scale variability within each block.

It is important to note that, between the time of the long range resource model and the time of mining, more data and information will become available. Additional information often comes in the form of more drillholes, blast holes, and geological observations that are combined for a final evaluation of the material to determine its economic value. At the time of mining, based on a determined cutoff grade, mined material is often categorized as either ore or waste. As additional data is collected, the distribution of values for that measurement becomes better understood and the estimates are not simply an average over a large volume. As the estimates become more variable, they provide more complete spatial information. This is referred to as the information effect (Rossi and Deutsch, 2014), and should be accounted for in the long range resource model.

Closely related to the information effect is the support effect. The support effect as described by Chilés and Delfiner (2012) is similar to the information effect in the sense that it describes a change in variance. The support effect relates to a change in a distribution as the scale of interest increases from a small scale to a larger scale. The predictive model is desired at the SMU scale. A large number of smaller scale samples will be averaged in a larger SMU volume; therefore extreme high and low values seen in small scale data will not be seen in SMU grades. This change in scale and dampening of high and low values results in a reduction in the variance of SMUs relative to the data. This must be considered in the modeling process to accurately portray the SMU distribution. A number of analytical methods and a purely numerical

approach are available to predict the change in a distribution from the data scale to the SMU scale. These methods rely on a variety of assumptions (Isaaks and Srivastava, 1989) (Journel and Huijbregts, 1978) (Matheron, 1985). Regardless of the methodology chosen, acknowledging and accounting for both the information effect and reduced variability at a larger scale are required aspects of a complete geostatistical modeling process.

1.3 Purpose of Quantifying and Transferring Uncertainty

One important goal of generating a geostatistical model is to accurately predict the total tons and grade of the in-situ mineral of interest, at the SMU scale, conditioned to the experimental data available. In most cases the available data constitutes a very small fraction of the volume being estimated, therefore making it impossible to provide an exact estimate of the resources and reserves. However, a carefully constructed model can generate a measure of uncertainty at each location, that is, a distribution of equiprobable values. This provides valuable decision support information, allowing the user to calculate not only a single estimated resource and reserve but a range of possibilities that may present themselves at the time of mining. A mine plan can be developed that is robust with respect to this uncertainty or steps can be taken to collect additional data and reduce uncertainty.

The alternative to quantifying uncertainty is to create a single deterministic model, with a single value at each location. There is undoubtedly uncertainty in predictions with incomplete data. A single deterministic block model of estimates does not permit understanding possible shortfalls or excess capacity that could result from inaccurate estimates. Quantified local uncertainty will allow for a risk analysis for evaluation of scenarios for financial and logistical planning with confidence.

Producing an accurate, precise, and unbiased geostatistical model of uncertainty that is properly conditioned to the input experimental data is not an easy task, and must be uniquely catered to each deposit as no two geologic systems are exactly identical. For this reason a variety of geostatistical

methods have been developed. Modeling techniques can be considered in two categories; a group of estimation techniques that provide a single deterministic model and a second group of more robust methodologies that generate quantified uncertainty for each location.

1.4 Methods for Resource and Reserve Calculations

Conventional resource and reserve calculations are performed on a single model at the SMU scale. This has long been the status quo in the mining industry. This requires utilizing an estimation geostatistical method or a similar technique like inverse distance estimation resulting in a single value for each block that can be evaluated as ore or waste for a straight forward and computationally inexpensive workflow. Ordinary Kriging is the preferred methodology due to demonstrated robustness and widespread application. Estimation methods inevitably result in smoothing of the distribution of estimates relative to the expected variability of the SMUs. The histogram smoothing of kriging is often more than that expected for the support effect. To counteract this, parameters could be adjusted to limit the search neighborhood for each estimate. Through an iterative process the desired distribution can be matched through tuning the search radius and number of data used in the kriging estimator. Tuning the kriging neighborhood fixes histogram smoothing but it can introduce significant conditional bias.

In contrast, quantifying uncertainty and providing a range of possible values for each SMU allows for an unbiased model that reproduces the appropriate histogram without the need for artificially reducing the search neighborhood. Four methods will be presented and discussed in this thesis, all of which generate a local distribution for each SMU being considered. There are pros and cons to each method regarding the complexity of the method, accounting for the support effect, and the assumptions that are required. When prepared carefully each method is capable of reproducing the SMU distribution expected in the future. The four methods are; MultiGaussian Kriging, Sequential Guassian Simulation, Uniform Conditioning and Indicator Kriging.

Each of these four methodologies is aimed at the same goal of generating a dependable distribution of uncertainty. MultiGaussian kriging and Sequential Gaussian simulation rely heavily on the well known and widely used quantile to quantile Guassian transform paired with the known properties of the Guassian distribution. Uniform conditioning also utilizes the Guassian transform but in conjunction with a more important assumption of bivariate Guassianity dictating change of support and variance reduction. Indicator Kriging is unique as a non-parametric method with no strong preference on the Gaussian distribution, but necessitates a complex workflow requiring additional variograms and substantial user input for interpolating and extrapolating local distributions.

1.5 Motivation & Goals

For all methods of quantifying local uncertainty, the long standing argument against a model of this type has been the increase in computational expense and complexity in the calculations. When a series of equiprobable values for each SMU must be generated and considered, the computational expense can be higher, however; computer processing speeds ands parallelization makes these calculations possible. The process for computing resources, reserves or any other important transfer function, including mine planning, must be performed on each model.

The effort required to process, say, one hundred models is significant. A variety of strategies for summarizing or localizing a model of uncertainty have been developed as a potential middle ground to avoid working with the entire model of uncertainty. These strategies include summary measures such as the probability to be within a certain percentage of the mean, extracting specific probability quantiles, ranking realizations, or the recently popular localization of uncertainty. Localization of uncertainty has gained significant attention of late and warrants some careful evaluation as a possible alternative to kriging. The lure of generating a single model at the SMU scale without excessive histogram smoothing is appealing, but this process can yield unsightly artifacts and questionable local precision. In general, specific summary measures may

be appropriate in some situations, but when possible the complete model of uncertainty should be utilized.

Managing a model of uncertainty remains an issue. Kriging has been widely implemented in commercial software; mining companies are familiar with kriging, and the process of establishing resource and reserve estimates using a single value is straightforward and widely used. The problem with modeling uncertainty is the transfer of this uncertainty through mine planning and decision making. Often this is avoided by calculating the expected value of each local distribution.

The status quo of considering only a single model despite the availability of more robust techniques is the motivation of this thesis. The goal of this thesis is to challenge that status quo through improved understanding of the four techniques available for modeling local uncertainty. This includes; improved volume variance reduction dictated by locally varying parameters, two new techniques for artifact reduced localization, and a case study to showcase practical implementation for modeling uncertainty to improve mine planning and decision making.

1.6 Outline

To begin, a review of implementation aspects for modeling selective mining units is required. The purpose of the selective mining unit in regards to resource and reserves evaluation, mine planning, and ore control must be considered. Also, the decision of appropriate SMU size must be assessed, guiding principles for this decision will be discussed. Lastly, each of the four methods available for modeling quantified uncertainty will be reviewed.

Building upon the reviewed modeling techniques, an investigation of variance reduction methods, that is, an understanding of how the variance of point-scale uncertainty reduces to represent the block scale is needed. This section will review currently available analytical and numerical techniques for variance these reduction calculations. More importantly, the local dependence of variance reduction will be explored through a simulation test. A simulation

test provides insight into the effect of conditioning data on variance reduction. Recommended best practice for both analytical and numerical variance reduction are presented for uncertainty quantification at the SMU scale.

Localization has gained significant attention for collapsing quantified SMU uncertainty into a single block model for simplicity. The third chapter will present and evaluate this technique. Two new techniques for artifact reduction in localized models are considered. The first of these techniques focuses on an optimization approach while the second relies on a slightly more flexible localization approach. The advertised advantages, and lingering issues of localization will be closely reviewed motivating a need to establish best practices for utilizing uncertainty.

The final section of this thesis will focus on practical implementation of SMU uncertainty. A practical comparison of available methods is warranted. Modeling a Copper deposit with each technique will show both the advantages and disadvantages of each method. The benefits of quantified uncertainty are demonstrated through the Lerchs-Grossman algorithm for pit shell determination. Improved decision making, risk analysis, and improved mining planning are possible through a deeper understanding of local uncertainty.

Chapter 2

Implementation Aspects of Local Uncertainty

2.1 Introduction

A number of steps are required to effectively model local uncertainty at the SMU block scale. In the case of long term resource and reserves evaluation, an appropriately sized SMU must be determined for modeling. Then, numerical models at the SMU scale must be constructed. Four methods are used that include: MultiGaussian (MG) Kriging, Sequential Gaussian Simulation (SGS) , Uniform Conditioning (UC) and Indicator Kriging (IK) . The goal of this chapter is to present the necessary background information regarding the generation of SMU uncertainty that is required for more detailed discussion of post processing and effective management of the model produced.

2.2 Selective Mining Units

In practice, the final tonnage and grade at the time of mining is dictated by many factors as well as additional information that will be gained prior to final selection. For this reason two separate models are required. For long term planning, a model is constructed at the SMU scale. At the time of mining; a higher resolution and better informed ore control model will be used to determine final ore and waste selection. Prior to mining, it is impossible to predict the high resolution variability and the subjective factors that will ultimately impact the final quantities of ore and waste.

2.2.1 The Information Effect

Some of the SMUs will be misclassified in the long term estimation process. Even at the time of mining, ore and waste selection is an imperfect process that relies on estimated ore grades and the spatial configuration of ore and waste. The increase in available information from the time of resource modeling to the time of mining can impact the classification of SMUs and changes the distribution of experimental data to better represent the true variability. The issues of misclassification, increased information and the potential dilution of ore grade based on selectivity is known as the information effect. This has been established in a number of references, including Rossi and Deutsch (2014) and Chilés and Delfiner (2012).

The addition of information leads to increased local precision and variation in the estimates. It is impossible to account for this variability accurately at a small scale across the entire domain with widely spaced data. For this reason long term modeling takes place at the SMU scale, where an average value over a larger volume of rock can be estimated more reliably. Even so, the SMU grades cannot be predicted with perfect accuracy; otherwise, mining could be more selective when closely spaced production sampling becomes available at the time of mining.

2.2.2 Optimal Size of Selective Mining Units

The appropriate SMU size has been a topic of some discussion. Ideally, the SMU scale model will generate an estimate of ore tons and grade similar to that which will be found at the time of mining. To accomplish this, the volume of an SMU must be determined with regard to the mining process that will take place. An SMU should represent the smallest volume of rock that can be selectively mined for separation of ore and waste based on the available information. Of course, at the time of mining additional grade information paired with geochemical and geometallurgical measures will impact these results. An average grade over an appropriate volume of rock should account for the impact of these additional factors and yield similar results as the final

ore control model.

There are multiple ways to determine the optimal SMU size. Some complex methods for simulating blast hole density and information have been suggested for this purpose (Leuangthong et al., 2004). Journel and Huijbregts (1978) suggest one half to one third of the data spacing as a guideline for the sides of each SMU as a general guideline. The vertical size of each block can be dictated by the mining method. In an open pit setting, SMUs with a height equal to that of the bench are practical for mine planning while in an underground mine a height equivalent to the size of anticipated lifts may be appropriate (Rossi and Deutsch, 2014). Practical experience and reconciliation with historical production are also considered when the professional chooses an SMU size. This size must be larger than the equipment bucket width and large relative to blast movement. This will allow adequate estimation and quantification of uncertainty at a scale relevant for future mining.

2.2.3 The Panel Scale

In addition to determining a reasonable SMU size for the task at hand some methods require selection of an appropriate panel size. A panel can be defined as a group of SMUs. In this thesis panels will be required for Uniform Conditioning as well as Localization. In the process of Uniform Conditioning panels are a rectangular volume which contains a number of SMUs. In localization, panels are often regularly shaped but some flexibility is permitted and SMUs belonging to a single panel are not required to be contiguous. In both cases the number of SMUs contained within each panel should be carefully determined. Although circumstances specific to each project may play a role in this decision some guidelines are suggested here. When regularly shaped, panels are being considered the SMU grid should nest neatly within the panel scale grid. The grids ought to have the same origin and SMUs should not be split between multiple panels. For any type of panel the number of SMUs is recommended to between ten to one hundred. This will provide enough SMUs in each panel that a well informed panel scale distribution can be compiled while maintaining a reasonable resolution at the panel scale.

2.3 Estimation Methods

Although not the focus of this thesis, estimation methods are common practice in mining thus some background information will be given. In an estimation framework, a single value is estimated for each unsampled location within the model. Estimates are normally a weighted average of samples in the vicinity of the unsampled location. The challenge in this methodology is determining an appropriate weighting scheme. Distance based methods are available, such as inverse distance or inverse distance squared but these methods do not incorporate details of data redundancy and spatial structure. Kriging, the preferred method, calculates weights based on a variogram model that quantifies the spatial relationship between all locations. Many references pertaining to many forms of Kriging are available for a more detailed explanation including Journel and Huijbregts (1978), Isaaks and Srivastava (1989), Deutsch and Journel (1997), Chilés and Delfiner (2012), and Rossi and Deutsch (2014).

Kriging is a standard technique in geostatistics for estimation and as an engine for modeling uncertainty. The goal of kriging is to calculate an estimate as a weighted average for each unsampled location. The optimal weights are determined to minimize the expected error variance. A variogram model quantifies the spatial dependence for the calculated error variance. There are many forms of kriging, but all are rooted in the same general linear estimator:

$$z^*(\mathbf{u}_\square) = \sum_{\alpha=1}^{n} \lambda_\alpha z_{\mathbf{u}_\alpha} \quad \forall\, \mathbf{u}_\square \in A \tag{2.1}$$

where $z(\mathbf{u}_\alpha)$ are the known data, λ_α are the optimal weights and $z^*(\mathbf{u}_\square)$ is the estimate at the unsampled location $\mathbf{u}_\square$, for all locations within the geologic domain A (Rossi and Deutsch, 2014). In practice, each SMU is estimated by weighting a number of nearby samples found in a local search neighborhood and employing a variation of the linear estimator described.

2.3.1 Simple Kriging

Simple Kriging is the most basic form of kriging. Imposing no constraints, Simple Kriging yields the minimum error variance estimate. Prior knowledge of the mean and a strict decision of stationarity are required. The known mean is removed and estimates are performed on the residuals. The mean is added to the estimated residuals for an unbiased estimate conditioned to the original data. In many practical settings it is unrealistic to assume a completely stationary domain which renders simple kriging impractical for most situations.

2.3.2 Ordinary Kriging

Ordinary Kriging permits a less strict definition of stationarity and an unknown mean. By constraining the sum of the weights to equal one, unbiasedness is achieved and the requirement of strict stationarity is relaxed; the assumption is more local than global. Ordinary Kriging is a non-stationary technique; only assuming a constant mean within the local search neighborhood for each estimate. Due to the imposed constraint, the minimized error variance from Ordinary Kriging is higher than that of Simple Kriging. Despite this, Ordinary Kriging is common in practical application for its non-stationary nature.

2.3.3 Histogram Smoothing and The Neighborhood Effect

All estimation methods yield an overly smooth distribution of estimates (Rossi and Deutsch, 2014). This results in under representation of the high and low values in the modeled domain giving an unrealistic estimate of overall tons and grade. By limiting the search neighborhood for each estimate histogram smoothing can be remedied at a cost. Through an iterative process, to determine the necessary number of data per estimate, the predicted SMU distribution can be reproduced. However, limiting the number of data available for each estimate can induce a significant conditional bias (McLennan and Deutsch, 2002) and the smoothing of the spatial distribution of estimates is not addressed.

2.3.4 Conditional Bias

In resource modeling an unbiased model is desired. Unfortunately, estimation techniques can be prone to conditional bias. As described by McLennan and Deutsch (2002), a model is considered conditionally biased when the true grade (Z_v) is not equal to the estimated grade (Z_v^*) for all grade ranges, z, that is:

$$E\{Z_v|Z_v^* = z\} \neq z \tag{2.2}$$

Conditional bias can be minimized by using a large search neighborhood for each estimate, at the cost of histogram smoothing. A limited search radius increases conditional bias by over estimating high values and under estimating low values to artificially increase the variability of the estimates. This can and should be checked by cross validation or jacknife.

When considering an estimated model for mine planning it is impossible to achieve both the correct distribution of block grades and a conditionally unbiased model (Isaaks, 1999). Consider the two options:

1. Take advantage of the neighborhood effect and limit the number of data per estimate to generate the correct distribution of block grades.

2. Minimize conditional bias using a large search radius and accept a smooth distribution of block grades.

If the first option is chosen, the tons and grade above cut off can be accurately estimated for mine planning but the estimated grades are conditionally biased. The second option minimizes conditional bias but the estimated tons and grade will not be accurate due to histogram smoothing. This is the oxymoron described by Isaaks (1999). Estimation techniques force the practitioner to choose between an unbiased model, or an accurate distribution of SMU grades. To achieve both, probabilistic methods could be considered.

2.4 Probabilistic Methods for Quantifying Uncertainty

Geostatistical modeling is commonly based on widely spaced data leading to inevitable uncertainty in estimation. Rather than accept inaccurate estima-

tion methods, probablisitic methods are available to quantify the uncertainty and generate a distribution of possible values for every unsampled location. MultiGaussian Kriging, Sequential Gaussian Simulation, Uniform Conditioning and Indicator Kriging are discussed here. These are the most widely used techniques in practice.

2.4.1 MultiGaussian Kriging

MultiGaussian Kriging relies heavily on the principles of the Gaussian distribution and the well known quantile to quantile normal score transform. The methodology is relatively straightforward. An overview of the steps is given here, the process is well documented in many references including (Rossi and Deutsch, 2014). A known distribution of grade from the composite data, is transformed to a Gaussian distribution with a mean of zero, and a variance of one. The Gaussian transformed distribution is considered as conditioning data for kriging that is applied within the framework of a multivariate Gaussian distribution.

Simple kriging is performed at the point scale across the domain. This process generates a conditional mean and variance in Gaussian units for each location. To generate the local conditional distributions in original grade units, post processing is required. The local Gaussian distribution can be back transformed to original units using a similar quantile to quantile transformation. The final result is a local distribution of uncertainty, in original units, for each unsampled location. Only point scale kriging is appropriate when considering Gaussian transformed data, therefore; the resulting local distribution is also at the point scale. Each local distribution can be upscaled to the appropriate SMU scale. Analytical methods such as the Affine Correction are often employed for this purpose and will be discussed in Chapter 3.

MultiGaussian Kriging is fast, straightforward and provides a measure of local uncertainty. Post processing relies on an analytical variance correction and no assessment of joint uncertainty is available. The measure of uncertainty generated by MultiGaussian kriging only pertains to a specific SMU, independent of uncertainty at surrounding locations. Joint uncertainty can

only be achieved through multiple realizations via a simulation technique like Sequential Gaussian Simulation.

2.4.2 Sequential Gaussian Simulation

Sequential Gaussian Simulation (SGS) makes similar assumptions as Multi-Gaussian Kriging but quantifies uncertainty through a set of realizations rather than a local conditional distribution for each SMU. For each location in a realization one simulated value is drawn from the local conditional distribution and then retained to condition subsequent locations. Considering previously simulated values preserves joint uncertainty between all locations. Performed at the data scale, this process correctly reproduces the histogram of the variable being modeled. Multiple realizations provide a more complete assessment of uncertainty. This technique has been discussed in many geostatistical texts, including: Deutsch and Journel (1997), Chilés and Delfiner (2012), Rossi and Deutsch (2014).

Sequential Gaussian Simulation, like MultiGaussian kriging, requires the data to be transformed to a standard normal (Gaussian) distribution. A location requiring a simulated value is then chosen at random. By solving the normal equations (similar to Simple Kriging), a conditional mean and variance are generated for the conditional distribution at this location. In Gaussian units, these two parameters define the local conditional distribution. A random value is chosen from this distribution and retained as a simulated value at this location. This simulated value is now incorporated into the set of data available for estimating the conditional mean and variance at surrounding locations. This process is repeated for the entire grid, retaining a simulated value at each location to complete a realization. The set of realizations is generated at the data scale and change of support is required to represent SMU scale uncertainty. Variance reduction, in regards to the support effect is accomplished by block averaging. This requires averaging the point scale simulated values within each SMU volume to generate a set of SMU scale realizations and will be discussed further in Chapter 3.

Sequential Gaussian Simulation is more complex than MulitGaussian Krig-

ing and requires additional computational time. Both the histogram and variogram are reproduced with statistical fluctuations. Histogram smoothing is avoided, yielding a more realistic portrayal of the deposit. Utilizing multiple realizations for resources and reserve evaluations requires updating standard practices and transferring uncertainty through to long range planning and evaluation. Rather than calculate resource and reserves in a conventional fashion with a single model, the set of all realizations must be considered.

2.4.3 Uniform Conditioning

Uniform Conditioning is unique, this technique estimates a local distribution of SMU grades within larger scale panels. The panel scale is a larger volume that contains a number of SMUs and that is more reliably estimated. A robust estimate, in original units, at the panel scale is the basis of Uniform Conditioning. The panel scale estimates are transformed to a Gaussian distribution. A bivariate Gaussian relationship between the SMU scale and Panel scale is assumed based on calculated change of support coefficients (Neufeld, 2005). From the bivariate relationship between the SMU and panel scales a mean and variance, in Gaussian space, can be calculated for the SMU distribution within each panel. Prior to back transformation, the proportion of the SMU distribution above a chosen cutoff can be calculated for each panel. The SMU scale Gaussian distribution can then be back transformed for a local distribution of uncertainty in original grade units at the SMU scale within each panel.

Uniform Conditioning is simple and requires little computational expense over kriging. In some situations this process may be too simple. As the name implies, Uniform Conditioning assumes that conditioning data are regularly spaced. This requirement paired with the globally defined change of support coefficients needed to define the relationship between the SMU distribution and the panel values leaves little room for local variation. Recent work by Emery (2008) suggests the use of locally varying change of support coefficients for a more accurate change of support.

The most notable inconvenience of Uniform Conditioning is the panel scale resolution of the model. Although the distribution of uncertainty represents

the SMU scale, no local precision is offered within each panel. The increasingly popular technique of localizing panel scale distributions offers histogram reproduction and resolution at the SMU scale through post processing (Abzalov, 2006). Localization aside, Uniform Conditioning offers a straight forward methodology for situations appropriate to the panel scale.

2.4.4 Indicator Kriging

The three aforementioned methods for quantifying uncertainty depend heavily on the Gaussian distribution. Indicator Kriging is non parametric. Rather than estimate the parameters of a Gaussian distribution, the distribution of a continuous variable is estimated via binary indicators (Journel, 1983). Indicators are kriged and local distributions are constructed through post processing to generate conditional distributions at the SMU scale.

Specified grade cutoffs define the binary transformation of the distribution. Grade cutoffs are chosen from the representative global distribution such that the series of indicators adequately describes the cumulative distribution function (CDF) . The number of indicators and the density of discretization is defined by the practitioner. A variogram model is required for each indicator. The set of variogram models must vary smoothly, and the spatial relationship of each indicator cannot be assumed entirely independent of the others. Once established, the set of variogram models is used to krige each indicator at the point scale.

Each kriged indicator yields an estimated point on the local conditional CDF for each unsampled location. Order relations corrections, interpolation between points and tail extrapolation are performed to construct the local distribution for the continuous variable. Similar to MultiGaussian Kriging, local distributions require variance reduction by an analytical technique to represent the SMU scale.

The most notable drawback of Indicator Kriging is the potential for order relations issues in local conditional distributions which can be fixed in post processing as described by Rossi and Deutsch (2014). Although work intensive, Indicator Kriging requires little computational expense over kriging and

provides a non-parametric approach to quantifying local uncertainty allowing the user to reduce reliance on the Gaussian distribution.

2.5 Recommendations

Each of the four methods discussed can generate a model of uncertainty, but the complexity of implementation and the final results differ. Determining which technique is appropriate can be based on prior experience or company policy. When the decision is left to the practitioner the options must be evaluated relative to the ease of use, computational expense and the reliability of resulting model. Cross validation and reconciliation with historical production data would also provide a basis to choose one option over the other.

2.5.1 Complexity and Computational Expense

The complexity of geostatistical modeling is often overshadowed by the challenge of geologic domaining, effective data management and preparation. In most cases, complexity is minimized when possible. MultiGaussian Kriging and Uniform Conditioning are the simplest of the methods described. Both require one kriging and a table look up to produce a measure of local uncertainty. Indicator kriging is more complex. Choosing appropriate cutoff values, modeling the entire suite of variograms and reconstructing local distributions requires substantial user input and often multiple iterations.

In terms of computational expense, simulation stands alone as the most memory and time intensive method. Through parallelization and increased computing power the additional expense is manageable. Once a set of realizations has been generated, the benefits of joint uncertainty coupled with histogram and variogram reproduction likely out weighs the cost in computational expense of simulating.

2.5.2 Joint Local Uncertainty

It is important to distinguish the two different uncertainty measures mentioned; local uncertainty and joint uncertainty. All of the above methods provide a distribution of possible values for each SMU, this is local uncertainty.

Only simulation provides joint uncertainty through multiple realizations. This implies that for any given realization the value estimated at an unsampled location is dependent on surrounding estimates. For this reason only a series of realizations can be used to quantify global uncertainty across the domain or in a given area of interest.

As an example, consider a planned cut (A') to be mined as part of the annual production at a mine within a larger domain. The designed cut is a collection of appropriately sized SMUs, one of which is highlighted (v) in Figure 2.1. Each method described can provide a distribution of possible values for the highlighted block ($F_v(\mathbf{u})$), but only through a series of realizations ($F^{\ell,\ldots,L}$) of cut A' is it possible to calculate the uncertainty in total tons and grade within a specific volume containing a number of SMUs. Uncertainty in total tons, grade, or other important factors can be calculated for nearly any volume larger than an SMU, including a single cut or the entire domain.

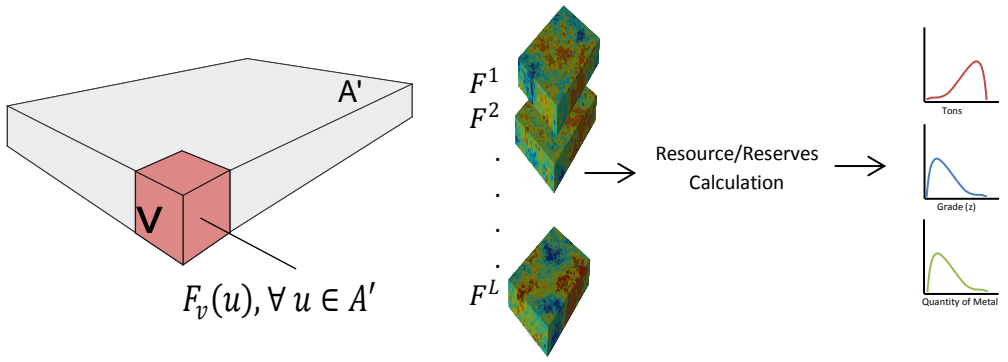

Figure 2.1: Measures of local uncertainty generate a distribution of values for block v while a complete set of simulated realizations make it possible to evaluate the uncertainty in a larger volume A'. Joint uncertainty through multiple realizations yields multiple responses for the variable of interest as shown in the sketch at right.

Considering joint uncertainty through a transfer function yields a distribution of results. In the case of cut A' this requires performing the calculations on each realization for a distributions of responses as shown schematically in Figure 2.1. Rather than accept estimates that are likely inaccurate, or a distribution only pertaining to a single block, joint uncertainty quantifies the possible outcomes for a given cut, or the entire domain, and can be transfered

through the planning process.

Considering uncertainty in planning and risk evaluation is ideal, but when exceptions arise and a single value or a specific local distribution is required, a series of realizations can provide these measures as well. A single block value can be extracted from a set of realizations to form a local distribution similar to the results of Indicator Kriging or MultiGaussian Kriging. If circumstances require a single estimate, the expected value of each local distribution can be calculated. Each of these measures can be calculated from a set realizations, but only simulation can generate joint uncertainty for evaluating uncertainty in volumes larger than an SMU.

2.5.3 A Common Format

Although only simulation provides joint uncertainty via realizations, a local distribution of uncertainty can be used in transfer functions for resource calculation and localization. To make this practice more convenient a standardized format is necessary. To effectively manage a local distribution it can be discretized into a series of quantiles ordered in a similar fashion to a series of realizations.

The ordering of gridded files is often specific to individual software platforms, the open source GSLIB format is used as a reference here. In the GSLIB convention a regular grid is defined in a specific order beginning at the lowest, Southwestern-most SMU. Blocks are listed increasing to the East, then row by row to the North, and finally level by level upward, i.e., x cycles fastest, then y, and finally z (Deutsch and Journel, 1997). In the case of multiple realizations this ordering is repeated for the total number of realizations within the set. This format can be extended to local distributions of uncertainty by ordering the first quantile for each SMU in this fashion, followed by a complete ordering of the 2nd quantile from each local distribution etc. This format is different than the standard output from existing GSLIB software for MultiGaussian Kriging, Uniform Conditioning and Indicator Kriging. Post processing programs for Uniform Conditioning, Indicator Kriging and MultiGaussian Kriging have been developed and updated to generate an additional

output file in the format described (see Software Appendix). The common format provides ease of use to the practitioner for flexible transfer functions regardless of the geostatistical method.

2.6 Conclusions

Estimation methods are commonly used within the mining industry. It is impossible to construct a conditionally unbiased model with the correct distribution of estimates due to histogram smoothing. Additionally, a deterministic model overlooks inevitable uncertainty in the estimates. Considering an unbiased model of uncertainty with the correct distribution of estimated values is a more appropriate for evaluating recoverable reserves. Uncertainty for long term planning is most valuable when available at both the local and global scale, this is possible only via simulation. Despite the marginal increase in time and complexity, the benefits of joint uncertainty support simulation as best practice.

In a practical setting, managing local distributions of uncertainty or multiple realizations can be challenging. For convenience, local distributions can be reduced to a single value by calculating the expected value for each SMU. This is not recommended. Similar to kriging, only considering the expected value of each distribution leads to issues of histogram smoothing and conditional bias. It is advantageous to consider the entirety of the SMU scale distribution through a transfer function.

Prior to processing results through a transfer function, three of the four methods discussed requires a change of support technique to accurately represent the SMU scale distribution. Uniform Conditioning is the exemption due to an inherent change of support process within the methodology. Fast and simple analytical methods such as the Affine or Indirect Lognormal corrections are often applied to the data scale results of MultiGaussian and Indicator Kriging. Multiple realizations are commonly upscaled from the simulated point scale to the SMU scale through block averaging. The effectiveness and global assumptions of some change of support technique have been questioned

and studied by some, including: Rossi and Parker (1993), Emery (2008), and Chilés and Delfiner (2012). This warrants further investigation into each variance correction technique. A careful assessment of how to best account for the support effect and accurately represent SMU scale uncertainty is required.

Chapter 3

Location Dependence of Change of Support Parameters

3.1 Change of Support

A valid model of uncertainty at the SMU scale requires a change of support model since MutiGaussian Kriging, Simulation and Indicator Kriging occur at the point scale. A change of support model specifies the upscaling to the appropriate SMU size. Uniform conditioning has a panel scale and an SMU scale change of support model embedded as part of the process while other methods require post processing.

Multiple methods have been developed for change of support. The Affine Correction and Indirect Lognormal Correction are analytical techniques dependent on a calculated variance reduction factor. The more complex Discrete Gaussian Model (DGM) depends on a bivariate Gaussian assumption between scales and a series of Hermite polynomials to fit the distribution and facilitate the relationship between scales. Lastly, a numerical approach is available to directly average the modeled data scale values within the larger SMU volumes. The goal of each technique is to accurately portray the modeled distribution at the SMU scale for resource evaluation, risk analysis and mine planning.

3.1.1 The Support Effect

The support effect describes the reduction in variability from the data scale to the defined SMU block scale. This is contrasted with the information effect. While the information effect predicts the increase in variance due to additional

data, the support effect accounts for a decrease in variability due to an increase in scale.

The support effect can be understood by averaging the point scale data values to the volume of interest. Chilés and Delfiner (2012) describes this by the following summarized properties:

1. The block mean grade, without consideration of a cutoff, is independent of block size.

2. The marginal distribution of block grades gets narrower around the mean as the support volume increases.

3. The marginal distribution of block grades tends to become Gaussian when the support tends to infinity in all directions.

The variance of the global and local distributions in a probabilistic model should accurately represent the grade information at the volume of interest. This should be considered in conjunction with the information effect. The increase in variance caused by the information effect can be predicted. It is important that a resource model at the SMU scale reproduce the same variability to confirm that the resources and reserves being estimated are reasonable.

3.2 Global Variance Reduction Methods

A geostatistical model at the data scale can be upscaled to represent SMU volume through post processing. This requires a reduction in variance for each SMU distribution. Standard variance reduction techniques assume a constant change in variance for all locations regardless of the local circumstances such as proximity to drillholes. A numerical example will show that variance reduction varies across a domain depending on the conditioning data that are present.

3.2.1 The Global Variance Reduction Factor

Established analytical methods for variance reduction include the Affine Correction and the Indirect Lognormal Correction. These methods are common for post processing results of MultiGaussian Kriging or Indicator Krig-

ing (Deutsch and Journel, 1997). Both methods start from data scale local distributions and apply variance reduction as a post processing step. The corrections rely on a variance reduction factor, f_{global}, that is calculated in expected value over the entire domain. This is the ratio of the block scale variance to the data scale variance (Rossi and Deutsch, 2014). In practice, the f_{global} value is calculated using the average variogram (Equation 3.1). The established relationship between the average variogram, $\bar{\gamma}$, and the variance reduction factor, f_{global}, is given below:

$$\bar{\gamma}(v,v) = \frac{1}{|v| \cdot |v|} \int_v \int_v \gamma(x-y)dxdy \tag{3.1}$$

The average variogram, $\bar{\gamma}$, is equal to the expected value of the variogram, γ, within the block v. This is used to approximate the global correction factor:

$$f_{global} = 1 - \frac{\bar{\gamma}(v,v)}{\sigma^2} \tag{3.2}$$

(Rossi and Deutsch, 2014)

In practice, Equation 3.2 is commonly used to evaluate the global volume variance correction factor. Although Equation 3.1 presents an integration approach to gammabar calculation, this is calculated numerically in a practical setting. This is accomplished by discretizing the SMU volume of interest into a number of points within the block. The distance from each discretized point within the block to all other points in the block is measured and a variogram value can be calculated using the variogram model fit to the experimental data. These values are averaged for an approximation of the average variogram. The approximated average variogram and the variance of the point scale data are used to calculate the to global variance correction factor.

3.2.2 The Affine Correction

The Affine correction is appealing for its simplicity. This method keeps the shape of the distribution exactly the same and reduces variance based on the

calculated global variance reduction factor, f_{global}, by transforming values of the sample distribution as shown in Equation 3.3.

$$z' = \sqrt{f_{global}} \cdot (z - m) + m \tag{3.3}$$

Where z is a value from the distribution requiring variance reduction, m is the mean of that sample distribution, f_{global} is the variance correction factor calculated in Equation 3.2, and z' is the transformed value corresponding to the distribution with a reduced variance.

Under the permanence of shape assumption, the affine correction is appropriate for situations where $f_{global} \geq 0.7$ (Journel and Huijbregts, 1978). More recent findings by Rossi and Parker (1993) suggest that even for small changes the Affine correction provides incorrect results. Despite its limitations the affine correction is commonly implemented for use as a simple post processing step to reduce variance. Despite these findings, the Affine correction using a global reduction factor is common in resource modeling (Vann and Guibal, 1998).

3.2.3 The Indirect Lognormal Correction

The Indirect Lognormal Correction assumes that both data and SMU scale distributions are lognormal with the same mean and different variances. Under this assumption change of support is accomplished in a series of three steps as described by Isaaks and Srivastava (1989) and Rossi and Deutsch (2014).
First, solve for the coefficients of a and b:

$$a = \frac{m}{\sqrt{f_{global} \cdot CV^2 + 1}} \left[\frac{\sqrt{CV^2 + 1}}{m} \right]^b \tag{3.4}$$

$$b = \sqrt{\frac{ln(f_{global} \cdot CV^2 + 1)}{ln(CV^2 + 1)}} \tag{3.5}$$

where CV is the coefficient of variation of the data scale distribution, m is the mean, and f_{global} is the variance reduction factor as calculated in equation

3.2. The calculated a and b values are used to transform quantiles of the SMU distribution by the equation:

$$q' = aq^b \tag{3.6}$$

Finally, a mean correction is required unless the two input distributions are truly lognormal:

$$q'' = \frac{m}{m'} \cdot q' \tag{3.7}$$

The indirect lognormal correction reduces the variance based on the calculated f_{global} factor in a predictable fashion. The assumption that both distributions are lognormal is limiting, but no artificial minima or maxima values are introduced as in the Affine correction.

3.2.4 The Discrete Gaussian Model

The required assumptions of the Indirect Lognormal and Affine Corrections are limiting. Sample distributions are rarely lognormal and the permanence of shape is unrealistic. The Discrete Gaussian Model (DGM) is a more robust alternative requiring neither of these assumptions. The Discrete Gaussian Model uses orthogonal Hermite Polynomials to fit and scale the sample distribution by a variance correction factor similar to f_{global}. An overview of the Discrete Gaussian Model will be given here. For a more comprehensive explanation see: Machuca-Mory et al. (2008) and Chilés and Delfiner (2012).

To achieve variance reduction via the Discrete Gaussian Model the data scale experimental distribution is fit to a standard normal (Gaussian) distribution by an anamorphosis function:

$$z(\mathbf{v}) = \phi(y(\mathbf{v})) \approx \sum_{p=0}^{\infty} \phi_p H_p(y(\mathbf{v})) \tag{3.8}$$

where z is a data in original units, y is value in Gaussian units, ϕ_p is the coefficient for the p-Hermite Polynomial value $H_p(y(\mathbf{v}))$ (Rossi and Deutsch, 2014). The relationship between the original distribution and the Gaussian transformed distribution is described by this anamorphosis function and the Hermite Polynomials fit to the original distribution. Change of support is accomplished by scaling the anamorphosis function coefficients by the change of support coefficient, r^p, to the appropriate volume, $\mathbf{v}$:

$$Z(\mathbf{v}) = \phi(y_v(\mathbf{v})) = \sum_{p=0}^{\infty} r^p \phi_p H_p Y(\mathbf{v}) \tag{3.9}$$

where r is determined based on the variance at the SMU scale as calculated using the average variogram:

$$\sigma_v^2 = \sigma_u^2 - \bar{\gamma}_{v,v} \approx \sum_{p=1}^{n} r^{2p} \phi_p^2 \tag{3.10}$$

This technique yields an SMU scale distribution that represents the variability predicted by the average variogram. The Discrete Gaussian Model is more robust than either the Affine or Lognormal Corrections because the Gaussian transform is general and the process naturally leads to a more Gaussian distribution as the scale is increased (Rossi and Deutsch, 2014).

The Discrete Gaussian Model is suited to predicting the change in variance when increasing the volume to the SMU or larger scale. Uniform Conditioning is unique because the estimates must be downscaled from panel estimates to the SMU scale distributions. Two different change of support coefficients are considered, r_V for the panels and r_v for the SMU distribution. These are incorporated into the Discrete Gaussian Model as shown in equation 3.9. It is assumed that the ratio between the change of support coefficients at each scale identifies the correlation coefficient, ρ, that describes the bivariate relationship between the normal scores of the two distributions (Figure 3.1).

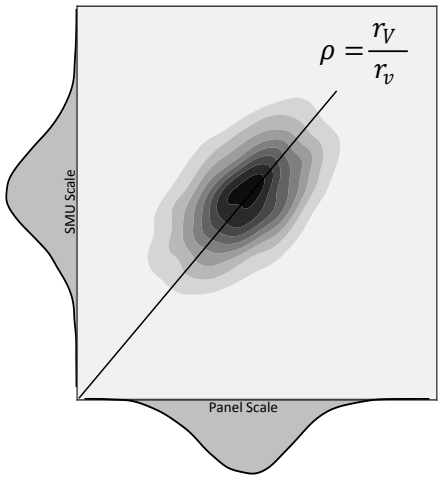

Figure 3.1: The SMU scale distribution in Gaussian units is inferred from normal score of the panel distribution using the correlation coefficient. It is assumed that the correlation coefficient is equal to the ratio of the two r values.

Similar to the variance reduction factor, the change of support coefficients used in the Discrete Gaussian Model are assumed constant throughout the domain. When local conditional distributions are considered the assumption of a global variance correction factor or change of support coefficient is problematic, this will be demonstrated by a simulation test.

3.3 Local Variance Correction

Each of the change of support techniques presented relies on a global assumption in some way. In the case of the Affine Correction and the Lognormal Correction this is the global variance correction factor. In the case of the Discrete Gaussian Model the change of support coefficients dictate the change in variance. Emery (2008) takes a close look at the DGM technique for the Gaussian case embedded in Uniform Conditioning, concluding that the change of support coefficient varies locally and is closely related to the simple kriging variance. The local variation of the change of support coefficients in the Gaussian case motivates further investigation of local variation for non-Gaussian

distributions in a Uniform Conditioning framework as well as the local dependencies of the variance correction factor, applied in the Affine and Lognormal Corrections.

3.3.1 A Numerical Approach to Variance Correction

The simulation based tests designed to challenge global change of support assumptions take advantage of the simple and dependable block averaging technique for variance reduction. Commonly applied in a simulation framework, realizations at a high resolution (the data scale) are upscaled by averaging simulated data scale values within each SMU volume.

By averaging simulated values within each SMU the high and low values are dampened, therefore appropriately reducing the variance at each location. This process is performed for each SMU, over all realizations. There is no dependence on a reduction factor, global assumption, or permanence of shape making this strategy the most robust technique for variance reduction.

3.3.2 Local Variance Correction for Analytical Methods

To test the consistency of the global variance correction and the relationship of the point and block scale dispersion variances for each SMU simulation is used. To accomplish this, a series of high resolution realizations, conditioned to synthetic data, are generated. To check the value of variance correction factor locally it is calculated for each SMU. This requires calculating the dispersion variance at both the point and block scale for each SMU in the domain.

Simulation Test of the Variance Correction Factor

To begin, a synethic dataset is generated via unconditional simulation using an isotropic spherical variogarm with a range of 100 for the normal scores: $\gamma = 0.1 + 0.9sph_{100}$. The realization is transformed to a lognormal distribution with a mean of one and a standard deviation of two. Data with variable spacing are extracted from this realization are displayed in Figure 3.2.

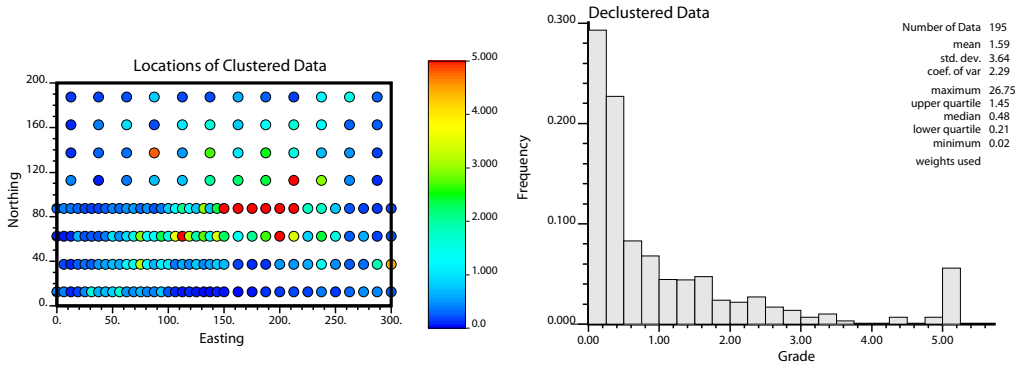

Figure 3.2: Location map and histogram of original data for simulation based example. Note: the high value above 5.00 represents all higher values. This is a synthetic dataset therefore no units are given for distance or grade values.

In addition to the location map a histogram is also displayed in Figure 3.2. The distribution of data is positively skewed with a declustered mean of 1.59 and a standard deviation of 3.64. Sequential Gaussian Simulation is used to create two hundred realizations conditional to the synthetic data. The same variogram used to generate the data is again used for simulation in Gaussian space. After simulating, realizations are back transformed to lognormal units and checked for histogram reproduction in Figure 3.3

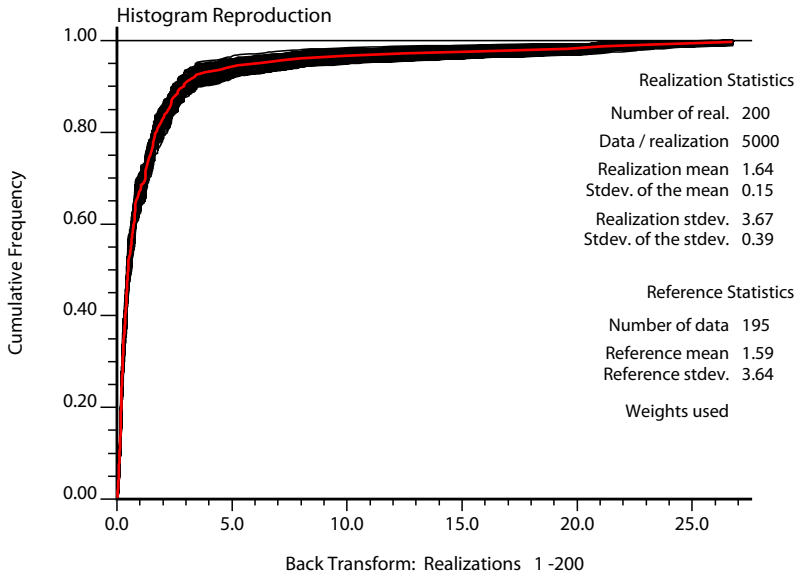

Figure 3.3: The distributions of the two hundred point scale realizations plotted against the weighted original data. Units are arbitrary for this synthetic example.

A single realization from the set of two hundred is displayed in Figure 3.4 showing the anticipated high grade region east of center.

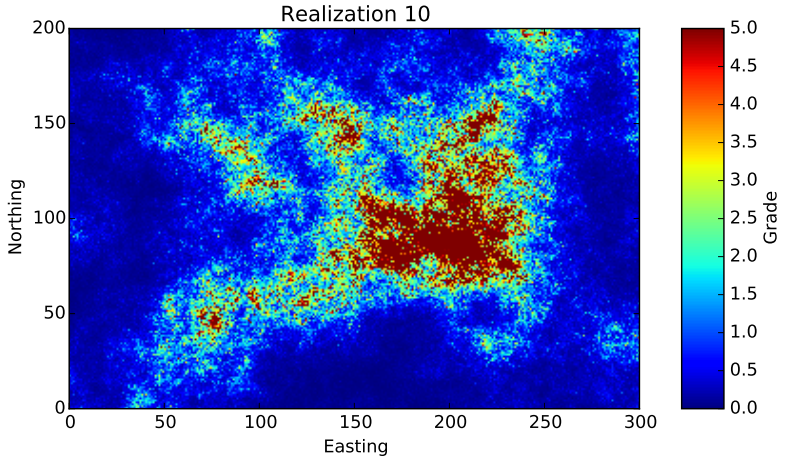

Figure 3.4: A single realization from the set of two hundred. Units are arbitrary for this synthetic example.

To calculate the dispersion variance at the point scale, realizations are post

processed using `postsim` (Deutsch and Journel, 1997) to calculate both the mean and variance at the point scale over the set of realizations. The point scale dispersion variance is block averaged over a 5x5 SMU volume. This yields the average point variance, $D^2(\cdot, G)$, within each 5x5 SMU (Figure 3.5).

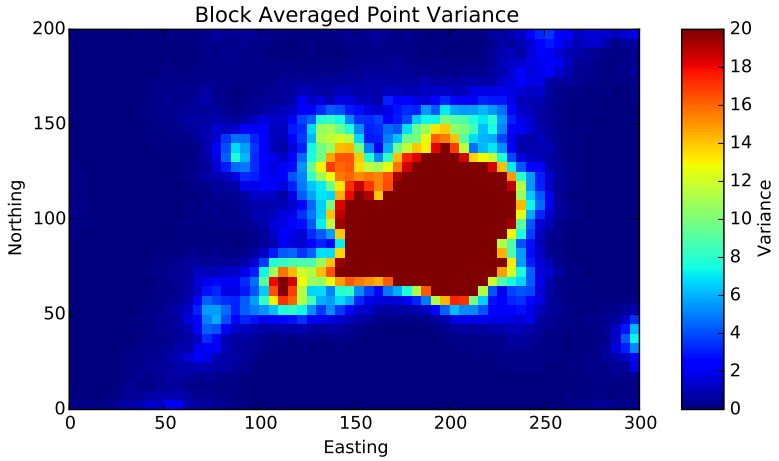

Figure 3.5: The block averaged point variance, $D^2(\cdot, G)$, for each SMU in the domain. Units are arbitrary for this synthetic example.

The block variance, $D^2(v, G)$, is calculated using the same set of simulated realizations. The results of the point scale simulations are block averaged to SMU scale. Realizations at the block scale are post processed to calculate the local distribution for each SMU. The block scale variance, $D^2(v, G)$, is extracted from each local distribution and plotted in Figure 3.6.

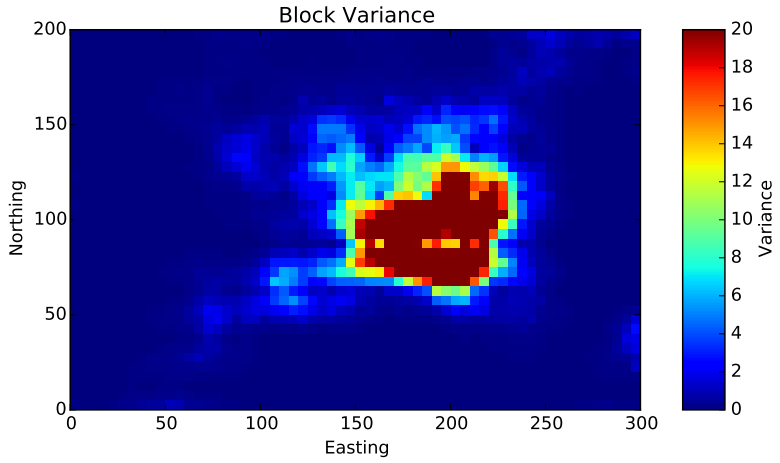

Figure 3.6: Block variance, $D^2(v, G)$, calculated for each SMU in the domain.Units are arbitrary for this synthetic example.

To find the local variance correction factor for each block, the ratio between the point scale variance and the block variance for each block is calculated for each SMU.This generates the distribution local variance correction factors.

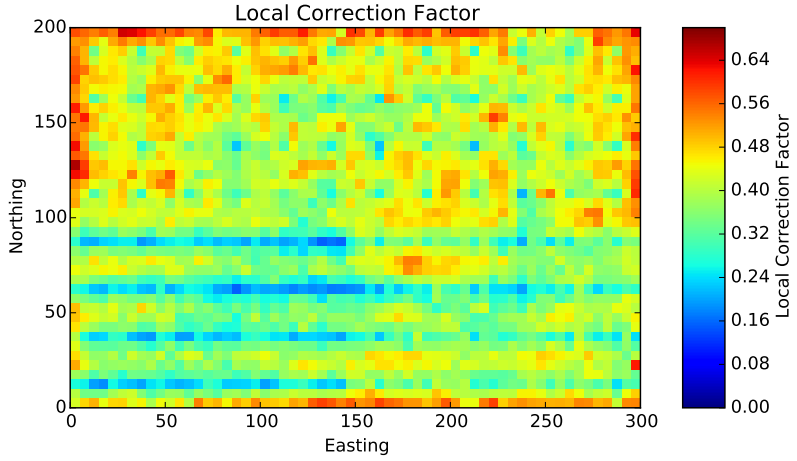

Figure 3.7: Map of Local Volume Variance Correction factor.Units are arbitrary for this synthetic example.

The distribution of values calculated for the local variance correction factor is displayed in Figure 3.8.

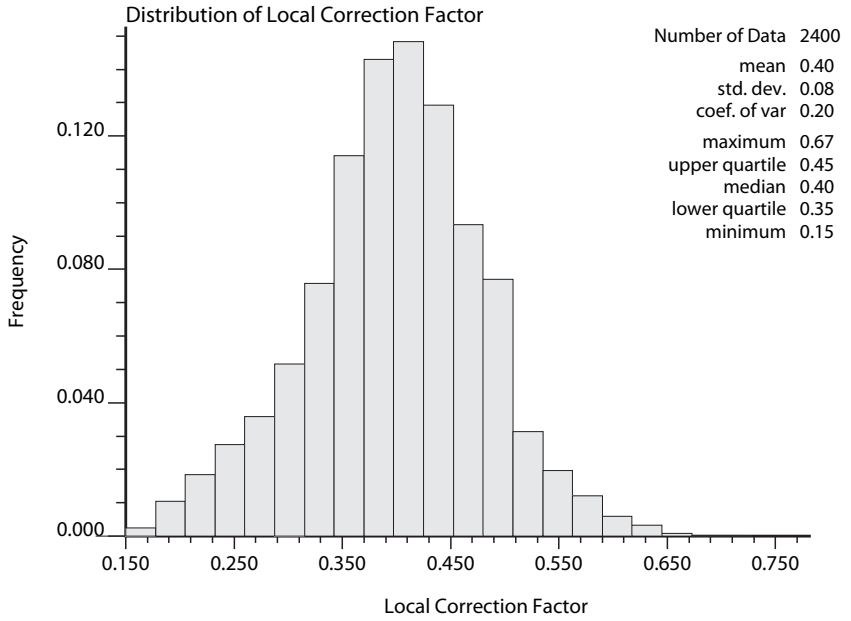

Figure 3.8: Distribution of Local Volume Variance Correction values. For comparison, the global correction factor is 0.76 calculated using `gammabar` (Deutsch and Journel, 1997).

The local variance correction factor, calculated from the conditionally simulated realizations is not consistent throughout the domain (Figure 3.7). The distribution of local variance correction values has a mean of 0.40, with a minimum of 0.15 and a maximum of 0.67. The global correction factor is also calculated, based on an inferred average variogram in original units for a value of 0.76 (Equation 3.2).

The local correction factor is lower than the approximated global correction factor and the spatial configuration shows higher values in areas of sparse data (Figure 3.7). A set of two hundred unconditional realizations, generated using the same reference distribution as the previous test, reveals that the local correction factor approaches the global correction factor when no conditioning data is present. The distribution below shows the calculated local correction factor calculated from the set of unconditional realizations:

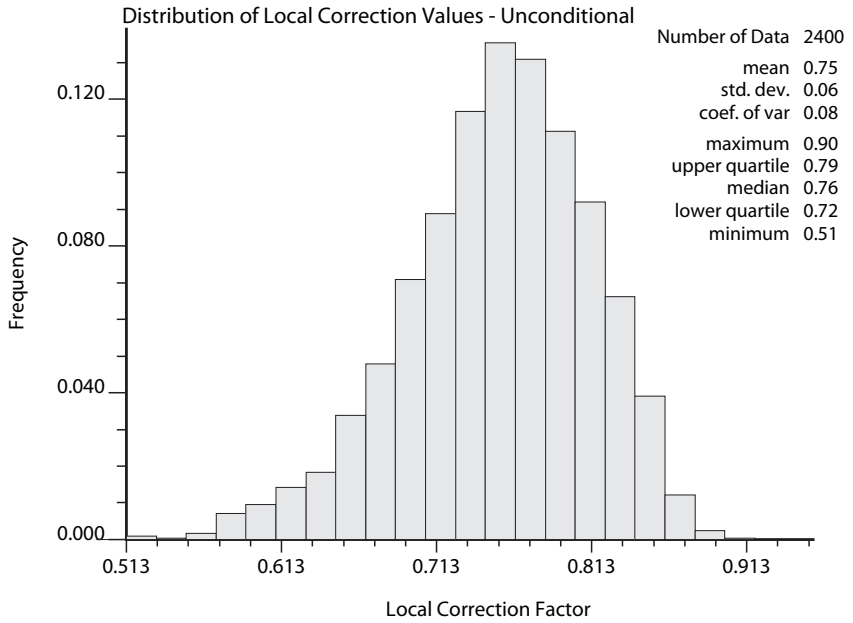

Figure 3.9: Distribution of Local Volume Variance Correction values for the unconditional example. For comparison, the global correction factor is 0.76 calculated using `gammabar` (Deutsch and Journel, 1997).

In this example the mean of the local correction factor values is 0.75, while the global correction factor is approximately 0.76. These findings show that the mean local correction factor is equivalent to the approximated global correction factor when no conditioning data is present.

The results of the example simulation tests (Figure 3.7) suggest that a local correction factor should be applied to the local distribution at each SMU to accurately predict uncertainty at the SMU scale in the presence of conditioning data. A closed form analytical solution to the local variance correction factor would be convenient; however, due to the non-linear transform to Gaussian space this is not practical. For an approximation of the local variance correction factor, the following steps are suggested:

1. Choose an a representative volume for a smaller scale study of local variance reduction and simulate at a high resolution.

2. Extract the point scale variance for each location.

3. Block average the point scale variance within each SMU.

4. Using the same set of realization, block average the simulated values and extract the SMU scale variance.

5. Calculate the local correction factor for each location based on the ratio of the average point scale variance to the SMU scale variance for each block.

6. Build a relationship of the local reduction factor to a known statistic such as: the mean, variance, grade, etc.

7. Use the defined relationship to approximate the local correction factor for each SMU in the model.

To implement the suggested approximation method, consider the example simulations previously generated (Figure 3.4). Having simulated a high resolution and calculated the point scale uncertainty the local correction factor is known for this small area. A relationship between the Simple Kriging variance and the local correction factor is approximated by a logarithmic function (Figure 3.10). Using the function fit to this relationship the local correction factor can be approximated for any location in this small domain or extended to a larger area, assuming the area tested is representative.

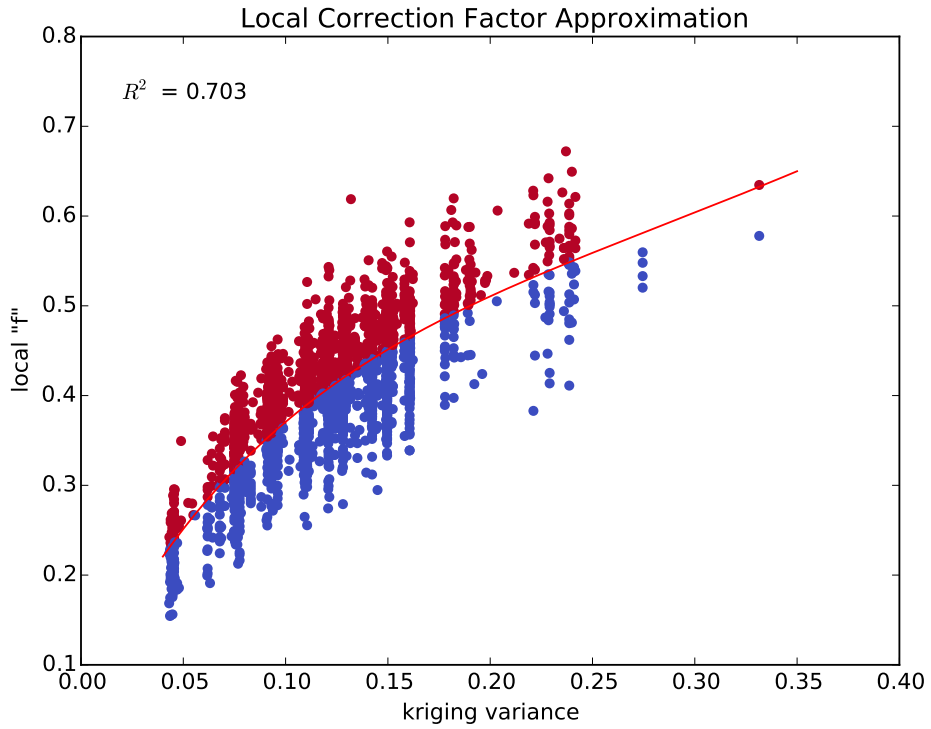

Figure 3.10: The local correction factor plotted against the simple kriging variance. The logarithm of a polynomial function has been fit to the plotted relationship for use as a proxy. A similar method can be used when calculating the local correction factor for the entire domain is impractical.

Approximating the local correction factor avoids the need to simulate the entire model at a high resolution which can be time consuming and computationally expensive. Moreover, if the entire domain were simulated then there would be no need for a heuristic fit for MulitGaussian or Indicator Kriging. Once calculated for each SMU the local correction factor can be incorporated into to the change of support methodology using the updated version of either `PostIK_Loc` or `PostMG_Loc`. Both Programs allow for volume variance correction using a gridded file of variance correction values corresponding to each SMU. The updated parameter files for each are available in the software appendix.

3.3.3 Local Variance Correction in Uniform Conditioning

Based on the findings presented above one would suspect that the change of support coefficients in the DGM framework also vary locally. This has been demonstrated for the Gaussian case by Emery (2008). It is rare that an inherently Gaussian distribution will present itself in a mining context; therefore it is necessary to investigate the local dependence of the change of support coefficients in a more realistic setting.

Testing for a Lognormal Case

For this example a lognormal distribution for a single panel is considered in conjunction with a series of possible SMU sizes. First, the correlation coefficient, ρ, between the panel and SMU scales calculated for each SMU volume considered is calculated numerically. Next, though a series of steps the change of support coefficients are determined for each scale and the correlation coefficient, ρ_{UC}, determined by Uniform Conditioning theory is calculated. The numerically calculated correlation coefficient is compared to the theoretically calculated values to test the reliability of the the UC approximation in the presence of varying amounts of conditioning data. Examples with 1, 2, 3, 4 and exhaustive (86 data) are tested (Figure 3.11).

To begin, a single unconditional realization of the 64x64x10 panel is generated using a spherical variogram model: $\gamma = 0.1 + 0.9Sph_{128}$. This realization will be used for extracting any conditional data used in the following examples, ensuring that the correct spatial relationship exists between the conditioning data.

To find the numerical correlation coefficient between the panel and SMU scales, one thousand realizations of the single 64x64x10 panel are generated at the point scale. Each realization is block averaged from the point scale to the panel scale, and also to the SMU scale. The bivariate relationship between the two scales of the same set of realizations can now be determined and summarized by calculating the correlation coefficient, ρ. This is done for an unconditional case and for each of the trials with conditioning data

(Figure 3.11).

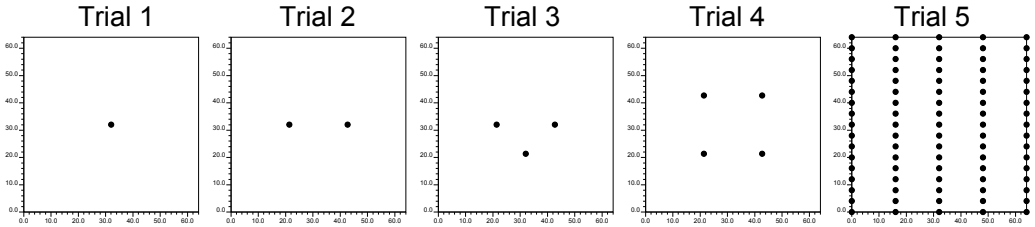

Figure 3.11: Configuration of conditioning data used in trials described.

Calculating the correlation coefficient via Uniform Conditioning theory in non-Gaussian units involves a seres of steps:

1. The average variogram is calculated using the known variogram model in Gaussian units (Equation 3.1). The Gaussian average variogram is denoted as $\bar{\gamma}_y$ in this series of steps to avoid confusion with $\bar{\gamma}$ which is the average variogram in logrnormal units. Using the Gaussian average variogram the correlation in Gaussian space, ρ_y, is calculated:

$$\rho_y = 1 - \bar{\gamma}_y \tag{3.11}$$

2. Determine the relationship between the the Gaussian distribution and the lognormal distribution of interest by solving for β^2:

$$\beta^2 = ln(1 + \sigma_z^2/m_z^2) \tag{3.12}$$

3. Use the calculated β^2 value to solve for the average variogram in the lognormal case, γ_z for both the SMU and panel scale.

$$\rho_z = \frac{m_z^2}{\sigma_z^2}[e^{\beta^2 \rho_y} - 1] \tag{3.13}$$

$$\bar{\gamma}_z = 1 - \rho_z \tag{3.14}$$

4. Using the average variogram in original units, $\bar{\gamma}_z$, the correlation coefficient for the panel to scale can be approximated by Uniform Conditioning theory. The GSLIB program, `DGM` is used for this purpose (Machuca-Mory et al., 2008) and results are tabulated in Table 3.1.

Results in Table 3.1 are based on a single panel, of dimensions 64x64x10. The SMU sizes considered all have a height of 10, and equal sides of 2, 4, 8, 16 and 32. These sizes are chosen so that the SMUs nest neatly within the single panel. The first trial of a Gaussian unconditional example demonstrates that the UC approximation to describe the bivariate relationship between scales is reasonable for this setting.

The same test is performed with lognormal distributions at the SMU and panel scale and the results do not match closely. Notice the theoretical correlation calculated using the r coefficients is lower when compared to the Gaussian example. Although not ideal, results for the Lognormal tests are consistent for unconditional and conditional data configurations when there are few data. Only when extensive conditioning data is used do the results change significantly. The final test, with an extensive grid of conditioning data reveals a disparity in the numerically calculated correlation coefficient and that approximated by Uniform Conditioning. This suggests that in the presence of substantial conditioning data the conditional SMU scale distribution predicted by change of support may be incorrect.

Table 3.1: Results of Testing the Theoretical Correlation Between Scales

Description	SMUs Per Panel	Numerical Correlation	UC Correlation
Gaussian	2	0.87	0.88
Unconditional	4	0.82	0.84
	8	0.79	0.82
	16	0.78	0.81
	32	0.77	0.79
Lognormal	2	0.83	0.90
Unconditional	4	0.75	0.86
	8	0.71	0.83
	16	0.69	0.83
	32	0.67	0.82
Lognormal	2	0.83	0.90
1 Conditioning data	4	0.75	0.86
	8	0.71	0.83
	16	0.69	0.83
	32	0.67	0.82
Lognormal	2	0.83	0.90
2 Conditioning data	4	0.75	0.86
	8	0.71	0.83
	16	0.69	0.83
	32	0.67	0.82
Lognormal	2	0.83	0.90
3 Conditioning data	4	0.75	0.86
	8	0.71	0.83
	16	0.69	0.83
	32	0.67	0.82
Lognormal	2	0.83	0.90
4 Conditioning data	4	0.75	0.86
	8	0.71	0.83
	16	0.69	0.83
	32	0.67	0.82
Lognormal	2	0.79	0.85
Exhaustive Conditioning	4	0.66	0.79
(86 data)	8	0.60	0.77
	16	0.58	0.76
	32	0.56	0.75

3.4 Conclusions

Change of support is a required step in any geostatistical modeling work flow for quantifying local SMU uncertainty. Conventional analytical techniques and the Discrete Gaussian model require global assumptions to reduce the variance of local conditional distributions. It is shown here, through simulated

examples, that variance reduction is influenced by the spatial configuration of conditioning data.

Analytical methods are commonly applied as a post processing step following MultiGaussian or Indicator Kriging. To appropriately scale local distributions and account for the change of support an approximation technique is suggested. The details of this process may vary in practical application, but it stands as an improvement upon the globally assumed variance correction factor which can lead to overstating local uncertainty.

In the case of Uniform Conditioning and the Discrete Gaussian Model; results show that bivariate relationship between the SMU and panel scales is difficult to generalize. This relationship is dependent on the shape of the distribution, and the conditioning data present in each panel. The test scenario for a single panel validates the theoretical approximation of this relationship for the Unconditional Gaussian case. Testing this relationship with a lognormal distribution for few conditioning data shows a noticeable but consistent difference between the actual bivariate relationship and that approximated by Uniform Conditioning. The numerically calculated correlation between scales and the approximated correlation become more dissimilar in the presence of substantial conditioning data within each panel. The results of this test emphasize the required assumption that data should be sparse and uniform for reliable results using this technique.

The caveats and approximations required for analytical methods and uniform conditioning may be appropriate when only the expected value of each local distribution is of interest. If the entire SMU scale distribution is to be utilized, the straight forward numerical approach of block averaging is recommended. Although more computationally expensive than a simple analytical solution, block averaging accurately reduces variance without a bias in the final uncertainty.

Chapter 4

Localization

4.1 Introduction

Localization is an alternative to estimation when a single model is required for resource modeling. Unlike estimation methods, localizing a model of uncertainty can reproduce the histogram without issues of conditional bias or smoothing. Initially this technique was developed as an extension of Uniform Conditioning (Abzalov, 2006). More recently, localization has been extended to Indicator Kriging (Hardtke et al., 2011), Sequential Gaussian Simulation (Boisvert and Deutsch, 2012) and MulitGaussian Kriging (Daniels and Deutsch, 2014).

Localization has been developed using the panel scale as a reliable middle step. The process is designed to approximate spatial grade distribution patterns at an SMU resolution within each panel (Abzalov, 2006). Panel scale distributions of uncertainty are only common in Uniform Conditioning. To create a flexible localization process the method has been altered to upscale SMU uncertainty to a panel. Within each panel SMUs are then ranked and assigned a grade. The idea is that the set of SMU grades reproduces the local distribution for that panel; therefore reproducing the global histogram. Despite the benefit of a single model and histogram reproduction there are drawbacks. The most noticeable issue is the unsightly edge-effect artifacts (Boisvert and Deutsch, 2012). Less obvious are the issues of local precision and questionable conditioning. The intent of this chapter is to provide an overview of the localization process and to present two approaches for artifact

reduction. Each method will be demonstrated through the same synthetic example to compare and contrast the different localized models with the model of uncertainty considered.

4.2 Methodology

Before beginning the process of localization it is assumed that a valid model of uncertainty has been generated and is deemed representative of the domain. The goal of localization is to replicate the same global histogram as the model of uncertainty in a single SMU scale model.

First, consider the three scales of importance in a localized model. There is the point scale, otherwise known as the data scale. This is the highest resolution and is the scale at which experimental data is collected. The SMU or block scale is the familiar scale at which resource evaluation takes place. Lastly, the panel scale is larger and each panel contains a number of SMUs. Each of the three scales is illustrated in Figure 4.1.

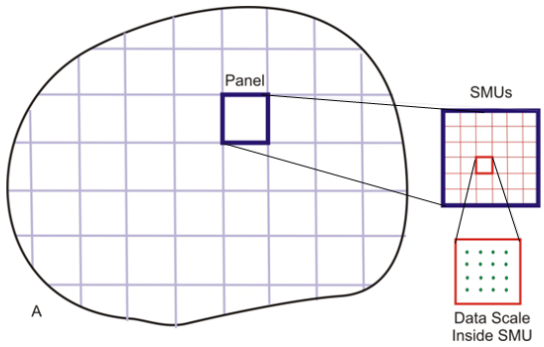

Figure 4.1: The three scales of importance to consider when localizing (Boisvert and Deutsch, 2012).

Prior to localizing, the panel scale must be defined. Rather be limited to the panel scale results from Uniform Conditioning, a more flexible option is to create panel distributions from an SMU scale model. Local SMU distributions, defined in the common format (described in Chapter 2), are assembled into panels as a first step in localization.

Conventionally, panels in both Uniform Conditioning and localization have been defined as a rectangular volume containing multiple SMUs. When lo-

calization is considered as a post processing step some flexibility is permitted. Three options are given in the GSLIB `localization` program: Regular, Semi-regular and random panel options are available as pictured in Figure 4.2. Panel size and the number of SMUs within each panel is determined by the user. Ideally, a single panel should contain between ten and one hundred SMUs to allow for reasonable discretization and representation of the panel scale CDF. Panel size is not affected by the type of mining, size of equipment or other considerations important to SMU size determination. With each style of panel definition, the number of SMUs per panel should nest properly with the chosen SMU grid.

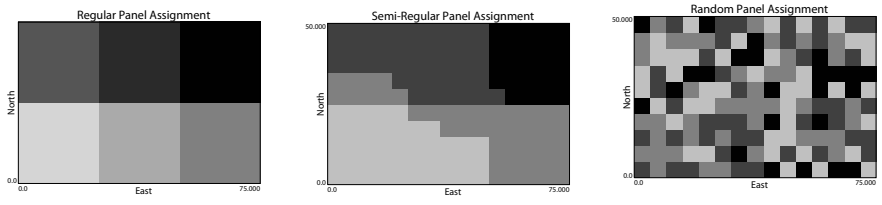

Figure 4.2: A simple example demonstrating each of the three panel definition options available.

Local distributions from each SMU within the panel are compiled to form a single CDF that quantifies the uncertainty for that panel. To summarize the panel scale model of uncertainty to a single model at the SMU scale, each panel distribution must be discretized into a series of grade classes. The process of compiling local distributions into a single panel CDF, and then discretizing into grade classes is shown in Figure 4.3.

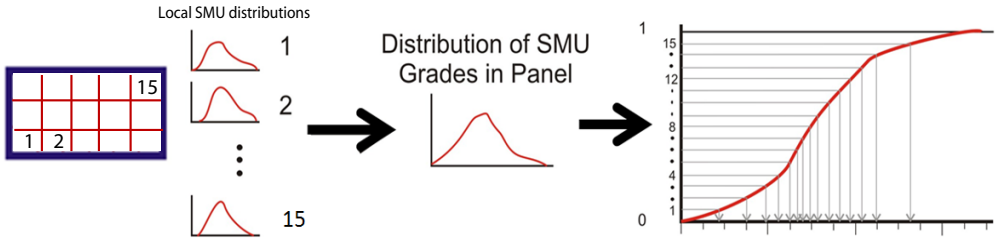

Figure 4.3: The local distribution of each SMU within the panel is compiled to a panel scale CDF, then discritized by grade class, a regularly defined panel containing fifteen SMUs is used in this example (Boisvert and Deutsch, 2012).

Grade class determination plays an important role in localization. The original methodology for localization uses grade classes of equal size (on the probability scale) to discretize the panel CDF illustrated in Figure 4.4. Once grade classes have been determined, the mean of each is calculated (Figure 4.4). These are the values assigned to SMUs within the panel. Boisvert and Deutsch (2012) suggest that instead of using the grade class mean, adding an element of randomness to the grade values chosen from each grade class captures the high and low extents of the CDF more accurately provides better histogram reproduction. Both types of grade class discretization will be demonstrated in the synthetic example to follow.

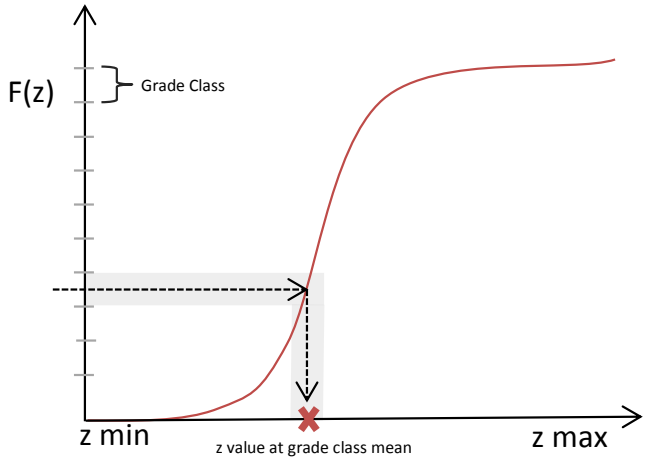

Figure 4.4: The CDF pictured represents a panel containing ten SMUs. One the y-axis the ten discretized grade classes are denoted by gray tick marks. The fourth grade class (in increasing order) is highlighted to represent the simple calculation and look up required that can be used to determine the grade class mean or a random value within the grade class.

The appropriate value from each grade class is assigned to an SMU based on its relative rank within the panel. Ranking is typically determined by Ordinary Kriging. This requires kriging independent from the rest of the modeling process. The kriged grades are not incorporated into the final model, therefore avoiding any histogram smoothing. The lowest value localized grade is assigned to the SMU of the lowest rank (lowest kriged grade) in the panel. The localized value from the second grade class is assigned to the SMU ranked second according to kriged grade. This is repeated until each localized grade is assigned to the corresponding SMU. The same process is repeated for all panels within the domain to generate a localized model as is portrayed in Figure 4.13.

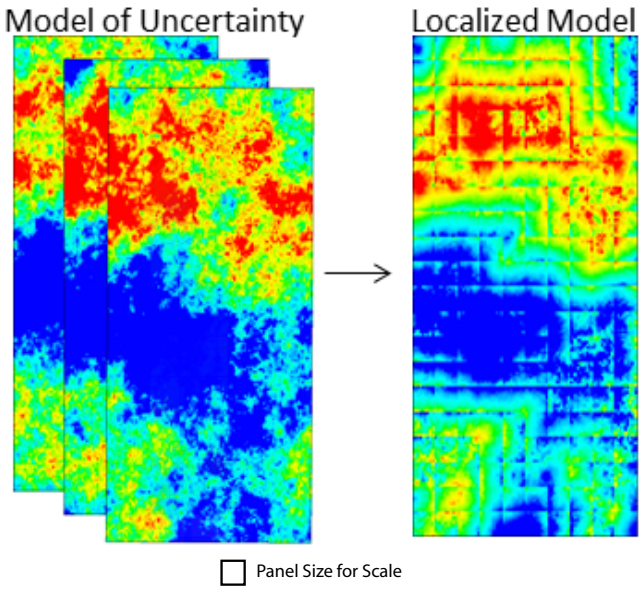

Figure 4.5: An example of set of realizations localized to a single model illustrating discontinuities at the boundaries of the regularly defined panels. Cooler colors represent lower grade values while warmer colors represent higher values. Units are arbitrary for this synthetic example.

Some observations can be made based on the localized model shown above. The localized model shows a similar configuration of high and low grade regions compared to the realizations that were localized. Although the spatial distribution of grade appears reasonable, at a small scale, the panel boundary artifacts are apparent throughout the model. These unattractive discontinuities at panel boundaries are the most obvious drawback of localization. The artifacts are caused by localization of each panel independent of surrounding information. In the presence of a trend that extends across multiple panels it is inevitable that the highest value SMUs in one panel will juxtapose lower value SMUs in a neighboring panel as dictated by the trend, causing artifacts at panel boundaries. A one dimensional example in Figure 4.6 helps to illustrate this problem.

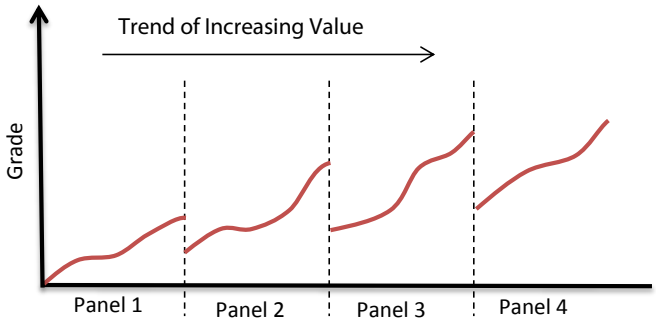

Figure 4.6: A sketch of four panels in a one dimensional localized model with a trend increasing from left to right. The red lines represent the localized SMU grades within each panel and show discontinuities at the panel boundaries.

The trend across all four panels in the sketch above shows how panel edge-effects are caused. With a trend of increasing grade from left to right in Figure 4.6 the lowest value is consistently assigned to the SMU furthest left and the highest value is always assigned to the far right SMU. This same effect occurs in two and three dimensional models causing the pervasive artifacts. Panel edge-effects are unavoidable in conventional localization. To address this issue two methods have been explored and implemented. Each of the techniques slightly alters the localization process yet still provides histogram reproduction at the SMU resolution.

4.3 Artifact Reduction

The two methods developed for artifact reduction generate different results. An optimization approach is presented as a post processing step for localized models based on a regular panel definition. This method reduces panel artifacts by minimizing the absolute difference between neighboring SMUs, regardless of panel boundaries. A second approach is implemented during the localization process by applying a more flexible definition of a panel. The details of each technique are given here followed by a synthetic example to showcase the flexible localization process with both artifact reduction techniques.

4.3.1 Optimization to Minimize Artfiacts

Artifact reduction by optimization is implemented using a greedy random restart method. The objective function is designed to minimize the difference between neighboring SMUs throughout the entire domain using the following objective function:

$$Minimize = \sum_{i=1}^{N} \sum_{j=1}^{n} |Z^*_{\mathbf{u}_i} - Z^*_{\mathbf{u}_{i,j}}| \quad i = 1, \ldots, N \; j = 1, \ldots, n \tag{4.1}$$

Where $Z^*_{\mathbf{u}_i}$ indexes over all of the SMUs in throughout the domain, N . The second term, $Z^*_{\mathbf{u}_i,j}$, indexes over the total number of SMUs, n, surrounding the location $\mathbf{u}_i$. Surrounding SMUs are considered regardless of panel boundaries. The optimization process begins by evaluating the objective function globally. Next, the grade values for two SMU values within the same panel are swapped. The objective function is again evaluated. If the objective function has decreased, the switch is kept. If the object function increases the SMU values are switched back to their original location. This process is repeated to reduce artifacts throughout the model. An example of a conventionally localized model after artifact reduction by optimization is shown in Figure 4.7.

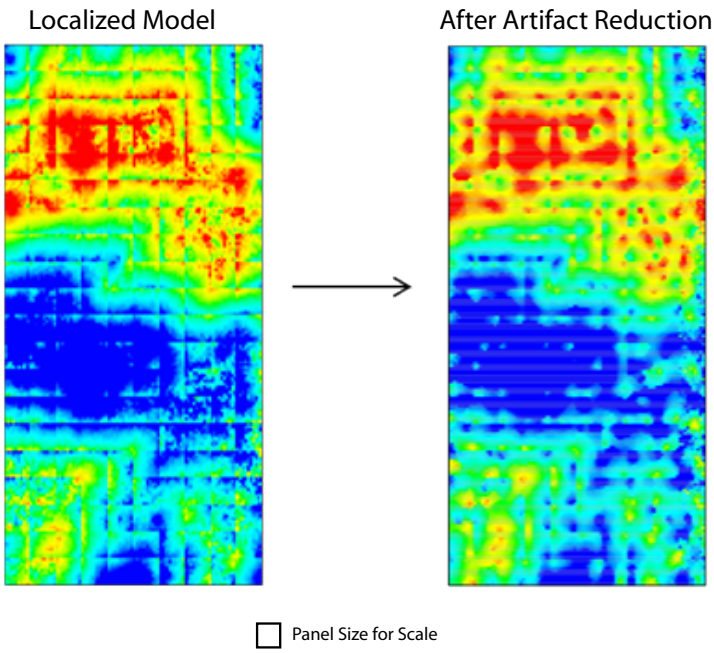

Figure 4.7: Artifact reduction performed on a conventionally localized model. Cooler colors represent lower grade values while warmer colors represent higher values. Units are arbitrary for this synthetic example.

The optimized model in Figure 4.7 shows improvement but the preexisting spatial pattern is still obvious and limits the optimization process. Further improvement can be gained by adjusting the initial localization methodology. The optimization technique applied here is dictated, in part, by the starting configuration of SMU grades within the panel and domain. If SMU grades are placed randomly within each panel during the localization process, the same model is produced at the panel scale while eliminating any preexisting structure or spatial patterns at the SMU resolution. This provides a more desirable starting position for the optimization algorithm. This method is applied to the same model as shown in Figure 4.7, the results are shown in Figure 4.8.

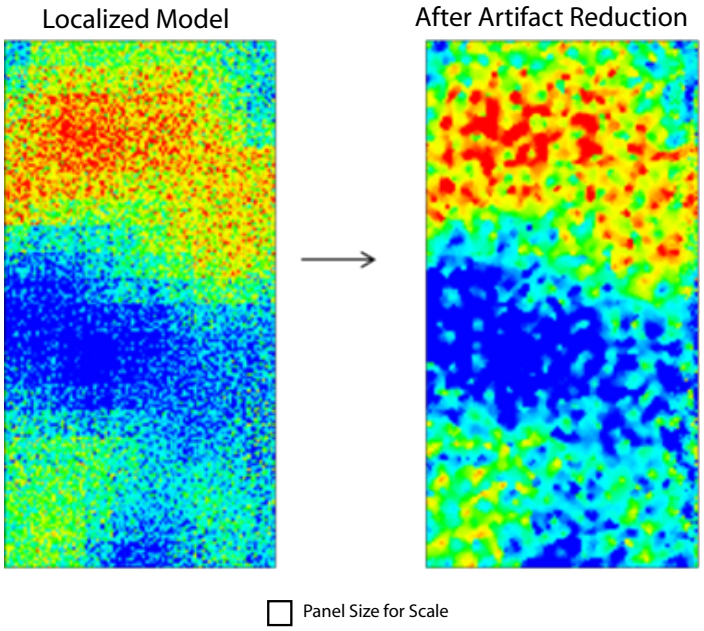

Figure 4.8: Artifact reduction performed on a randomly localized model. Cooler colors represent lower grade values while warmer colors represent higher values. Units are arbitrary for this synthetic example.

The artifact reduced model in Figure 4.8 shows an improvement over the previous example. Artifacts at panel boundaries have been reduced. Although free of artifacts targeted by the optimization process the model remains unrealistic. Due to the fact that SMU values can only be rearranged within a single panel the high and low are clustered with those of neighboring panels through optimization. This results in a model that has areas of low grade in regions that ought to show more consistent high grade. The same is true for areas that ought to show consistent low grade. Artifact reduction by random panel definition was attempted to reduce artifact while avoiding the spotted appearance described.

4.3.2 Flexible Panel Definition for Artifact Reduction

An alternative approach to artifact reduction is the random panel definition option described earlier and illustrated in Figure 4.2. The panel scale serves an important purpose in localization as a middle ground between the model of

uncertainty and an estimated single model; however, there is no requirement that panels must be defined as a regular rectangular shape of contiguous SMUs. By randomly assigning each SMU to a panel there are no distinct panel edges to generate the edge artifacts.

This approach to artifact reduction requires little change to the localization methodology. As before, the local distribution of values for each SMU in the panel contributes to the panel scale CDF. The location and configuration of the SMUs that make up a panel changes. Rather than a panel forming a regular shape, the panel consists of SMUs scattered throughout the domain and the localization process is performed as before. This is implemented in the GSLIB style `localization` program by giving each SMU a panel identification number. When it comes time to assemble a panel scale CDF, the distribution from each SMU with that identification number is compiled into a single panel. This slight change to the methodology requires no additional post processing and provides the histogram reproduction that localization is known for. An example of this is included in the synthetic example below.

4.4 A Synthetic Example

A synthetic example has been prepared to illustrate the localization and artifact reduction techniques discussed. This example will not cover the details of SGS implementation but instead begins with a set of one hundred realizations that will be localized. The set of realizations will be localized using both center point and randomly chosen grade values. Artifact reduced localization techniques will also be applied to the same set of realizations to demonstrate the improved appearance.

4.4.1 Localization of Simulated Realizations

To begin, a set of one hundred conditionally simulated realizations at a 3x3x10 meter SMU scale has been generated. The realizations have been created with a major direction of continuity at 45 degrees from north and a minimum direction of continuity 135 degrees from north. Two slices of a single realization

from this set are visible in Figure 4.9.

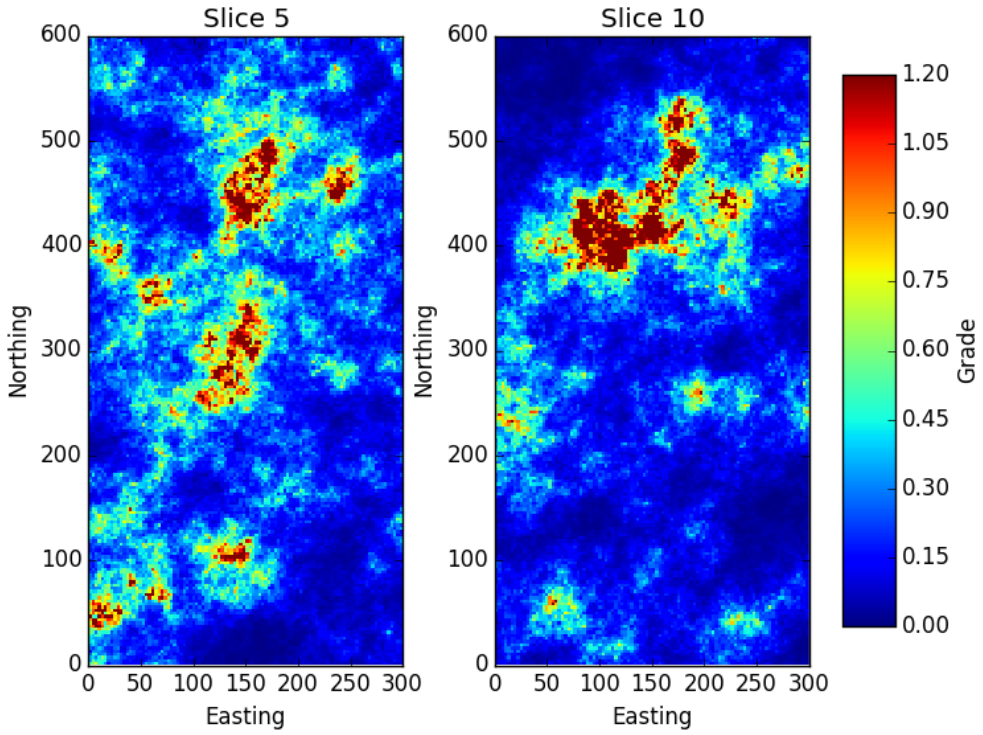

Figure 4.9: Two slices from a single realization from the set of one hundred realizations. Units are arbitrary for this synthetic example.

Conventional parameters are used for the first localized model created from this set of one hundred realizations. Panels are defined at a 30x30x10 volume so that one hundred SMUs nest neatly within each panel. Once the panel scale distributions have been assembled, CDFs will be discretized using regularly spaced grade classes, and ranking is determined by Ordinary kriging. For reference, two slices from the kriged ranking model are shown in Figure 4.10.

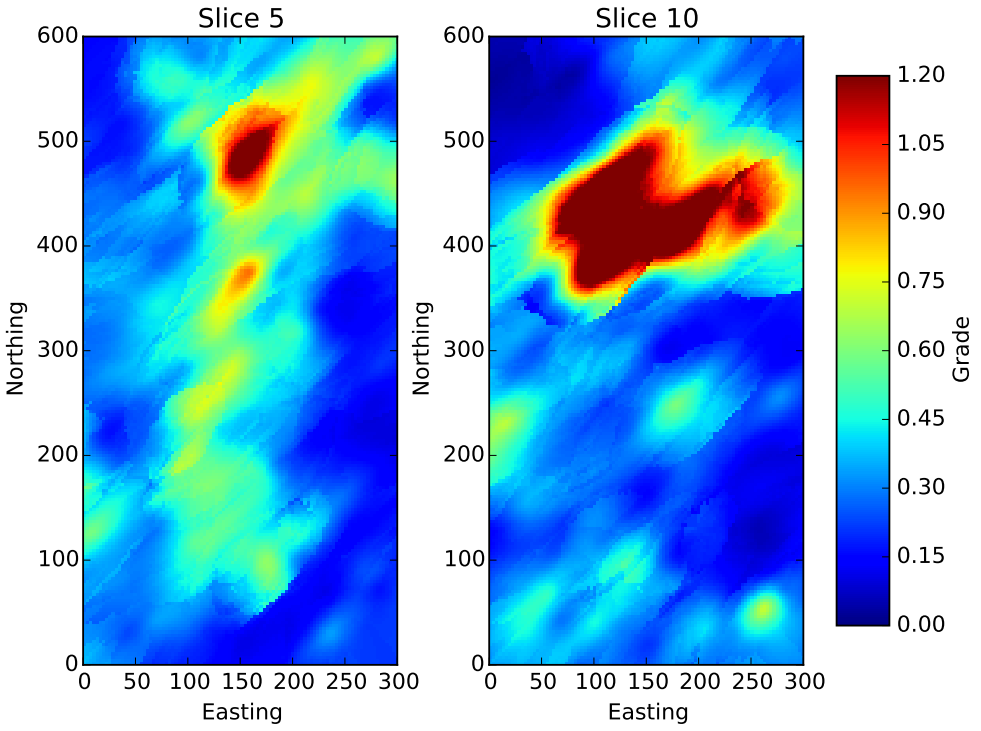

Figure 4.10: Two slices from the Ordinary Kriging model used as a ranking scheme for localization.Units are arbitrary for this synthetic example.

The localization process is fast, taking only minutes to localize the one hundred realizations of 400,000 SMUs each. Once completed, histogram reproduction can be checked against the simulated realizations. In Figure 4.11, the CDF of each realization is plotted in black while the localized model is overlaid for comparison in red.

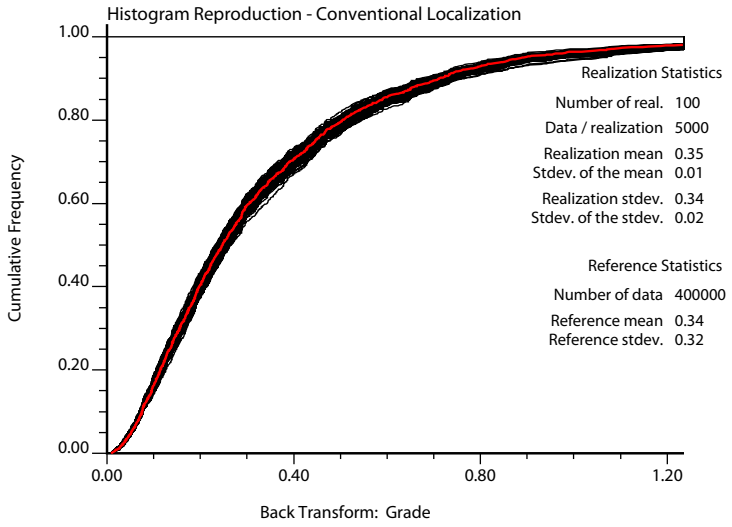

Figure 4.11: Histogram reproduction of the conventionally localized model (red) is shown in comparison to the one hundred simulated realizations (black). Units are arbitrary for this synthetic example.

The localized model (red CDF) appears reasonable in comparison to the simulated realizations (black) upon visual inspection, but the calculated standard deviation of 0.32 (variance of 0.10) is lower than expected. The random sampling of grade classes described by Boisvert and Deutsch (2012) is implemented to improve upon this issue. Randomly choosing a value within each grade class yields better reproduction of the panel scale distribution through more effective sampling of the upper and lower tails. The results of the localized model using random sampling is shown in a similar plot, Figure 4.12.

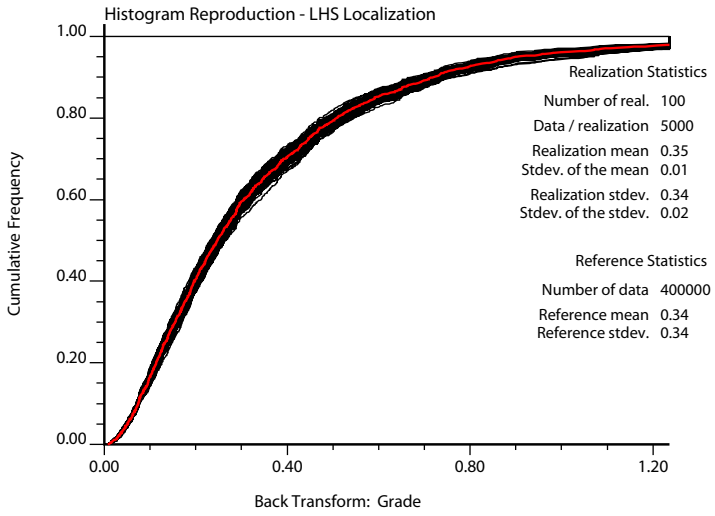

Figure 4.12: Histogram reproduction of the localized model using the random sampling scheme (red) is shown in comparison to the one hundred simulated realizations (black). Units are arbitrary for this synthetic example.

The random value sampling strategy for determining localized values improves the overall histogram reproduction of the localized model. The standard deviation of 0.34 (variance of 0.12), and visual inspection of the CDF comparison shows improvement. Plotted slices of the model reveal the unattractive panel edge artifacts described by Boisvert and Deutsch (2012) remain. Two slices from the conventionally localized model illustrate this issue in Figure 4.13.

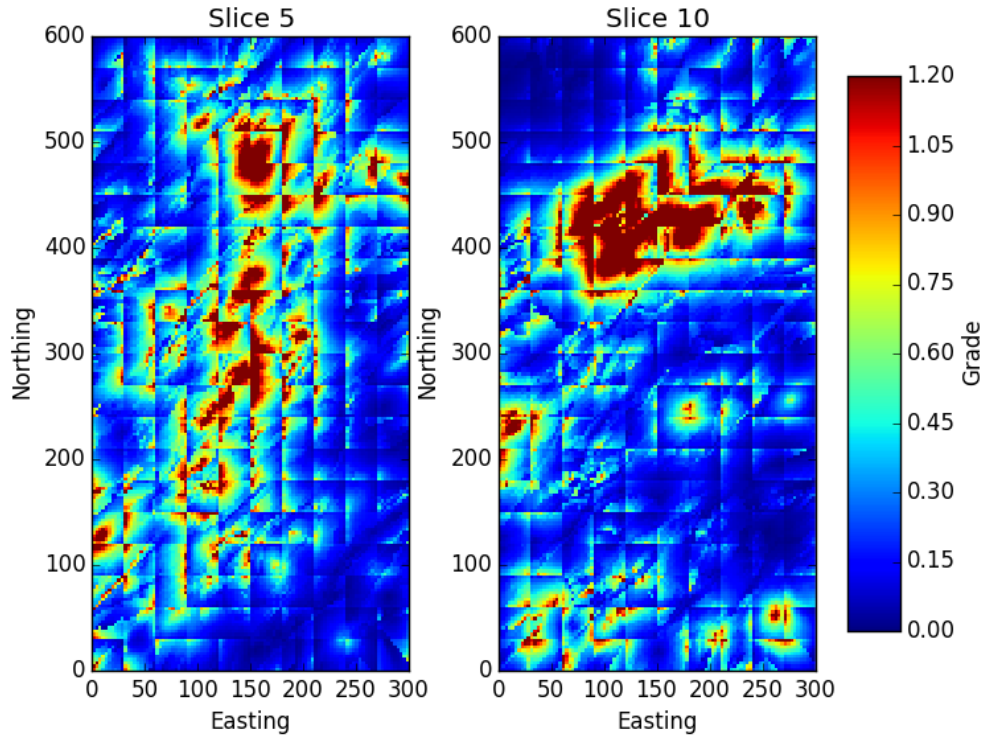

Figure 4.13: Two slices from the localized model created using random grade class sampling. Units are arbitrary for this synthetic example.

The two slices plotted from the localized model in (Figure 4.13) are at the 3x3x10 SMU scale but the 30x30x10 panel scale remains overprinted throughout. The artifacts give the model an unrealistic appearance. This issue is will be addressed via the artifact reduction techniques.

4.4.2 Artifact Reduction of the Localized Model

To effectively compare the conventionally localized model and the artifact reduced models, parameters are kept consistent where possible. Dimensions of both panels and SMUs are maintained for artifact reduction via optimization. Artifact reduction by random panel assignment maintains the same SMU dimensions and the same ratio of one hundred SMUs per panel, but in a random configuration.

First, artifact reduction by optimization is generated. For best results the localized model is re-created using a random set of values for SMU ranking within each panel. At the panel scale, this model remains identical to the

localized model shown in Figure 4.13 and will generate the same global distribution seen in Figure 4.12. At the SMU scale, the two models appear very different and any spatial pattern within each panel has been erased by using a random ranking scheme. Now, the optimization algorithm can be applied.

The number of iterations required to effectively reduce panel artifacts is determined by the user. The goal of this process is to reduce artifacts for visual improvement only. It is difficult to quantify exactly how many iterations are required to erase panel artifacts. In this example, the objective function does not reach an effective minimum until ten million iterations are (about twenty-five per SMU). This amount of iterations seems like a large number but the process is fast. The optimization process can be monitored by visual inspection and by plotting the change in the global objective function against iterations completed as shown in Figure 4.14.

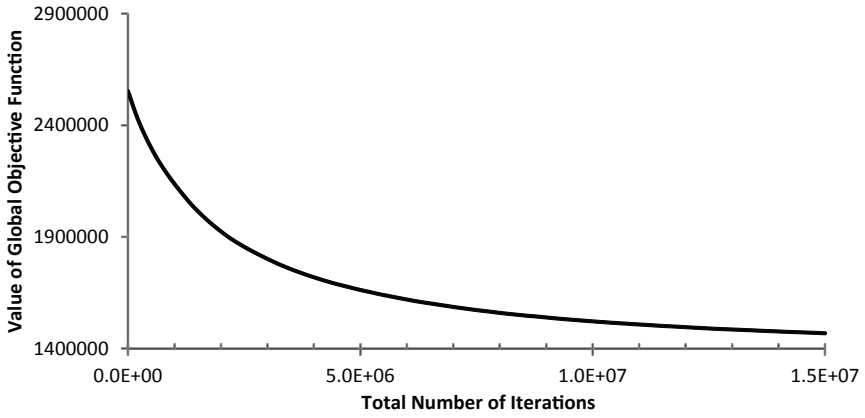

Figure 4.14: The global object function plotted over a large number of iterations for the example model containing 400,000 SMUs.

In practice, the optimization process can be run until panel artifacts have been reduced to the practitioner's satisfaction. Two slices of the reduced artifact model in Figure 4.15 show that the distinct panel edges visible in the conventionally localized model are no longer present.

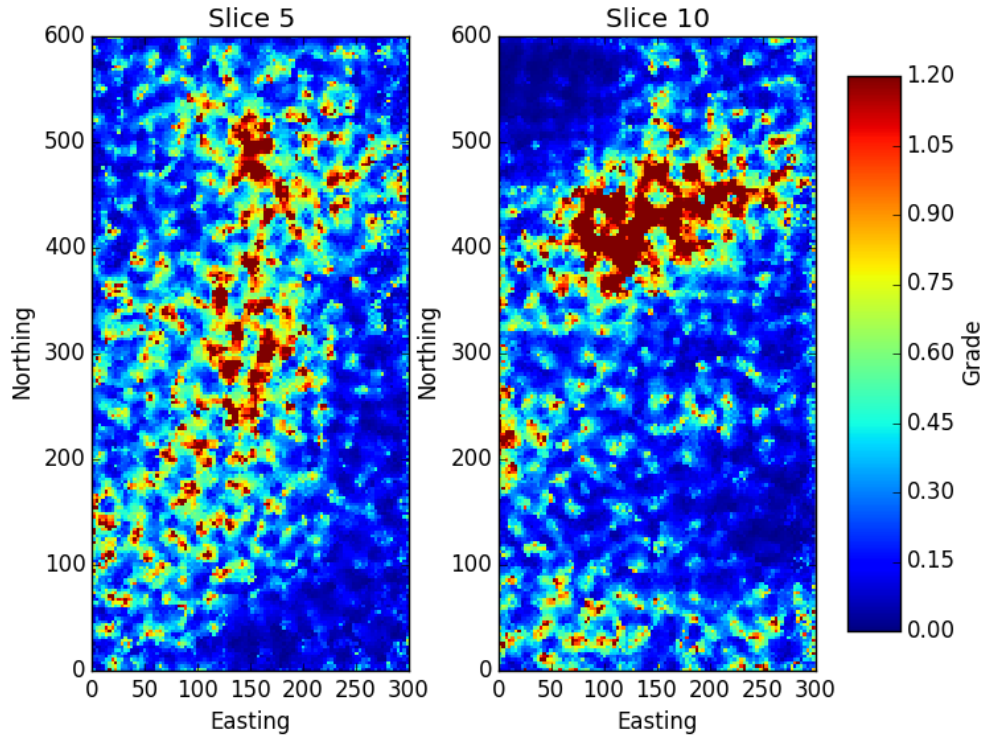

Figure 4.15: Two slices from the artifact reduced model by optimization. Units are arbitrary for this synthetic example.

Although free of panel edge effects, the optimized model shows high and low grade clusters in regions that ought to display more consistent values. To preserve the correct CDF within each panel, SMU values are only able to be re-arranged within that panel. This makes it impossible to smoothly portray larger scale trends that extend across multiple panels expected in a geologic setting.

The alternative approach to artifact reduction is random panel definition. This also requires re-creating the localized model but with no additional post processing or optimization. By randomizing which SMUs belong to each panel it is unlikely that any of the SMUs within a single panel will be contiguous, eliminating panel structure that can cause artifacts. The one hundred SMUs in each panel are ranked by Ordinary Kriged grade from the same model portrayed in Figure 4.10. Figure 4.16 shows the global histogram of the model created using random panels, demonstrating that the histogram is well repro-

duced.

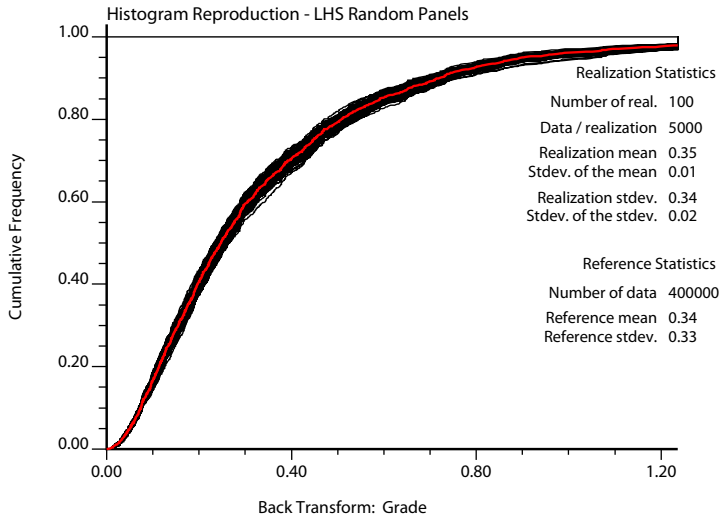

Figure 4.16: Artifact reduction by random panel assignment. Units are arbitrary for this synthetic example.

Two slices of the localized model are shown in Figure 4.17. The same two slices of the conventionally localized model, artifact reduced by localization and this example are distinctly different. This model shows no panel artifacts, and is void of the local grade inconsistencies seen in the optimized model. This artifact reduced model presents the smoothest appearance, most similar to the Ordinary Kriged model in Figure 4.10.

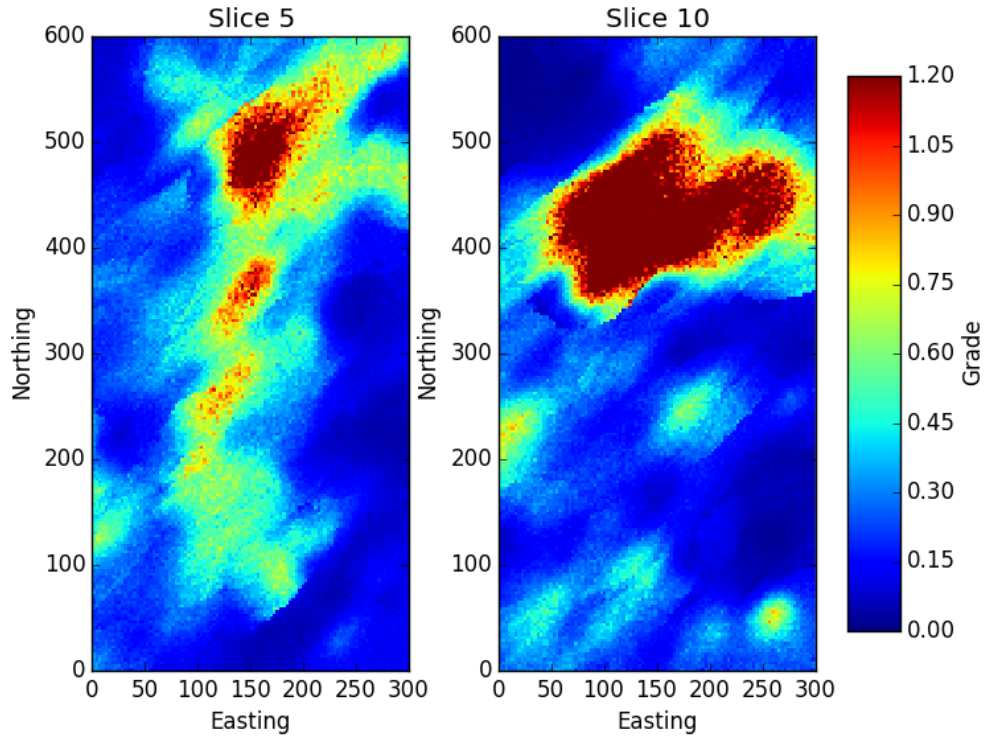

Figure 4.17: Artifact reduction by random panel assignment. Units are arbitrary for this synthetic example.

Despite the smooth large scale features, at a small scale this model displays at a pixelated texture similar to a simulated realization. Aside from the slight visual issue the main drawback of this method is the spatial distribution of grade throughout the domain. The highest grades of each panel scale distribution are often assigned to a single area within the model where the highest Ordinary kriged grades are located. This clusters all of the highest values in a single area. It can be assumed the same issue exists for the lowest grades within the model. In this example, this effect has caused the areas of high grade to the north to grow and intensify while the areas to the south which show high grade in both the conventional and optimized localization models (Figures 4.13 and 4.15 respectively) are reduced to mid grade by the random panel artifact reduction technique.

The artifact reduction techniques presented here are improvements upon conventional localization. Both techniques effectively reduce the unattrac-

tive discontinuities that occur at panel edges, but each comes at a cost. The optimization process adds an additional post processing step and yields inconsistent grade in an unrealistic fashion. The random panel approach generates a model with a pixelated appearance while compromising the spatial distribution of high and low grade values within the domain. Despite the drawbacks, each of the methods shown provides good histogram reproduction. The mean and variance for each of the models created is tabulated in Table 4.1 for comparison.

Table 4.1: Results of Panel Artifact Reduction

Model	Mean	Variance
Conditional Simulation	0.35	0.12
Localized model I	0.34	0.10
Localized model II*	0.35	0.13
Artifact Reduction - Optimization*	0.34	0.10
Artifact Reduction - Optimization	0.35	0.13
Artifact Reduction - Random Panels	0.36	0.13
Artifact Reduction - Random Panels*	0.34	0.11

* Denotes model created using random grade class sampling

It is clear from the table of results and plotted CDFs, that the localized models provide good histogram reproduction but randomly sampling from within grade classes can be considered best practice. Even with confidence in histogram reproduction, local precision remains an issue. Localizing a panel scale distribution and assigning SMU grades by a ranking scheme does not guarantee local precision. This simply creates a spatial pattern that mimics the model used for ranking. The issue is exaggerated by re-arranging SMUs within a panel through optimization for artifact reduction. Artifact reduction approaches presented here offer a visual improvement to the conventionally localized model.

4.5 Conclusions

Localization is appealing. The benefit of histogram reproduction from a single model is desirable. Application of artifact reduction techniques improve upon

the results visually but highlight the issues of local precision. Questionable local precision also suggests that a localized model is likely not conditioned to data values present. Even so, localization provides an effective methodology for reducing a probabilistic model of uncertainty to one model for situations that may value histogram reproduction over local precision.

The pros and cons of localization should be carefully considered prior to implementation in a practical setting. The simplicity of a single high resolution model is undeniable, but it may not be reasonable. In most cases the limited number of data present make deterministic models unrealistic for resource evaluation and risk analysis.

Chapter 5

Practical Implementation

5.1 Introduction

This chapter will focus on practical application of geostatistical modeling for SMU uncertainty in the case of a large copper porphyry deposit. Models of uncertainty will be constructed at the SMU scale to demonstrate the steps involved and for comparison of results. Results from multiple models will be contrasted at both a global and local scale along with resource calculations. In this example MultiGaussian (MG) kriging, Sequential Gaussian Simulation (SGS) and Uniform Conditioning (UC) will be used to create models of uncertainty for resource evaluation. The goal of this study is to review each of the three methods and apply recommendations made in a practical setting.

The data used for this study has been collected from the JA deposit in the Highland Valley Porphyry system near Kamloops British Colombia and was provided to the University of Alberta more than a decade ago for the mine design project class. Like many porphyry systems, the ore in this deposit is common along veins, fractures and faults. All of these features are vertically continuous and preferentially oriented with a strike of Northeast - Southwest (Aziumuth:135). The dataset includes both Copper and Molybdenum values. For simplicity, only Copper will be modeled in this case study. It is assumed that the same techniques can be adapted for modeling molybdenum but will not be demonstrated here. Further information about the deposit, geologic setting and history of exploration is available in Casselman et al. (1995).

5.2 Data

The data provided for this case study consists of 10649 samples collected from near vertical drillholes. A domain of 2100x1800x750 meters, with an origin at 34210 East 27025 North and 600 (m) Elevation for this practical demonstration. The locations of drillholes form a large elliptical area of interest with a long axis striking Northwest-Southeast as seen in Figure 5.1.

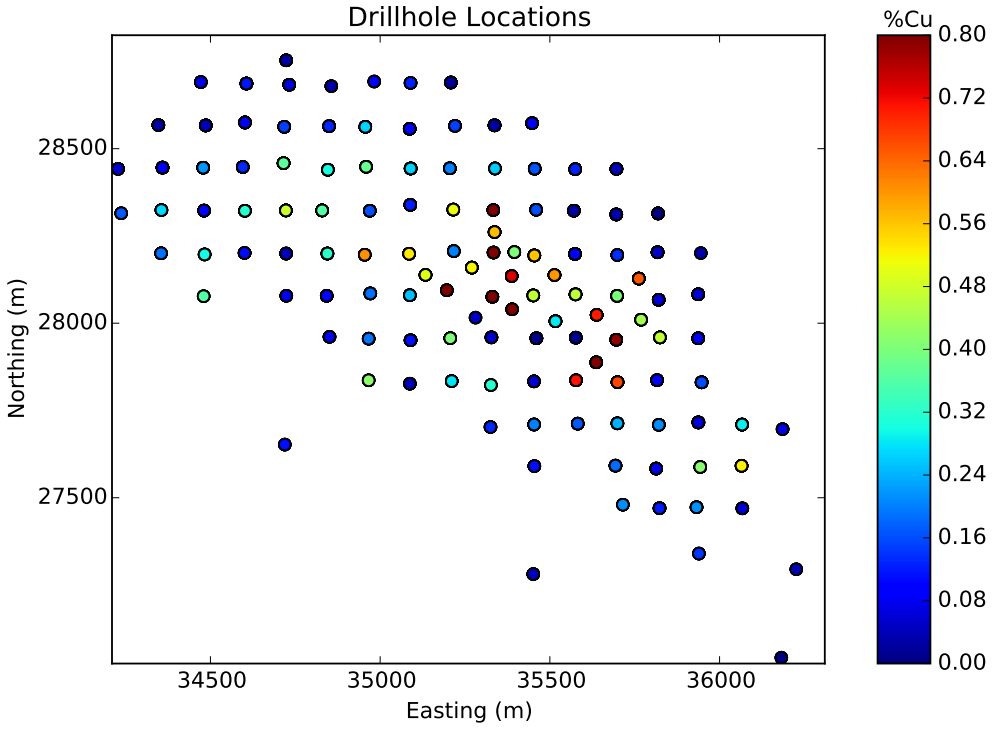

Figure 5.1: A two dimensional map showing the location of drill holes in the domain. Drill holes are colored by the grade from the bottom sample of each drill hole.

Similar to many porphyry systems this deposit contains a large amount of low grade samples. Based on this observation it is assumed open pit mining is an appropriate extraction method. The data have been averaged to fifteen meter fixed length composites which is deemed an appropriate length because it is a reasonable bench height. Original data was provided at three meter intervals, therefore a single composite is the average five samples (in most cases). From this point forward composited data will be used for all

calculations, modeling and model checking.

Although the drillhole spacing is somewhat regular throughout the domain, areas of inconsistent spacing exist and declustering is required. In this study, a cell size equivalent to the drillhole spacing in sparsely sampled areas is applied, per the guidelines suggested by Rossi and Deutsch (2014). A cell size of 50x50x50 meters is used and declustering weights are calculated. Applying the declustering weights yields the declustered histogram in Figure 5.2.

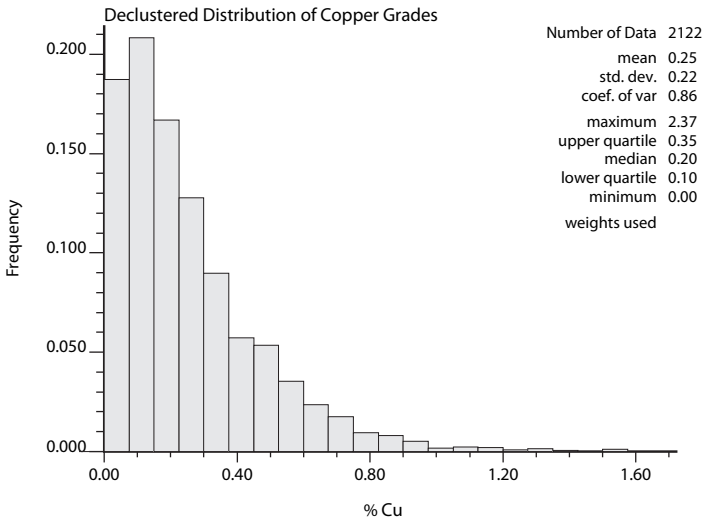

Figure 5.2: The declustered histogram of Copper grades present in the composite data set.

The histogram in Figured 5.2 shows that the distribution of copper grades is positively skewed with a declustered mean of 0.25 (naive mean of 0.26) and a standard deviation of 0.22. This is considered the reference distribution that is representative of the data that will be used for geostatistical modeling at the point scale (the scale of the data).

For all models generated in this study, an SMU size of 15m x 15m x 15m will be considered. As discussed in Chapter 2, the decision of SMU size is dependent on many factors. A number of equipment and planning based factors would ultimately influence this decision. This SMU size is considered to be consistent with the composite length in the vertical direction and provides reasonable resolution at the SMU scale. Additionally, SMUs could be grouped to form panels of size 75m x 75m x 15m as necessary.

5.3 Variography

The spatial relationship of the composited data is quantified by the variogram in both original and Gaussian units. The original units variogram is required for change of support and kriging in original units. The variogram in Gaussian units is required for MG Kriging and SGS. For both cases, the variograms are modeled in three directions: vertical, and two directions in the horizontal plane. Directions of maximum and minimum continuity in the horizontal plane are chosen at an Azimuth of 135 and 45 degrees respectively.

Due to poorly behaved and difficult to model experimental variograms in both horizontal directions the pairwise relative variogram was used here. The pairwise relative experimental variograms are standardized and modeled to a sill of 0.70 (sill determined using the `prsill` program) as seen in Figure 5.3. More information about the pairwise relative variogram can be found in Babakhani and Deutsch (2013).

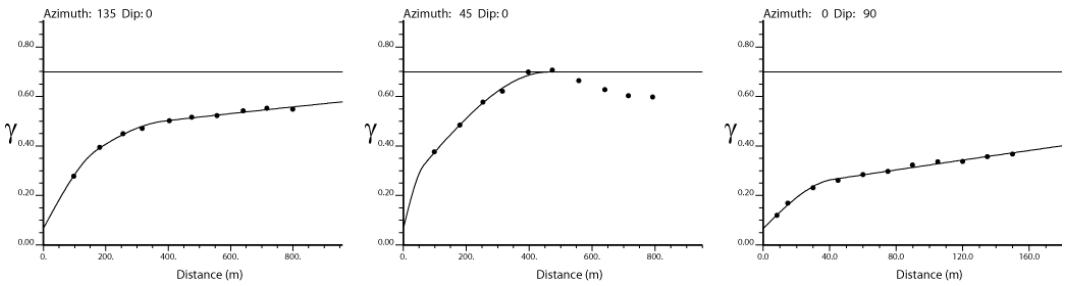

Figure 5.3: The pairwise relative variograms calculated and fit with three spherical structures to reach a sill of 0.70.

Table 5.1: Parameters of Variogram Model in Original Units.

Type of Structure	Contribution	Max Range (m)	Min Range (m)	Vert Range (m)
Nugget Effect	0.067			
Spherical	0.156	175.2	65.8	41.7
Spherical	0.222	390.3	461.4	684.0
Spherical	0.254	2611.5	464.6	727.7

An azimuth of 135 degrees is considered for the maximum direction of continuity, and 45 degrees azimuth for the minimum direction of continuity with 0 degrees for both plunge and dip.

Variograms for the three directions described are plotted in Figure 5.3 are modeled with automated fitting software using three spherical structures defined in Table 5.1. This is the three dimensional variogram model that will be scaled to the correct variance of the univariate distribution for change of support and kriging in original units. Geostatistical modeling in Gaussian space requires an additional set of variograms calculated using the normal scores of composited data. Experimental variograms are calculated and fit with a three dimensional model as shown in Figure 5.4.

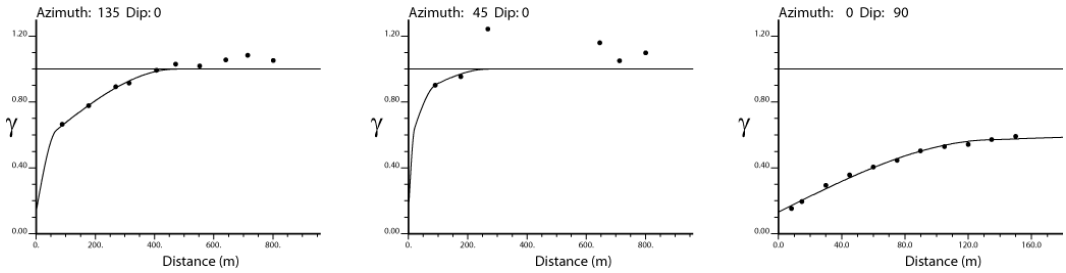

Figure 5.4: Normal score variograms for the the composited data in Gaussian units fit with three spherical structures and a nugget effect of 0.132.

Table 5.2: Parameters of Variogram Model in Gaussian Units

Type of Structure	Contribution	Max Range (m)	Min Range (m)	Vert Range (m)
Nugget Effect	0.132			
Spherical	0.396	69.1	21.1	132.6
Spherical	0.287	475.0	93.9	2060.8
Spherical	0.186	476.7	271.6	2378.3

An azimuth of 135 degrees is considered for the maximum direction of continuity, and 45 degrees azimuth for the minimum direction of continuity with 0 degrees for both plunge and dip.

The variogram model, in Gaussian units, is defined by three structures in Table 5.2. This will be used for both MG Kriging and SGS.

In both sets of variograms the extensive vertical continuity of the porphyry system is made clear. This is obvious in the vertical variogram which does not reach the sill in either the Pairwise Relative or Gaussian variogram model. This observation, paired with the known geologic setting suggests an areal trend is present and well defined by the data. The presence of an areal trend makes

this domain non-stationary and may warrant subdividing the data or explicitly modeling a trend. Ideally, rock codes and in-depth geologic observations could provide more information to subset the data to create a series of smaller, more staionary domains. In this example, due to the information available and methodology focus of this case study, the deposit will be modeled as a single domain. The non-stationarity of the domain should be considered when reviewing small fluctuations in statistical results.

In addition to the non-stationarity of the domain the use of two different variogram models may contribute to variation in the results. Normal scores variograms are typically better behaved than variograms in original units, but in this case the pairwise relative variogram performs better and provides a more continuous variogram model. In this case, the Normal scores of the composited data are used to construct the variogram in Gaussian units. An alternative that may yield a more continuous variogram would be to translate the experimental pairwise relative variogram to Gaussian space. This process was not pursued at the time of modeling.

5.4 Geostatistical Modeling

Three geostatistical techniques for modeling uncertainty will be applied here for resource evaluation. These include, MultiGaussian Kriging, Sequential Gaussian Simulation and Uniform Conditioning. Indicator Kriging will not be applied due to the complexity and time consuming requirements of binning, variogram modeling, and order relations corrections required to achieve the correct global distribution of estimates. Prior to modeling the correct reference distribution at the SMU scale is established.

5.4.1 The SMU Scale Reference Distribution

It is known from the information effect that greater precision and local detail will be available at the time of mining. It is also known that the scale of relevance for mining is larger than the pseudo point scale of the drillholes. For this reason, geostatical modeling takes place at the SMU scale. To provide a

reference histogram at the SMU scale, the declustered histogram of composite data must be adjusted for variance reduction. This is due to the support effect which describes the dampening of high and low values within each SMU block for an overall decrease in variance. The most robust method available for this task is the Discrete Gaussian Model (discussed in Chapter 3). The Pairwise Relative Variogram model is re-standardized to a sill of 1 and used to scale the distribution in Figure 5.2 appropriately. The resulting histogram is displayed in Figure 5.5.

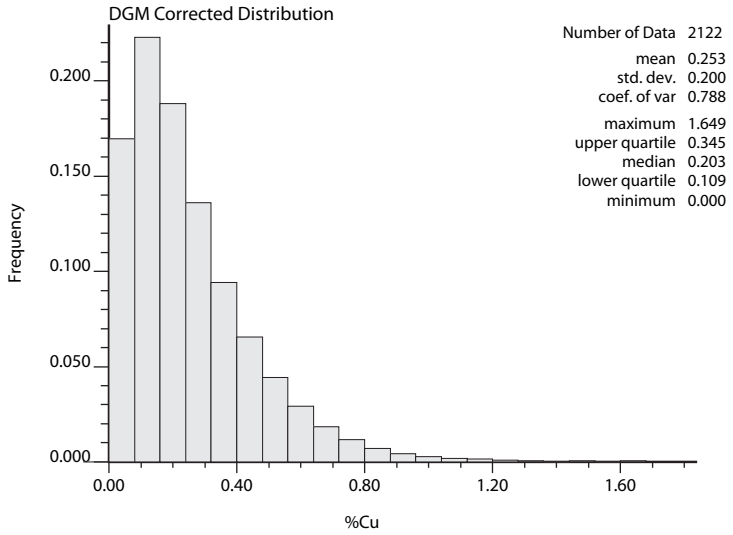

Figure 5.5: The distribution of declustered composite data, scaled to the SMU volume to be used as a reference histogram for model validation.

After upscaling to the SMU volume, the reference histogram gives a mean of 0.25 (%Cu) and standard deviation of 0.20 (variance of 0.04). These are the statistics that will be targeted for validating the global distribution of estimates generated via geostatistical modeling.

5.4.2 MultiGaussian Kriging

The first method considered, MultiGaussian Kriging, is performed in accordance with the description in Chapter 2. Kriging parameters include using a search ellipse of 500x300x600 meters oriented with the longest axis in the vertical direction and the longer of the two horizontal axes at an Azimuth of 135 degrees corresponding to the maximum direction of continuity. A minimum of

four and a maximum of forty data are considered for each estimate. Normal scores of the composited data are kriged over the entire domain, at the point scale, by Simple Kriging with a stationary mean of zero.

The local distributions defined by a conditional mean and variance at each location must be back transformed from Gaussian units to original units. Before post processing, the grid of kriged estimates is clipped. Due to the Northwest striking elliptical shape of this deposit a large amount of empty space exists in the Northeast and Southwest quadrants of the domain. Estimates far from data will be generated equal to the mean. This large number of estimated locations can slow down post processing and generate misleading results once back transformed to original units since there are no conditioning data. For this reason, all kriged estimates more than 300 meters from a drillhole are set to a null value.

Back transformation and volume variance correction are handled simultaneously in the `PostMG_Loc` program developed (see Software Appendix). Back transformation is performed using the look up table created during the initial normal score transform of the composited data. One hundred quantiles of each local distribution are back transformed.

Change of support is applied using the Affine correction. The Affine Correction using a local variance reduction factor is the best option despite the permanence of shape assumption due to the complexity and CPU cost of applying the DGM at each location. The local variance correction factor is calculated using high resolution simulations for the entire domain similar to the procedure presented as an example in Chapter 3. The distribution of calculated local correction values is shown in Figure 5.6.

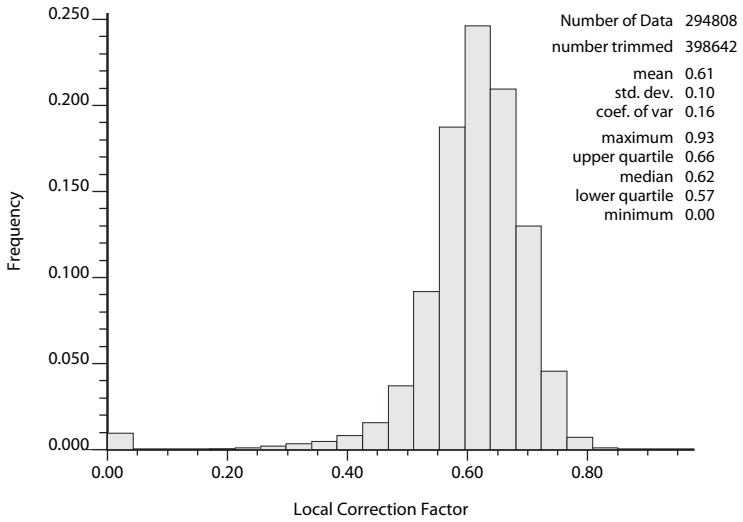

Figure 5.6: The distribution of local correction values calculated using the methodology presented in Chapter 3. For reference, the global correction factor is 0.845 calculated using `gammabar`

The mean value of this distribution is lower than the global correction factor of 0.84 (calculated using `gammabar`). This difference in the local and global correction factor can be attributed to the presence of conditioning data as noted in Chapter 3. The spatial configuration of these results is portrayed in Figure 5.7 which shows slice 10 from the gridded set of local correction values.

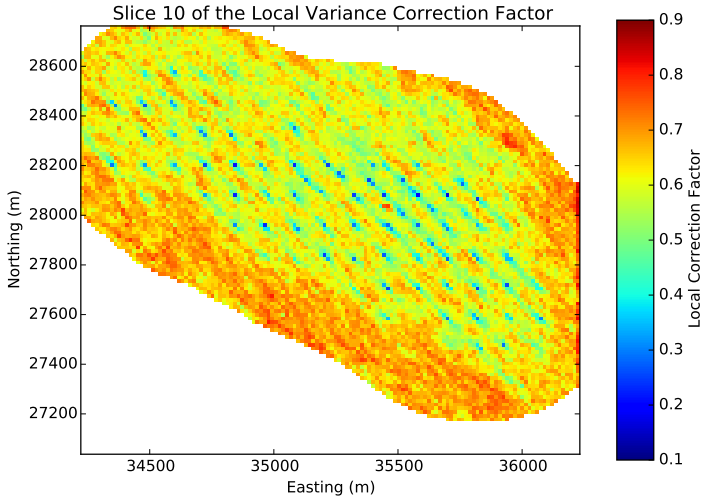

Figure 5.7: Slice 10 of the gridded local correction factor values shows high values in regions of sparse data and lower values proximal to conditioning data

The plotted local correction factor in Figure 5.7 shows near zero values at data locations which increase and approach the global correction factor in areas far from conditioning data.

The clipped and back transformed SMU scale results are exported in the common format described in Chapter 2 for resource evaluation to follow. The estimates calculated are portrayed in the histogram seen in Figure 5.8.

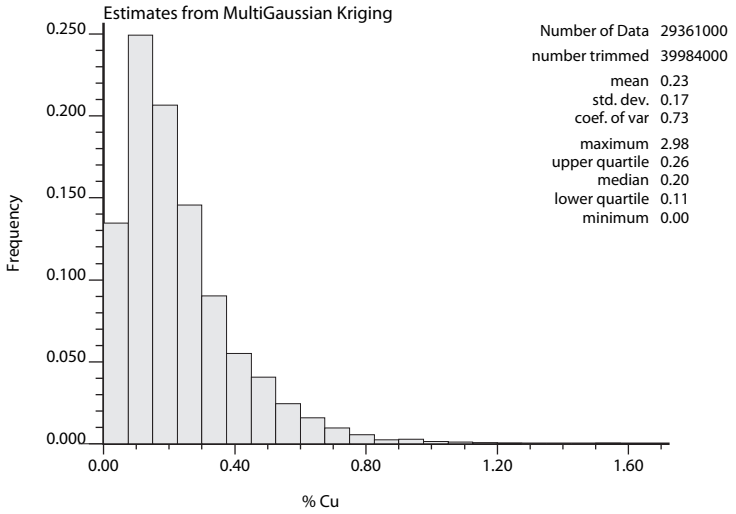

Figure 5.8: The distribution of estimates at the SMU scale after back transformation and variance correction. This histogram includes one hundred quantiles from each local conditional distribution.

The global distribution of estimates from MultiGaussian Kriging closely reproduces the original histogram scaled to an SMU volume in Figure 5.5. To confirm that the model is reasonable a plot of the expected value of each local distribution is shown in Figure 5.9.

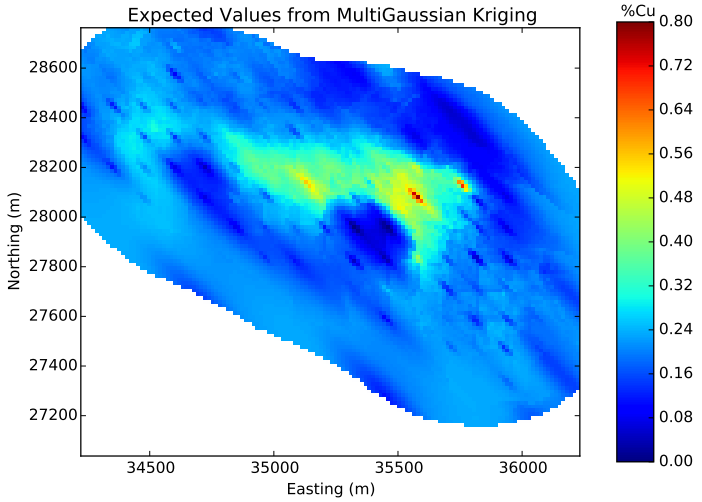

Figure 5.9: Slice number 10 from the defined grid showing the expected value from each local distribution estimated by MultiGaussian Kriging after back transformation and change of support.

The plot of expected values in Figure 5.9 shows a similar spatial distribution of high and low grade compared to original data in Figure 5.1. This plot presents an overly smooth representation of the grades due to the expected value calculation. A single realization from Sequential Gaussian Simulation will present a more realistic view of the variability that is present throughout the domain.

5.4.3 Conditional Simulation

Sequential Gaussian Simulation is used to generate a second model of uncertainty for comparison with the model generated via MultiGaussian Kriging. The same set of normal scored data values and Gaussian variograms are used for simulation as were used in Multi Gaussian Kriging. Parameters for simulation include the same search radius and number of data per estimate for consistency.

Simulation is performed at the data scale. This can be inconvenient over multiple realizations due to time, computational expense, and management of large files. To mitigate these issues realizations are generated at a 5x5x15 meter grid resolution. This provides an adequate discretization to average the

results to the larger 15x15x15m SMU size. Despite the adjustment in grid definition, this remains a large number of locations. To further reduce the number of cells requiring an estimate only grid cells within the area of interest will be given a simulated value. Only cells within 300 meters of a drill hole will be estimated. All other cells will be given a null value for all realizations.

One hundred realizations are generated at the grid spacing described within the area of interest. Change of support is accomplished by block averaging the composite scale simulated values within each SMU volume. This results in one hundred values from each local SMU distribution, equivalent to the one hundred quantiles calculated by MultiGaussian Kriging. The SMU scale CDF from each realization is checked against the reference distribution of composites in Figure 5.10.

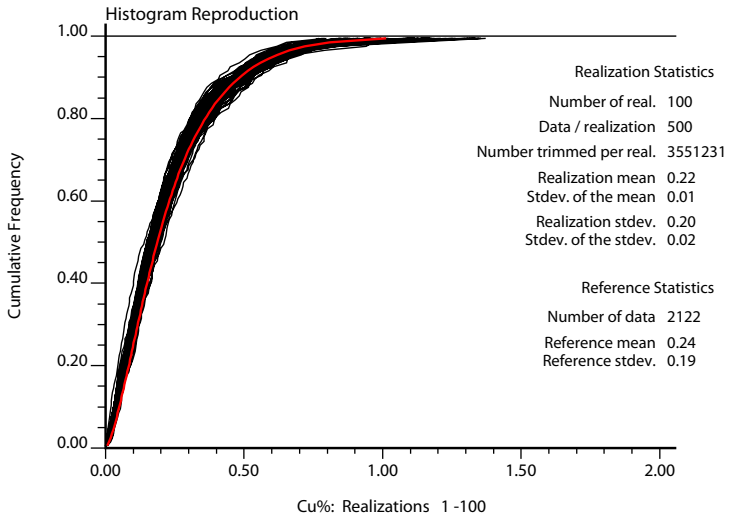

Figure 5.10: Each black CDF represents a single realization and the overlaid red CDF represents the original distribution of composites for comparison.

Visualizing each realization is not practical. For this reason one slice from a randomly chosen realization is given as an example. A single slice from realization twenty-five, at the SMU scale, is displayed in Figure 5.11. Realization twenty-five was chosen arbitrarily.

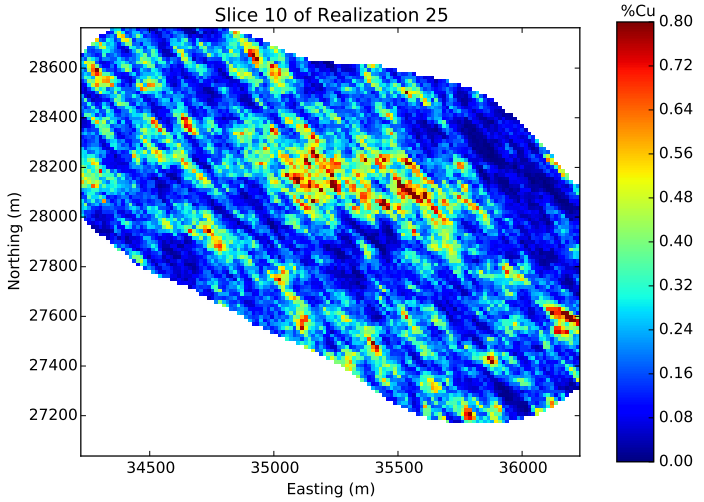

Figure 5.11: Slice ten from realization twenty-five at the SMU scale.

The plot displayed in Figure 5.11 shows more realistic variability compared to the smooth map of expected values from MultiGaussian kriging in Figure 5.9. It is important to note that the grade values displayed in this plot are only from one realization, out of one hundred. This is not an expected value or best estimate but instead shows a realistic spatial distribution of possible values. One benefit of simulation is that the spatial relationship can also be verified by checking the variogram from each realization, shown in Figure 5.12.

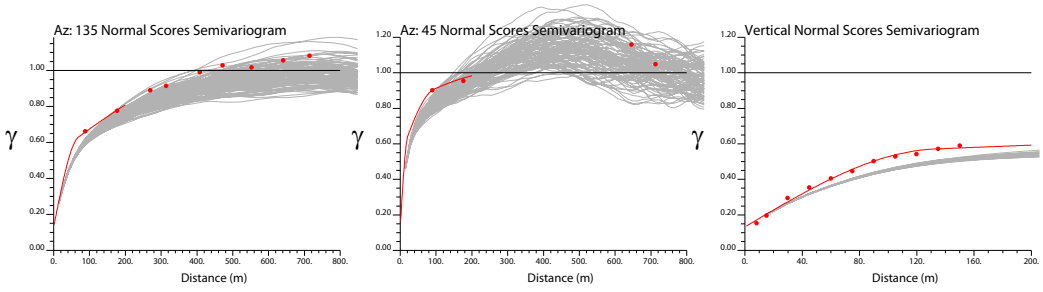

Figure 5.12: The variogram of each realization is plotted in gray for all three directions. The experimental variogram calculated from origianl data is plotted by red dots and the variogram models fit to points are the red lines for comparison.

Simulated realizations permit reasonable variogram reproduction. By checking the global histogram the practitioner can be assured the correct distribution is reproduced while variogram reproduction confirms this grade is distributed correctly throughout the domain. The results of conditional simulation suggest the model of uncertainty is representative for this domain, and will be contrasted with results from both MultiGaussian Kriging and Uniform Conditioning later.

5.4.4 Uniform Conditioning

Uniform Conditioning is different from the other geostatistical techniques applied above. The normal score transformation is only needed to fit the DGM at the SMU and Panel scale for local resources; all kriging and average variogram calculations use the variogram in original units. To begin, kriging takes places over a panel scale grid defined by 75x75x15 meter panels. Kriging parameters include a larger search radius of 2000x500x700 meters to accomodate the longer range of the pairwise relative variogram model in original units. This elliptical search radius is oriented with the longest axis in the horizontal plane at an Azimuth of 135 degree and the shortest also in the horizontal plane at an Aziumuth of 45 degrees. Panels are discretized for block kriging to account for the panel scale volume support, the resulting distribution of estimates is seen in Figure 5.13.

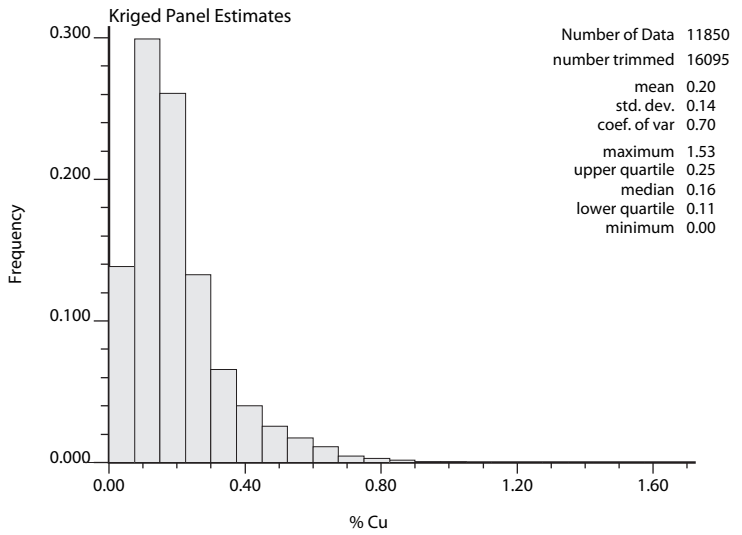

Figure 5.13: The distribution of panel scale estimates (after clipping to the area of interest) used for Uniform Conditioning.

The distribution of panel scale estimates shows that that variability has been reduced significantly to standard deviation of 0.14. This is due to both the increase in scale as well as the smoothing effect of kriging with a large search radius. The kriged panel scale estimates are used as the input for Uniform Conditioning. Additional inputs for Uniform Conditioning include the variance of the kriged panel scale estimates and the variance of the SMU distribution which can be inferred using the average variogram. The `gammabar` program (Deutsch and Journel, 1997) calculates the SMU scale variance of 0.0395 while the panel scale variance is calculated directly from the kriged estimates for a value of 0.0196. This is a 18.4% (SMU) and 59.5% (panel) reduction compared to the original distribution of composited grades. Next, the Hermite polynomial coefficients are fit to the distribution using the `pre_uc` program (Neufeld, 2005) as the last preparation for Uniform Conditioning.

The GSLIB style program (`UC_Loc`) is used to perform the Uniform Conditioning. This is an newly updated version of the Uniform Conditioning program developed by Neufeld (2005). The update includes an additional export file that tabulates a defined number of quantiles per panel distribution. Recall that the distributions calculated are at the SMU scale, but portrayed on a panel grid. The program `UC_reorder` has been developed, and is used here,

to re-organize the exported quantiles into the common format at an SMU grid definition. More information about software alterations is available in the Appendix. The results from Uniform Conditioning are displayed in Figure 5.14.

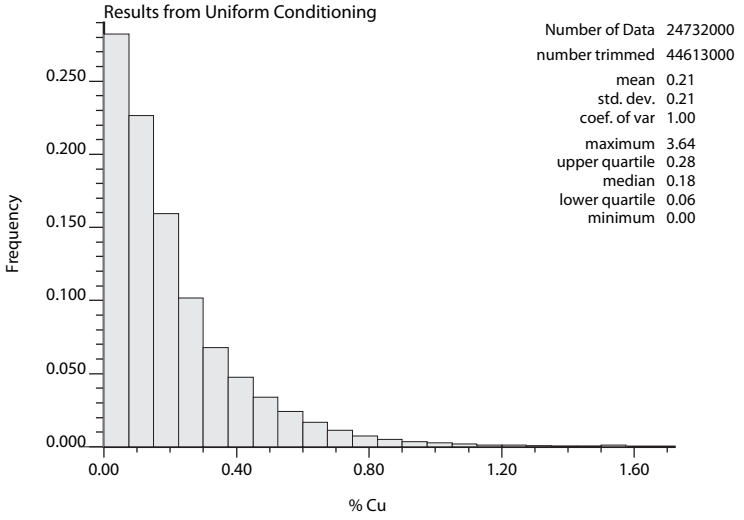

Figure 5.14: The distribution of one hundred quantiles for each local distribution for the SMU grid.

The global distribution of results shown above closely reproduces the target mean and variance at the SMU scale. Although the SMU scale distribution suggests that this model is appropriate for mine planning, a visual inspection of Figure 5.15 shows that even at an SMU grid it is obvious that precision is limited to the panel scale, but only shows resolution at the panel scale.

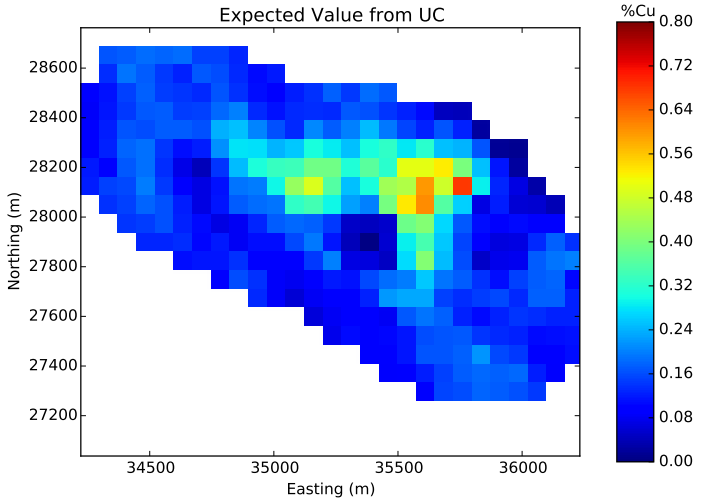

Figure 5.15: This plot shows the expected value of each local SMU distribution calculated by Uniform Conditioning. This map is plotted on an SMU grid.

The plot in Figure 5.15 shows the expected value for each local SMU distribution. The expected value calculation generates a smoothed map and distribution of values. Rather than calculate the expected value of each distribution, localization is an option. Despite the advertized benefits of a higher resolution model via localization, the issues of conditioning and local precision within each panel are problematic. This suggests the results are best utilized at SMU support with panel resolution as shown in Figure 5.15.

5.5 Results and Analysis

In this case study, each modeling technique has generated a distribution of uncertainty at the SMU scale for each location within the area of interest. The standard approach to model checking is global histogram reproduction, visual inspection, and variogram reproduction (in the case of simulation). Global and local results from each model are contrasted here in regards to resource evaluation. First an overview of the results from each model is given, followed by a look at local results for select locations and finally the overall resource is evaluated for each model at the global scale.

5.5.1 Global Scale

The global results of each model are reported at the SMU scale. To be consistent, these results are compared to the original distribution of composites at SMU volume support in Figure 5.5. The mean and variance of each model, along with the SMU scale original data are tabulated in Table 5.3.

Table 5.3: Global Results of Geostatistical Modeling at the SMU Scale

Description	Global Mean (%Cu)	Global Variance
Upscaled Original Data	0.253	0.0400
MultiGaussian Kriging	0.225	0.0289
Conditional Simulation	0.222	0.0408
Uniform Conditioning	0.199	0.0396

The mean and variance reported for each model are similar. Slight fluctuations in these statistics can be attributed to multiple factors. The reduction in global mean can be attributed to the large number of estimates at the borders of the model. Additionally, the vertical trend that exists in this system creates a non-stationary domain which can lead to inconsistencies. Estimates from MultiGaussian Kriging are clipped at an SMU scale while panel estimates are clipped at a larger scale prior to Uniform Conditioning, this has the potential to influence results by creating a slightly different area of interest for each model. Despite the differences, global scale reproduction of first and second order statistics suggest that all three of the models effectively represent the deposit.

5.6 Local Scale

Comparing and contrasting each local distribution across an entire domain between all three models is not practical. It can be assumed some variation between the three models exists on a local scale. To summarize, the CDF from three SMUs within each model have been selected. A high value SMU with the grid coordinates of: x= 87 y= 65 and z= 21; a medium value SMU with coordinates: x= 63, y= 64 and z= 12 , and a low value SMU at x= 74, y= 63 and z= 11, are displayed in Figure 5.16.

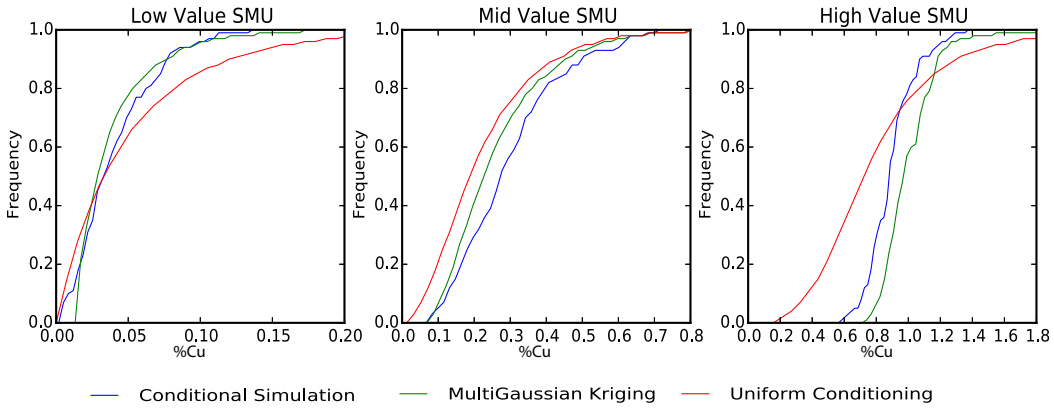

Figure 5.16: Three local conditional distributions from the same three locations in each model are plotted here. Accompanying statistics are available in Table 5.6, Table 5.5, and Table 5.4.

Local distributions selected from each model are constructed from the one hundred quantiles or simulated values modeled at that location. The similar shape and variance displayed for each local distribution across all three models suggests that local scale results are consistent. The low value location displays a small variance for each, while the medium and high value locations show higher variance. First and second order statistics along with grade and tonnage above a specified cutoff is given for each CDF in Table 5.6, Table 5.5, and Table 5.4.

Table 5.4: Results for High Value Location

			Above Cutoff 0.6 % Cu		
Model	Mean	Variance	Mean (%Cu)	Tons	Metal (Tons)
SGS	0.892	0.0231	0.899	8930	8030
MG	1.00	0.0282	1.00	9110	9110
UC	0.798	0.159	0.992	5920	5880

Table 5.5: Results for Mid Value Location

			Above Cutoff 0.4 % Cu		
Model	Mean	Variance	Mean (%Cu)	Tons	Metal (Tons)
SGS	0.294	0.0213	0.524	1820	955
MG	0.26	0.0199	0.516	1460	752
UC	0.222	0.0225	0.53	1090	579

Table 5.6: Results for Low Value Location

			Above Cutoff 0.1 % Cu		
Model Type	Mean	Variance	Mean (%Cu)	Tons	Metal (Tons)
SGS	0.0396	0.000769	0.116	365	42.3
MG	0.038	0.000817	0.134	365	48.9
UC	0.0506	0.00287	0.158	1280	201

The conditional mean and variance values tabulated in Table 5.6, Table 5.5, and Table 5.4 show that some variation in the results is present. Results are similar for both MultiGaussian Kriging and Sequential Gaussian Simulation. This is expected as these two methods are the most similar. Uniform Conditioning presents smooth CDFs which give a higher variance, particularly in the high value SMU selected. The difference in these results may be due to proximity to conditioning which Uniform Conditioning does not account for. Other factors may also contribute to these differences, including: tail extrapolation, Gaussian transformation and random number generation (in the case of simulation).

5.7 Resource Evaluation

Quantifying the total tons and grade of material present is an important step in long term planning for a deposit. Calculating these metrics based on an estimated model the process is simple. A cutoff grade, determined based on overhead cost and market value of the ore, is used to define which SMUs are ore and waste based on a single value at each location. From the SMUs evaluated as ore a total tonnage and average grade above cutoff can be easily calculated. In the presence of uncertainty, this methodology must be adapted.

In this step the value of multiple realizations is made clear. When evaluating the resource in a given deposit using multiple realizations the uncertainty can be transferred through to the final metrics. When only local uncertainty is present, in the case of both MultiGaussian Kriging and Uniform Conditioning, the expected value must calculated prior to grade and tonnage calculations. This is demonstrated for each of the models created. Grade - tonnage curves for each model in Figure 5.17 show the change in tons and average grade above

a series of cutoffs specified on the horizontal axis. Tonnage estimates presented here are calculated based on a specific gravity of 2.7 ($tons/m^3$).

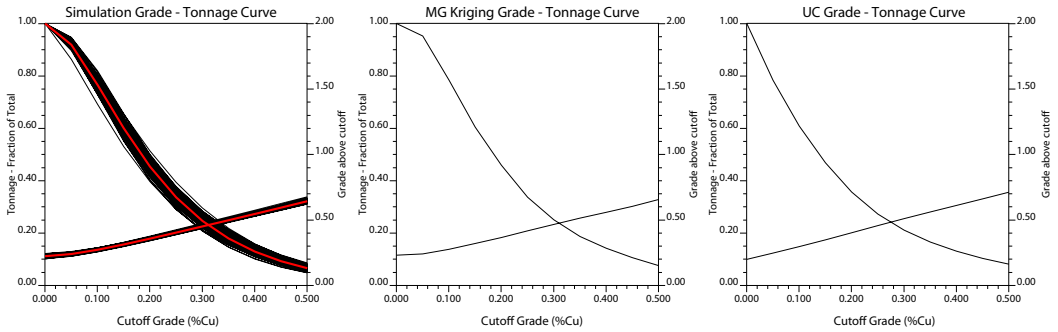

Figure 5.17: Grade tonnage curves are plotted from each model. Each realization of the simulated model is plotted separately with the average plotted in red. For MultiGaussian Kriging and Uniform Conditioning only the expected value is plotted.

Statistics accompanying each of the plotted grade tonnage curves are available in Table 5.7, Table 5.8 and Table 5.9 for the simulated realizations, expected value of MultiGaussian Kriging and expected value from Uniform Conditioning.

Table 5.7: Resource Evaluation of Simulated Model

			Above Cutoff		
Type	Cutoff	% Total Tons	Mean (%Cu)	Tons	Metal (Tons)
p10	0	100	0.21	2.66E+09	5.59E+08
	0.1	74	0.26	1.97E+09	5.12E+08
	0.2	42	0.35	1.12E+09	3.91E+08
	0.3	22	0.44	5.85E+08	2.58E+08
	0.4	11	0.53	2.93E+08	1.55E+08
Expected Value	0	100	0.22	2.66E+09	5.85E+08
	0.1	77	0.27	2.05E+09	5.53E+08
	0.2	45	0.36	1.20E+09	4.31E+08
	0.3	25	0.45	6.65E+08	2.99E+08
	0.4	13	0.54	3.46E+08	1.87E+08
P90	0	1.00	0.23	2.66E+09	6.12E+08
	0.1	79	0.28	2.10E+09	5.89E+08
	0.2	48	0.37	1.28E+09	4.73E+08
	0.3	27	0.46	7.19E+08	3.31E+08
	0.4	14	0.56	3.73E+08	2.09E+08

The one hundred realizations of the simulated model yield one hundred equiprob-

able grade tonnage curves. This provides valuable uncertainty for decision making based on the tons and grade calculated. There are multiple ways to interpret the information from the set of realizations. In this example the P10 and P90 quantiles have been evaluated along with the expected value. This provides information for planning in regards to a reasonable best, worst and average of the subsurface resource present.

Table 5.8: Resource Evaluation of MutiGaussian Kriging

		Above Cutoff		
Cutoff	% Total Tons	Mean (%Cu)	Tons	Metal (Tons)
0	1	0.23	2.68E+09	6.14E+08
0.1	0.97	0.23	2.60E+09	6.08E+08
0.2	0.63	0.27	1.67E+09	4.56E+08
0.3	0.15	0.40	3.94E+08	1.58E+08
0.4	0.059	0.49	1.58E+08	7.70E+07
0.5	0.018	0.59	4.83E+07	2.83E+07

Table 5.9: Resource Evaluation of Uniform Conditioning

		Above Cutoff		
Cutoff	% Total Tons	Mean (%Cu)	Tons	Metal (Tons)
0	1	0.20	2.69E+09	5.32E+08
0.1	0.61	0.30	1.63E+09	4.83E+08
0.2	0.36	0.40	9.57E+08	3.84E+08
0.3	0.21	0.51	5.66E+08	2.88E+08
0.4	0.13	0.61	3.50E+08	2.14E+08
0.5	0.081	0.71	2.16E+08	1.54E+08

Results from MultiGaussian Kriging and Uniform conditioning are calculated at each location based on the portion of a local distribution above the cutoff grade being considered. Although easy to calculate, there is an obvious lack of information in comparison to the values calculated from the set of realizations. The results of global resource evaluation vary, especially at higher cutoff grades. Uniform Conditioning presents the highest average grades with lower tonnage estimates while Simulation Presents the lowest grades even when considering the expected value. The differences in these results are large. Possible reasons for these differences are the non-stationary nature of this domain and the use of two different variogram models. Even so, it is difficult to de-

termine which model is correct until mining takes place and reconciliation can be performed. It is undeniable that Simulation provides much more information that either MultiGaussian Kriging or Uniform Conditioning by providing quantified uncertainty rather than a single estimate for resource evaluation.

5.8 Conclusions

Geostatistical modeling for long term planning and resource evaluation is challenging. The case study presented here demonstrates the practical implementation of three available techniques for modeling uncertainty. All three techniques take advantage of the robust Gaussian distribution and share similar assumptions of stationarity. The differences in these models become apparent in the final steps of resource evaluation. The significant differences present, especially at higher cutoff grades, can be attributed to differences in methodology and non-stationarity. Careful separation of geologic domains for separate modeling could improve upon the difference in results.

MultiGaussian Kriging and Uniform Conditioning are faster and simpler than Conditional Simulation, but the advantages of multiple realizations are many. The spatial relationship between simulated values for each realization can be checked through variogram reproduction. The ability to block average point scale realizations to the SMU scale avoids the pitfalls of analytical change of support methods and generates the correct histogram at both scales of interest. These factors instill confidence in the model of uncertainty by providing additional methods for model checking and validation.

The findings of this case study suggest that all three methods can generate the correct distribution of estimates and local uncertainty. Only through multiple realizations can uncertainty be transferred to the decision making process. The local uncertainty generated through MultiGaussian Kriging and Uniform Conditioning can be queried for information regarding a specific location, but does not provide a measure of uncertainty for any larger volume. Only through simulation can uncertainty be quantified over the entire domain. This allows for resource evaluation on each realization which can then be used

to accurately calculate the best, worst and average expectations for the subsurface deposit. This is valuable for economic decision making, determining pit limits, and even in long term mine planning.

Chapter 6

Conclusion

Assessing uncertainty in a resource model is an important task. Often the data available for geostatistical modeling constitute less than one-trillionth of the total volume of the deposit. This small amount of data makes deterministic modeling impractical and motivates the quantification of uncertainty through geostatistical modeling. Long range resource modeling takes places at the scale of the selective mining unit. This larger volume than the data scale is appropriate for resource evaluation. Multiple methods have been presented here for quantifying uncertainty at the SMU scale along with associated post processing techniques. The benefits and downfalls of each approach have been highlighted and a number of practical developments have been made in this thesis.

6.1 Topics Covered and Contributions

The issues of conditional bias and histogram smoothing that result from estimation techniques have been discussed. These downfalls of estimation demonstrate the need for a measure of uncertainty in a geostatistical model. An overview of four common techniques for quantifying uncertainty at the SMU scale are presented in Chapter 2. These modeling techniques are: MultiGaussian Kriging, Sequential Gaussian Simulation, Multiple Indicator Kriging and Uniform Conditioning. The benefits and challenges of each technique are summarized considering complexity and computational expense. While Indicator Kriging can be regarded as the most complex; simulation is the most compu-

tationally expensive. Despite the time and computational requirements, the findings of this research suggest that the benefits of multiple realizations out weigh the cost of this technique simulation; no other technique provides joint uncertainty and freedom from location dependent change of support parameters.

The review of the change of support techniques including analytical methods, the Discrete Gaussian Model and numerical block averaging is given in Chapter 3. Analytical methods such as the Affine and Indirect Lognormal Corrections rely heavily on a globally calculated variance correction factor. Inquiry into the location dependence of this parameter using a simulation test reveals that it is, in fact, strongly location dependent. The ratio of the block and point scale variances is dependent on proximity and value of conditioning data. Unfortunately, a closed form analytical solution for calculating the local correction factor is impractical. An approximation was proposed using known factors such as the kriging variance. Similarly, the embedded change of support technique in Uniform Conditioning is affected by conditioning data. It is demonstrated through another simulation test and inference of the theoretical correlation coefficient at panel and SMU scales, that the change of support parameters of Uniform Conditioning are also location dependent. The inconsistencies in both of these techniques suggest that the robust numerical approach of block averaging simulated realizations is the most effective and reliable method available. This further supports the case for simulation as best practice.

Managing uncertainty represented by multiple realizations can be challenging. One alternative is localization. This process reduces SMU uncertainty to a single value while avoiding the pitfalls of estimation. Although there are no issues of histogram smoothing, localization generates a resource model with artifacts poor local precision and unclear conditioning to available drill hole data. Panel edge artifacts are an unattractive side effect of localization. Two techniques for minimizing these artifacts are presented in Chapter 4. A flexible localization methodology has been developed to extend this process and artifact reduction techniques, to each of the four modeling techniques presented.

Despite the improved flexibility and developments towards artifact reduction the issues of local precision and conditioning make localization of questionable value for long term planning.

The final chapter of this thesis includes a practical demonstration of three geostatistical modeling techniques using data collected from a well established porphyry deposit. This section showcases MultiGaussian Kriging, Sequential Guassian Simulation and Uniform Conditioning. This provides summary and overview of implementation details for each method in a realistic setting to quantify the resource. Through modeling and validation processes, the three geostatistical techniques generate consistent results at both the global and local scale. In the final stage of resource classification the benefits of joint uncertainty are made clear. The ability to transfer information through the resource evaluation process for a measure of uncertainty in the final results demonstrates the value of multiple realizations. This information is beneficial in an economic feasibility, mine planning or risk assessment setting supporting simulation as best practice.

Through each of these topics addressed the goals put forth in Chapter 1 have been met. The status quo of a single model has been challenged through careful review of techniques for modeling uncertainty which avoid the need to choose between over smoothing or conditional bias of estimates. Understanding of the local dependence of change of support parameters has been improved for analytical methods as well as for the embedded technique within Uniform Conditioning. In addition, a suggested methodology for approximation of the local correction factor has been put forth. Localization has been addressed through an explanation of the methodology and a synthetic example to highlight the benefits and issues surrounding this new approach to generating a single model. Artifact reduction of localized models is now a possibility through either optimization or random panel definition as outlined in Chapter 4. Lastly, a case study has demonstrated the modeling process for three methods highlighting the benefits of Sequential Gaussian Simulation paired with robust change of support by block averaging. The joint uncertainty portrayed by multiple realizations provides valuable information in resource evaluation

as shown by the summarized P10, P90 and Expected value measures which can lead to better informed decision making than would be possible using a single model.

6.2 Future Work

Through the investigation of available methods for accurately quantifying SMU uncertainty some contributions have been made yet, future work remains. Specifically, topics including the local correction factor, artifact reduction in localization and tools for mine planning over multiple realizations require further attention.

The local variance correction factor, implemented using the Affine Correction, is an improvement upon the assumption of a global stationary parameter in variance reduction. For a single SMU the local correction factor is the ratio of the block variance and the average point variance of that SMU. The large difference between this local correction factor and the global correction factor is not completely understood. The global correction factor is theoretically correct for reducing the global distribution; but its relationship to the local correction necessitates further work.

Artifact reduction in localization remains a topic in need of further development. Improvements have been made in artifact reduction of panel edge effects. Both methods presented effectively reduce artifacts for a visually improved model but each method yields a side effect. The local clustering effect of optimization is unrealistic and the concentration of high and low grades in specific areas caused by random panel localization is undesirable. There may be way to reduce panel artifacts while honoring larger scale trends without disrupting the spatial distribution of estimated values.

A final issue that deserves more attention is the implementation of multiple realizations in mine planning. The use of loss functions for optimization over multiple realizations has been explored by some, including: Schofield and Rolley (1997) but are not widely implemented in practice. The benefits of multiple realizations for quantifying uncertainty are clear but the tools of how

to fully utilize the set of in a mining context need further development in software and practice.

6.3 Recommendations

The goals of presentation, discussion and implementation of methods for quantifying SMU uncertainty have been accomplished here. Additionally, change of support techniques have been analyzed and implemented with respect to local variation and conditioning data. The improved flexibility of localization paired with artifact reduction provides a framework for generating a single model when required. Despite the improvements in local change of support, and flexible localization, the findings of this thesis support conditional simulation, paired with block averaging for change of support, as best practice in resource modeling to quantify uncertainty for improved resource evaluation.

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Chapter 7

Appendix - Software

7.1 Introduction

A number of GSLIB style software programs have been updated and developed to support the research presented in this thesis. The conventions used and software developed built upon the work of others including: Deutsch and Journel (1997), Lyster and Deutsch (2005), Neufeld (2005), and Boisvert and Deutsch (2012). This appendix provides a description of the purpose, methodology and parameters required for each of the programs developed to assist in generating models of SMU uncertainty.

7.2 Updated Postprocessing Programs

To generate results in the common format described in chapter 2, alterations to postprocessing programs for Uniform Conditioning, MultiGaussian Kriging, and Indicator Kriging were necessary. The required postprocessing for each method varies but the goal is to generate results at the SMU scale ordered in the same fashion by grid location. As an example, consider the small grid of 4 SMUs and a distribution discretized by one hundred quantiles for each location. The first quantile is given for each SMU in the grid by standard GSLIB convention, followed by the second quantile value for each grid location etc, until one hundred quantiles have been accounted for. This file is formatted similar to one hundred realizations for the a same grid. An example in this format is given in Figure 7.1 using descriptive labels where quantile values ought to be.

```
1     local distributions
2     1
3     quantile values
4     SMU 1-quantile 1
5     SMU 2-quantile 1
6     SMU 3-quantile 1
7     SMU 4-quantile 1
8     SMU 1-quantile 2
9     SMU 2-quantile 2
10    SMU 3-quantiel 3
11    SMU 4-quantile 4
12    .
13    .
14    .
15    SMU 1-quantile 100
16    SMU 2-quantile 100
17    SMU 3-quantile 100
18    SMU 4-quantile 100
```

Figure 7.1: A schematic example of the common format for a small grid of four SMUs and local distributions discretized by one hundred quantiles.

The above example demonstrates how the results of each modeling technique can be exported through the postprocessing programs created. This format can be used as the input for localization or another appropriate transfer function for a model of local uncertainty.

7.2.1 Uniform Conditioning

Results from Uniform Conditioning are generated at SMU scale volume support over a Panel grid. This requires post processing to reorganize results into the common format. Prior to performing Uniform Conditioning, Kriging at the panel scale is required (Recomended GSLIB software: `kt3dn` (Deutsch and Deutsch, 2012)) followed by calculation of Hermite Polynomial coefficients using `pre_uc` (Neufeld, 2005). The output from each of these is required for Uniform Conditioning. The `UC` program developed by Neufeld (2005) has been updated to include an additional export file that contains a specified number of quantiles per panel distribution. The parameter file for the updated program, called `UC_Loc` is given below.

The SMU scale output file from `UC_Loc` must be re-formatted and ordered properly prior to input for `Localization`. A small post-processing utility program, `UC_reorder`, is provided for this purpose. Input parameters required include the grid definition for both panel and SMU scales, the number of

```
                  Parameters for UC
                  *****************

START OF PARAMETERS:
pre_uc.out                    -file with the anamorphosis coefficients
 2                             -   column with the coefficients
 0.00   4.32                   -z minimum and maximum (from point data)
 0                             -type of estimate:  0 = Z, 1 = Y
kt3d.out                       -file with kriged estimate
 1                             -   column with estimate
 0.0     1.0e21                -   trimming limits
   0.1  1.0  0.1               -minimum cutoff, maximum cutoff and increment
1                              -uc type:  0=hermite polynomials, 1=integral
uc.out                         -file for output
 0                             -output file format:  0 Geo-EAS, 1=CSV
uc_loc.out                     -file for output to localize
200                            -number of quantiles
```

Figure 7.2: Input paramters for `UC_Loc`

.

quantiles used and trimming limits.

```
                   Parameters for reorder
                   **********************

  START OF PARAMETERS:
  uc_loc.out                -file with panel distributions for localizing
  1                         -  column for distribution by quantile
  -1.0      1.0e21          -  trimming limits
  200                       -  number of quantiles per panel
  160   5225.5    50.0      -panel size:  nx,xmn,xsiz
  180   3625.5    50.0      -             ny,ymn,ysiz
    1      0.0    1.0       -             nz,zmn,zsiz
  uc_reorder.out            -file for output
   80   5205.0   10.0       -SMU Output size: nx,xmn,xsiz
   90   3605.0   10.0       -                 ny,ymn,ysiz
    1      0.0    1.0       -                 nz,zmn,zsiz
```

Figure 7.3: Input parameters for `UC_reorder`

The output file, `UC_reorder.out`, contains the results from Uniform conditioning reorganized into the common format for use in localization or further post processing.

7.2.2 Indicator Kriging

Standard Postprocessing of indicator kriging results accomplishes back transformation from to original units while correcting order relations. This process is well established and has been made widely available by Deutsch and Journel (1997). Updates to this program include an additional input file for a local variance correction factor and an additional output option for export in the

common format. No additional reorganization is required.

```
                      Parameters for POSTIK
                       *********************
START OF PARAMETERS:
ik3d.out                          -file with IK3D output (continuous)
postik.out                        -file for output
1   0.25                          -output option, output parameter
3                                 -number of thresholds
0.520 0.2350 1.2290               -the thresholds
2                                 -volume support?(0=none,1=global,2=local)
2                                 -   type? (1 = affine, 2 = IL)
0.65                              -   global "f" for vvc option 1
flocal.out                        -   gridded local "f" for vvc option 2
data.dat                          -file with global distribution
3   0    -1.0   1.0e21            -   ivr,  iwt,  tmin,  tmax
0.0    11.2                       -minimum and maximum Z value
3 0.35                            -lower tail: option, parameter
3 0.35                            -middle    : option, parameter
3 0.35                            -upper tail: option, parameter
200                               -maximum discretization
1                                 -localizing option 1-yes, 0-no
IKloc.out                         - output file, quantiles for localizing

       option 1 = E-type
              2 = probability and mean above threshold(par)
              3 = Z percentile corresponding to (par)
              4 = conditional variance
```

Figure 7.4: Input parameters for `PostIK_Loc`

The updated program called `PostIK_Loc` requires only a few additional input parameters: the number of quantiles per local distribution, the gridded file containing the local correction factor and options for type of variance correction as well as the additional common format output. The additional output file is generated using the common format for localization or further postprocessing.

7.2.3 MutiGaussian Kriging

Kriging at the point scale, in Gaussian units, requires post processing for back transformation and change of support. Developed by Lyster and Deutsch (2005) the `Post_MG` program back transforms the conditional mean and variance to export specified quantiles from each local distribution along with the conditional mean and variance in original units. This program has been updated to include volume variance correction and export an additional common format file.

```
                    Parameters for POSTMG
                    *********************
START OF PARAMETERS:
kt3d_ns.out                     -file with kriged NS mean and variance
1  2                            -columns with NS mean and variance
PostMG.out                      -file for output
nscore.trn                      -file with input transformation table
15                              -probability to be in +/-XX% of mean
5                               -number of quantiles to keep
0.05  0.10  0.50  0.90  0.95   -the probability values
0
locMG.out                       -option for localization output, file name
200                             - number of quantiles to keep for localizing
1   0.65                        - option and global "f" for vvc, see below
flocal.out                      - gridded local "f" for option 2

         Options for Volume Variance Correction
           - 0 = no volume variance correction
           - 1 = global "f" affine correction, "f" value req.
           - 2 = local "f" affine correction, gridded "f" file req.
```

Figure 7.5: Input parameters for `PostMG_Loc`

The input parameter file for `PostMG_Loc` pictured above shows the additional inputs and options required. A binary 0/1 option dictates whether or not an additional output file is created in the common format with the specified number of quantiles. Global and local variance correction options are available using the Affine Correction.

The updated software presented here allows the practitioner to generated results from each of these three geostatistical methods in the common format. This permits results from each of these methods, or simulation to be localized using the flexible `Localization` program.

7.3 Localization

The original methodology for localization presented by Abzalov (2006) has been adapted to a more flexible methodology for models of uncertainty in the common format described. The software presented here permits localization of results from MultiGaussian Kriging, Sequential Gaussian Simulation, Indicator Kriging and Uniform Conditioning. Parameters for localization are given below.

```
                    Parameters for Panel Definition
                    ******************************

START OF PARAMETERS:
60   2.5   5.0             -SMU grid definition: nx,xmn,xsiz
40   2.5   5.0             - ny,ymn,ysiz
 1   0.5   1.0             - nz,zmn,zsiz
0    IKmodel.out           -1=indicator for clipping model, 0=no IKmodel
1                          -1=regular panel grid, 2=semi-regular, 3=random
12   12.5    25.0          - nx,xmn,xsiz Panel grid for options 1 & 2
8    12.5    25.0          - ny,ymn,ysiz
1     0.5    1.0           - nz,zmn,zsiz
20                         - target number of SMUs per panel
paneldef.out               -file for Panel Definition output

                    Parameters for localizing
                    *************************

kt3drank.out               -file with localizing variable at SMU scale
1                          - column for localizing variable
-1.0      1.0e21           -trimming limits
IKloc.out                  -Data to be localized - See note
200                        -# of realizations or Quantiles
1                          -1 = regular sampling, 2 = LHS
6921                       -random # seed
LocalizationIK.out         -file for output
```

Figure 7.6: Input parameters for `Localization`

Input parameters for `Localization` are separated into two sections. The first section determines the type of panel definition. The SMU grid definition is required regardless of panel definition style while panel grid definition is only required for regular panels. A target number of SMUs per panel is required for either of the more flexible panel definition options. Additionally, an option for clipping by a binary indicator grid is offered. The input gridded file for the binary clipping option assumes a that locations given a 0 are within the area of interest and locations with a zero are given a null value.

The second section of the input parameters defines the localizing process. Inputs are simple, requiring only the input file, trimming limits, the number of realizations (or quantiles), a binary option for the type of grade class definition and a random number seed for random sampling of grade classes.

This program operates by first sorting values by panel number. Once SMU values have been grouped by panels the local distribution for each panel can be constructed. This CDF is discretized into a number of grade classes, equal to the number of SMUs. At this point either the mean of each grade class or a random value from the grade class is drawn, depending on the configuration

of input parameters. Localized values are assigned to the appropriate SMU within the panel based on kriged grade from the secondary input file. Results are exported in a standard GSLIB grid format (Deutsch and Journel, 1997). The issues surrounding localization, discussed in Chapter 4, should be carefully considered before implementing this process for resource evaluation.